Wedge-local fields in integrable models with bound states (partly with D. Cadamuro, arXiv:1502.01313)

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March 18th 2015, Berlin

Towards more 2d QFTs

Goal

Construct Haag-Kastler nets (local observables) for integrable models with bound states (factorizing S-matrices with **poles**).
Non-perturbative, non-trivial quantum field theories in $d = 2$.

- Sine-Gordon, Bullough-Dodd, $Z(N)$-Ising...

Methods and partial results

Conjecture the S-matrix with **poles**, construct first **observables localized in wedges**, then prove the existence of local observables indirectly.

- **Weakly commuting** fields: $\widetilde{\phi}(f) = z^\dagger(f^+) + \chi(f) + z(J_1 f^-), \widetilde{\phi}'(g)$ (c.f. Lechner '08, $\phi(f) = z^\dagger(f) + z(J_1 f^-)$ for S-matrix without poles).
- Wedge-algebras: $\mathcal{A}(W_\mathrm{L}) = \{e^{i\widetilde{\phi}(f)} : \mathrm{supp}\, f \subset W_\mathrm{L}\}$, $\mathcal{A}(W_\mathrm{R}) = \{e^{i\widetilde{\phi}'(g)} : \mathrm{supp}\, g \subset W_\mathrm{R}\}$. $\widetilde{\phi}(f)$ and $\widetilde{\phi}'(g)$ strongly commute? Arguments for **modular nuclearity**.

Overview of the strategy

- Haag-Kastler net $(\{\mathcal{A}(O)\}, U, \Omega)$: **local observables** $\mathcal{A}(O)$, spacetime symmetry U and the vacuum Ω.
- **Wedge-algebras first**: construct $\mathcal{A}(W_\mathrm{R}), U, \Omega$, then take the intersection

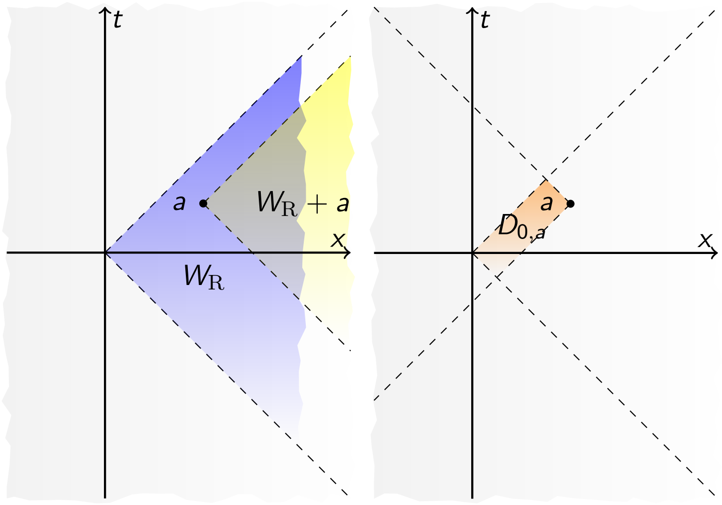
$$\mathcal{A}(D_{a,b}) = U(a)\mathcal{A}(W_\mathrm{R})U(a)^* \cap U(b)\mathcal{A}(W_\mathrm{R})'U(b)^*$$

  The intersection is large enough if modular nuclearity or wedge-splitting holds.
- **Wedge-local fields**: a pair of operator-valued distributions ϕ, ϕ' such that $[e^{i\phi(f)}, e^{i\phi'(g)}] = 0$ if $\mathrm{supp}\, f \subset W_\mathrm{L}, \mathrm{supp}\, g \subset W_\mathrm{R}$.

Examples: scalar analytic factorizing S-matrix (Lechner '08), twisting by inner symmetry (T., '14), diagonal S-matrix (Alazzawi-Lechner '15)...

More example? **S-matrices with poles**.

Standard wedge and double cone



Factorizing S-matrix models (Lechner, Schroer)

- **Input**: meromorphic function $S : \mathbb{R} + i(0, \pi) \to \mathbb{C}$,

$$\overline{S(\theta)} = S(\theta)^{-1} = S(-\theta) = S(\theta + \pi i), \quad \theta \in \mathbb{R}.$$

- S-symmetric Fock space: $\mathcal{H}_1 = L^2(\mathbb{R}, d\theta)$, $\mathcal{H}_n = P_n \mathcal{H}_1^{\otimes n}$, where P_n is the projection onto S-**symmetric** functions:

$$\Psi_n(\theta_1, \cdots, \theta_n) = S(\theta_{k+1} - \theta_k)\Psi_n(\theta_1, \cdots, \theta_{k+1}, \theta_k, \cdots, \theta_n).$$

- Zamolodchikov-Faddeev algebra: S-symmetrized **creation and annihilation operators**

$$z^\dagger(\xi) = Pa^\dagger(\xi)P, \quad z(\xi) = Pa(\xi)P, \quad P = \bigoplus_n P_n,$$

$$(a^\dagger(\xi)\Psi_n)(\theta_1, \cdots, \theta_{n+1}) = \xi(\theta_1)\Psi_n(\theta_2, \cdots, \theta_{n+1}).$$

If S has **no pole**, then $\phi(f) = z^\dagger(f^+) + z(J_1 f^-)$ is a wedge-local field (Lechner '04).

The bound state operator

S: two-particle S-matrix, **simple poles** at $\frac{\pi i}{3}, \frac{2\pi i}{3}$, bootstrap equation

$$S(\theta) = S\left(\theta + \frac{\pi i}{3}\right) S\left(\theta - \frac{\pi i}{3}\right).$$

P_n: S-symmetrization, $\mathcal{H} = \bigoplus P_n \mathcal{H}_1^{\otimes n}$, $\mathcal{H}_1 = L^2(\mathbb{R})$,

$$\mathrm{Dom}(\chi_1(f)) := H^2\left(-\tfrac{\pi}{3}, 0\right)$$
$$(\chi_1(f))\xi(\theta) := \sqrt{2\pi|R|} f^+\left(\theta + \tfrac{\pi i}{3}\right)\xi\left(\theta - \tfrac{\pi i}{3}\right),$$
$$R = \operatorname*{Res}_{\zeta = \frac{2\pi i}{3}} S(\zeta),$$

where $H^2(\alpha, \beta)$ is the space of analytic functions in $\mathbb{R} + i(\alpha, \beta)$ such that $\xi(\cdot - \gamma i)$ is uniformly bounded in L^2-norm, $\gamma \in (\alpha, \beta)$, and f^+ is analytic.

$$\chi_n(f) = nP_n\left(\chi_1(f) \otimes I \otimes \cdots \otimes I\right) P_n,$$
$$\chi(f) := \bigoplus \chi_n(f).$$

Wedge-local fields and weak commutativity

The CPT operator: $(J_n\Psi_n)(\theta_1,\cdots,\theta_n) = \overline{\Psi_n(\theta_n,\cdots,\theta_1)}$.
New field:

$$\widetilde{\phi}(f) := \phi(f) + \chi(f) = z^\dagger(f^+) + \chi(f) + z(J_1 f^-),$$

and the **reflected field**: $\widetilde{\phi}'(g) := J\widetilde{\phi}(g^j)J$, $g^j(x) = \overline{g(-x)}$.

Theorem (Cadamuro-T. arXiv:1502.01313)

For real $f, g, \mathrm{supp}\, f \subset W_{\mathrm{L}}, \mathrm{supp}\, g \subset W_{\mathrm{R}}$, *then*

$$\langle\widetilde{\phi}(f)\Phi, \widetilde{\phi}'(g)\Psi\rangle = \langle\widetilde{\phi}'(g)\Phi, \widetilde{\phi}(f)\Psi\rangle, \quad \Phi,\Psi \in \mathrm{Dom}(\widetilde{\phi}(f)) \cap \mathrm{Dom}(\widetilde{\phi}'(g)).$$

Strong commutativity? Question of domains.

Summary

- input: two-particle factorizing S-matrix with **poles**
- **new field** $\widetilde{\phi}(f) = \phi(f) + \chi(f)$
- weak commutativity
- modular nuclearity (by assuming strong commutation)
- features of $\widetilde{\phi}(f)$: no polynomial Reeh-Schlieder property, no energy bound, non-temperateness

Open problems

- **strong commutativity**
- non-scalar models (Sine-Gordon, Z(N)-Ising...): weakly commuting fields (with D. Cadamuro), strong commutativity and modular nuclearity more difficult