INdAM Workshop: Geometric Challenges in Isogeometric Analysis

Istituto Nazionale di Alta Matematica "Francesco Severi"
Piazzale Aldo Moro 5, 00185 Rome, Italy
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Program:

	MONDAY January 27	TUESDAY January 28	WEDNESDAY January 29	THURSDAY January 30	FRIDAY January 31
9.30 – 10.00		**Reif**	**Peters**	**Qian**	**Sangalli**
10.00 – 10.30					
10.30 – 11.00		*Break*	*Break*	*Break*	*Break*
11.00 – 11.30		Marussig	Mourrain	Kosinka	Möller
11.30 – 12.00		Hiemstra	Kapl	Grošelj	Ratnani
12.00 – 12.30		Wei	Takacs	Pelosi	**Zhang**
12.30 – 13.00		Sabin	Prautzsch	Lyche	
13.00 – 14.30	*Lunch*	*Lunch*	*Lunch*	*Lunch*	*Lunch*
14.30 – 15.00	**Sorokina**	**Jüttler**		**Dokken**	
15.00 – 15.30					
15.30 – 16.00	*Break*	*Break*		*Break*	
16.00 – 16.30	Toshniwal	Giannelli		Patrizi	
16.30 – 17.00	Villamizar	Mitter		Sampoli	
17.00 – 17.30	Sande	Mantzaflaris		Sestini	

Keynote Presentations

Differences and similarities of different approaches to locally refined splines over hierarchical meshes

Tor Dokken (SINTEF, Norway)

Hierarchical meshes are attractive in Isogeometric Analysis as additional degrees of freedom can be inserted locally in a uniform way in regions where needed. Three are three major approaches to locally refined splines over hierarchical meshes: Truncated Hierarchical B-splines (THB), T-spline (TS) and Locally Refined B-splines (LRB). For all three approaches the structures of polynomial elements produced are similar. In most cases the spline space of THB or TS can be exactly reproduced by LRB, in some case the spline spaces produced LRB over a given THB or TS element mesh are slightly bigger than those of THB and TS, In this presentation we address the differences and similarities of the three different approaches, as well as look at how the extra flexibility offered by LRB can be used for reducing the number of B-splines covering and element. We will also show the positive effect this has on condition numbers of mass and stiffness matrices.

Joint work with Ivar Stangeby.

DPB-splines: the decoupled basis of patchwork splines

Bert Jüttler (Johannes Kepler University Linz, Austria)

Patchwork spline spaces (PSS), which were introduced as a generalization of hierarchical spline spaces, make it possible to employ more flexible refinement strategies. This provides advantages for geometric modeling and numerical simulation via isogeometric analysis. In this talk, we focus on particular PSS defined via partitions of the d-dimensional unit cube into patches and associated tensor-product spline space for each of them. The spline spaces possess uniform degree p and maximum smoothness, but potentially different knots. Under certain assumptions on this hierarchy, we show how to construct Decoupled Patchwork B-splines (DPB-splines) that span the corresponding PSS. More precisely, we generate a basis for the space formed by all C^{p-1} smooth functions that admit patch-wise representations in the associated spline spaces. Based on decoupled tensor-product B-splines, we obtain a basis that is algebraically complete, forms a convex partition of unity and preserves the coefficients of the local B-spline representations. Furthermore, we present an adaptive refinement algorithm for surface approximation that satisfies the required assumptions and hence generates patchwork spline spaces possessing a DPB-spline basis.

Joint work with Nora Engleitner.

Smooth polynomial spaces with semi-regular layout for design and analysis

Jörg Peters (University of Florida, USA)

To be directly useful for both shape design and thin shell analysis, a surface representation has to satisfy three properties: be representable in the CAD surface standard, be generically yield a good highlight line distribution, and offer a refinable space of functions on the surface.

This talk discusses recent constructions that aim at satisfying all three criteria. The focus is on constructions that convert quad-dominant meshes with irregularities (semi-regular meshes) to a smooth polynomial surfaces; and in particular on an approach that yields a parameterically smooth main body, completed by a suitably small geometrically smooth cap.

Isogeometric analysis of fourth-order PDEs on triangulations

Xiaoping Qian (University of Wisconsin-Madison, USA)

This talk presents our recent work on isogeometric analysis of fourth-order PDEs through triangular Bézier splines.

We show C^1 triangulation Bézier splines can lead to optimal convergence in isogeometric analysis of von Karman equations. We also show numerical results on triangulation Bézier spline based analysis of biharmonic equations. Such C^1 triangular Bézier elements are especially advantageous for problem domains of concave shapes where mixed form finite element formulation can lead to erroneous results.

We then present our ongoing geometric challenges in apply C^1 triangular Bézier elements in shape optimization: how to ensure mesh validity when boundary shape varies during optimization.

Spline methods in simulation

Ulrich Reif (Darmstadt University of Technology, Germany)

The emergence and growth of Isogeometric Analysis (IGA) has led to a new interest in spline methods for simulation in science and engineering. While more and more fields of application are conquered and ever new variants of the method are developed, also some fundamental problems become visible: In particular, just as for traditional Finite Element methods, the meshing of the domain of simulation can be a most complicated task. As matters stand, there is no easy way how to convert a boundary representation, as offered by standard CAD processes, to a volumetric representation, as requested by IGA. Thus, the central goal “to bridge the gap between CAD and simulation” is not yet achieved.

In the first part of the talk, we discuss least squares methods for elliptic PDEs as an alternative spline-based method. This approach does without meshing, maintains the high approximation power of spline spaces, and admits rigorous error estimates for the computed solutions with respect to the max-norm. In the second part, we present a new method to convert a given boundary representation, as standard in CAD, to an IGA-compatible one-patch parametrization of the domain.

Efficient solvers for high-degree IGA

Giancarlo Sangalli (University of Pavia, Italy)

The concept of k-refinement was proposed as one of the key features of isogeometric analysis, “a new, more efficient, higher-order concept”, in the seminal work [1]. The idea of using high-degree and continuity splines/NURBS as a basis for a new high-order method appeared very promising from the beginning, and received confirmations from the next developments. The k-refinement leads to several advantages: higher accuracy per degree-of-freedom, improved spectral accuracy, the possibility of structure-preserving smooth discretizations are the most interesting features that have been studied actively in the community. At the same time, the k-refinement brings significant challenges at the computational level: using standard finite element routines, its computational cost grows with respect to the degree, making degree raising computationally expensive. This presentation gives an overview of some recent results that extend what we did in [2].

Joint work with Monica Montardini, Mattia Tani, and other collaborators.

[1] T.J.R. Hughes, J.A. Cottrell, and Y. Bazilevs. Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. *Comput. Methods Appl. Mech. Engrg.*, 194:4135–4195, 2005.

[2] G. Sangalli and M. Tani. Matrix-free isogeometric analysis: The computationally efficient k-method. *Comput. Methods Appl. Mech. Engrg.*, 338:117–133, 2018.

Bernstein-Bézier techniques for linear differential operators on splines

Tatyana Sorokina (Towson University, USA)

Bernstein-Bézier techniques for analyzing continuous harmonic splines in n variables are developed. Dimension and a minimal determining set for special splits are obtained using the new techniques. We show that both dimension and bases strongly depend on the geometry of the underlying partition. In particular, the angles in the triangulation play an important role. We construct quadratic harmonic conforming FEMs on Clough-Tocher refinements and other special partitions.

Material transport simulation in complex neurite networks using isogeometric analysis and machine learning techniques

Jessica Zhang (Carnegie Mellon University, USA)

Neurons exhibit remarkably complex geometry in their neurite networks. So far, how materials are transported in the complex geometry for survival and function of neurons remains an unanswered question. Answering this question is fundamental to understanding the physiology and disease of neurons. In this talk, we present an isogeometric analysis (IGA) based platform for material transport simulation in neurite networks. We model the transport process by reaction-diffusion-transport equations and represent geometry of the networks using truncated hierarchical tricubic B-splines. We solve the Navier-Stokes equations to obtain the velocity field of material transport in the networks. We then solve the transport equations using the streamline upwind/Petrov-Galerkin (SU/PG) method. Using our IGA solver, we simulate material transport in three representative and complex neurite networks. The simulation has been further speeded up significantly using machine learning techniques. Together, our simulation provides key insights into how material transport in neurite networks is mediated by their complex geometry.

[1] A. Li, X. Chai, G. Yang, and Y.J. Zhang. An isogeometric analysis computational platform for material transport simulations in complex neurite networks. *Mol. Cell. Biomech.*, 16(2):123–140, 2019.

[2] A. Li, R. Chen, A.B. Farimani, and Y.J. Zhang. Reaction diffusion system prediction based on convolutional neural network. Scientific Report, under review, 2019.

Invited Presentations

Multilevel preconditioners for isogeometric analysis with (T)HB-splines

Carlotta Giannelli (University of Florence, Italy)

The talk will present the construction of additive multilevel preconditioners, also known as BPX preconditioners, for the solution of the linear system arising in isogeometric adaptive schemes with (truncated) hierarchical B-splines. We show that the locality of hierarchical spline functions, naturally defined on a multilevel structure, can be suitably exploited to design and analyze efficient multilevel decompositions. By obtaining smaller subspaces with respect to standard tensor-product B-splines, the computational effort on each level is reduced. We prove that, for suitably graded hierarchical meshes, the condition number of the preconditioned system is bounded independently of the number of levels. A selection of numerical examples to validate the theoretical results and the performance of the preconditioner will be also shown.

Joint work with Cesare Bracco, Durkbin Cho, and Rafael Vázquez.

B-spline representations of super-smooth cubic Powell-Sabin splines

Jan Grošelj (University of Ljubljana, Slovenia)

In this talk, we investigate subspaces of the full C^1 cubic Powell-Sabin spline space that are obtained by imposing additional smoothness conditions. The full space is well-defined on general triangulations and consists of splines that admit a B-spline-like representation constructed in a geometrically intuitive way. Such splines have a large number of degrees of freedom and we present some approaches for their reduction by prescribing C^2 smoothness conditions at particular vertices or across particular edges of the Powell-Sabin refinement. Furthermore, we consider how to recombine the basis functions of the original B-spline representation in order to adjust them to the reduced subspaces. Finally, with the aim to narrow the gap between globally C^1 and C^2 splines, we restrict ourselves to the structured domain partition with three-directional triangulations and show how additional geometric symmetries help in further reduction of degrees of freedom.

Joint work with Hendrik Speleers.

Untrimmed splines: towards analysis suitable CAD

René R. Hiemstra (Leibniz University Hannover, Germany)

Current CAD technologies describe geometry by means of the boundary representation or simply B-rep. Boolean operations, ubiquitous in computer aided design, use a process called trimming that leads to a non-conforming description of geometry that is un-editable and incompatible with all downstream applications, thereby inhibiting true interoperability across the design-through-analysis process.

In this talk I discuss techniques that address this problem by providing interactive control over the boundary surface parameterization, leading to watertight, editable and conforming descriptions of geometry, alleviating the need for trim. The methodology is based on recent advances in topological vector field design and processing. First, a smooth frame field [1] is computed, by minimizing an appropriate energy functional on a background mesh of the initial B-rep. Frame-field singularities, which together satisfy the topological invariant known as the Poincaré-Hopf theorem, are automatically placed and can be modified by the user. The frame field is used as a guide for the re-parameterization [2] of the initial B-rep into a conforming watertight and editable spline description that is suitable for design as well as analysis.

[1] D. Panozzo, E. Puppo, M. Tarini, and O. Sorkine-Hornung. Frame fields: Anisotropic and non-orthogonal cross fields. *ACM Trans. Graph.*, 33(4), 2014.

[2] F. Kälberer, M. Nieser, and K. Polthier. QuadCover. Surface parameterization using branched coverings. *Comput. Graph. Forum*, 26(3):375–384, 2007.

C^1-smooth isogeometric spline spaces for trilinearly parameterized multi-patch volumes

Mario Kapl (RICAM, Austria)

On the one hand, multi-patch volumes are required to describe complex physical domains, which in general cannot be represented just by one single patch. On the other hand, globally C^1-smooth functions are needed to solve fourth order PDEs, such as the biharmonic equation, the Cahn-Hilliard equation or problems of strain gradient elasticity analysis, via their weak form and a standard Galerkin discretization as mostly applied in isogeometric analysis. In this talk, we aim at combining both needs by presenting a framework for the construction of a C^1-smooth isogeometric spline space over trilinearly parameterized multi-patch volumes. The C^1-smooth space is generated as the direct sum of simpler subspaces corresponding to the single patches, faces, edges and vertices of the volumetric multi-patch domain. The constructed basis functions are either given by a closed form representation or can be obtained by solving a small linear system.

Joint work with Katharina Birner and Vito Vitrih.

Converting CAD models to triangular spline surfaces

Jiří Kosinka (University of Groningen, the Netherlands)

The standard representation of CAD (computer aided design) models is based on the boundary representation (B-reps) with trimmed and (topologically) stitched tensor-product NURBS patches. Due to trimming, this leads to gaps and overlaps in the models. While these can be made arbitrarily small for visualisation and manufacturing purposes, they still pose problems in downstream applications such as analysis and 3D printing.

It is therefore worthwhile to investigate conversion methods which (necessarily approximately) convert these models into water-tight or even smooth representations. After briefly surveying existing conversion methods, we will focus on techniques that convert CAD models into triangular spline surfaces of various levels of continuity.

Multivariate simplex splines: an example

Tom Lyche (University of Oslo, Norway)

We briefly review multivariate simplex splines in $\mathbb{R}^n$, and use them for any n to construct a basis for the Alfeld split of smoothness C^1.

Fast matrix formation for isogeometric analysis based on hierarchical splines

Angelos Mantzaflaris (INRIA Sophia Antipolis Méditerranée, France)

In this talk we present methods to improve the efficiency of computations with hierarchical splines that are constructed over tensor-product spline spaces. We focus on the problem of computing the Gram matrix of the basis. Typically, computations involve numerical integration using tensor-product Gauss quadrature. However, it is known that an element-wise assembly of the Gramian of tensor product B-splines is sub-optimal in dimension bigger than one.

We present efficient algorithms for this computation using the background tensor structure. In particular, we extend the Kronecker formula that is known for the Gramian of tensor-product spaces to a Hadamard formula for the Gramian of hierarchical bases. This implies an efficient algorithm for the computation of the matrix, that does not involve a multivariate quadrature over the elements.

The challenges of deriving analysis-suitable geometries from trimmed CAD models

Benjamin Marussig (Graz University of Technology, Austria)

Trimmed CAD models arise from so-called Boolean operations, which are a fundamental technology of geometric modeling. A central component of these operations is the computation of surface-to-surface intersections (SSI) of tensor product patches. In general, SSI leads to gaps and overlaps between surfaces due to the high complexity of the exact intersection. In other words, a mathematically sound connection along an intersection is missing.

In CAD, tailored visualization techniques mitigate this issue. For analysis purposes, however, a link between the individual surfaces of a trimmed object has to be established. This link may be achieved by conventional meshing processes or in an isogeometric fashion.

This talk outlines the isogeometric concepts for deriving analysis-suitable CAD models. They either (i) aim to resolve the problems of trimmed geometries in a pre-processing step before the analysis, or (ii) intend to enhance the simulation process so that it can cope with the flaws of these models. The different advantages and challenges of both strategies are discussed.

Local multigrid solvers for adaptive isogeometric analysis in hierarchical spline spaces

Ludwig Mitter (Johannes Kepler University, Austria)

We propose local multigrid solvers for adaptively refined isogeometric discretizations using (truncated) hierarchical B-splines. Smoothing is only performed in or near the refinement areas on each level, leading to a computationally efficient solving strategy. The proposed solvers have provably robust convergence with respect to the number of levels and the mesh sizes of the hierarchical discretization space. We also give some numerical experiments confirming the theoretical findings.

Joint work with Clemens Hofreither and Hendrik Speleers.

High-order isogeometric methods: curse or blessing?

Matthias Möller (Delft University of Technology, the Netherlands)

In this talk we will address certain practical aspects that come with the use of high-order isogeometric methods.

These include the interplay of truncation and round-off errors. High-order methods enable unparalleled accuracy compared to low-order approximations but require a much more careful tuning of the optimal number of degrees of freedom since excessive refinement will lead to sub-optimal solutions with largely dominating round-off errors. We also address the use of non-standard number systems, namely, Posits that are particularly designed to provide better accuracy and are less prone to round-off errors than IEEE-754 floating-point numbers.

We will furthermore shed some light on efficient solvers and, in particular, discuss a p-multigrid method that is robust with respect to the approximation order and the grid size. Numerical examples for several non-trivial two-dimensional single and multi-patch domains with tensor-product B-Splines and THB Splines will be presented.

Geometrically continuous splines for simulations and shape reconstruction

Bernard Mourrain (INRIA Sophia Antipolis Méditerranée, France)

We study the space of geometrically smooth spline functions that satisfy differentiability properties across shared edges of a mesh. We present new and efficient constructions of basis functions of the space of G^1-spline functions on quadrangular meshes, which are tensor product B-spline functions on each quadrangle and with B-spline transition maps across the shared edges. The basis function construction relies on a local analysis of the edge functions, does not depend on the global topology of the mesh and produces functions, which are attached to the vertices, edges and faces. We illustrate it by the fitting of point clouds by G^1-splines on quadrangular meshes and in IsoGeometric Analysis for the solution of diffusion equations.

Joint work with Ahmed Blidia, Gang Xu, and Angelos Mantzaflaris.

Adaptive refinement with locally linearly independent LR B-splines

Francesco Patrizi (SINTEF, Norway)

Adaptive refinements are frequently used for balancing accuracy and computational costs. B-spline spaces are formulated as tensor products of univariate B-spline spaces and therefore cannot address local refinements. In order to break the tensor structure of the underlying mesh and achieve local refinements, new formulations of multivariate B-splines have been introduced during the last decades. One of these is the Locally Refined B-splines, or LR B-splines. The definition of LR B-splines is broadly similar to the standard B-splines even though they address local refinements. Furthermore, LR B-splines satisfy the same properties of classical B-splines, such as positivity, local support, piecewise polynomials and partition of unity (when using positive scaling weights). However, a particular structure of the mesh is required to guarantee local linear independence. Even though such a characterization in terms of meshing constraints is provided in the literature, no truly adaptive refinement strategy was available to produce LR-meshes with such a structure. In this talk we describe the first refinement of this kind. Then we exploit this strong feature in quasi-interpolation and isogeometric analysis.

Joint work with Carla Manni, Francesca Pelosi, and Hendrik Speleers.

An immersed-isogeometric model based on THBox-splines

Francesca Pelosi (University of Rome Tor Vergata, Italy)

In the design of isogeometric methods and in numerical simulation, an accurate representation of computational domains is one of the key features. The use of an immersed boundary approach in isogeometric analysis allows us to construct complex single patch domains by suitably combining trimming with geometry mappings.

In this talk we present an isogeometric immersed model to solve partial differential problems. The model does not need additional degrees of freedom in the final system of equations. The method is free of user defined penalties and stabilization parameters. Flexible and adaptive geometry representations are achieved by employing hierarchically nested splines spaces. In particular, we focus on truncated hierarchical box splines (THBox-splines) defined over regular triangulations. Several numerical examples demonstrate the optimal convergence of the adaptive scheme for the numerical solution of PDEs.

Joint work with Carlotta Giannelli, Tadej Kanduč, and Hendrik Speleers.

Rational quadrilateral spline orbifolds

Hartmut Prautzsch (Karlsruhe Institute of Technology, Germany)

Any closed surface of genus g can be cut open into, e.g., a $4g$-gon and the hyperbolic plane can be tessellated by hyperbolic shifts of $4g$-gons. This implies that G^k smooth spline surfaces of genus g can be parametrized over some hyperbolic $4g$-gon, where the hyperbolic shifts correspond to the rational linear transition maps defining its G^k joints. Such splines with a projective structure, i.e. with only rational linear transition maps have been called orbifold splines by Wallner and Pottmann [5].

Orbifold splines – albeit under different names – have also been studied by others [1, 2, 3, 4] and except for Peters' splines [4] they are all built from rational triangular patches. Contrary to this, Peters' splines are integral quadrilateral orbifold splines, i.e. orbifold splines that consist of integral quadrilateral patches. However their patch layout cannot be arbitrary if they should be differentiable.

In this talk, I will show how one can built rational quadrilateral orbifold splines with arbitrary patch layout and projective structures for them. Further, I discuss how to deal with intrinsic difficulties in constructing such splines. In particular, the quadrilateral splines presented here are G^k smooth although their parametrizations are missing C^0 continuous transitions which is possible only with rational patches but not with integral ones unless we use degree raising reparametrizations.

[1] C.V. Beccari and M. Neamtu. On constructing RAGS via homogeneous splines. *Comput. Aided Geom. Design*, 43:109–122, 2016.

[2] H. Ferguson and A. Rockwood. Multiperiodic functions for surface design. *Comput. Aided Geom. Design*, 10:315–328, 1993.

[3] X. Gu, Y. He, M. Jin, F. Luo, H. Qin, and S.-T. Yau. Manifold splines with single extraordinary point. In: *Proceedings of the 2007 ACM symposium on Solid and physical modeling (SPM '07)*. ACM, New York, pp. 61–72, 2007.

[4] M. Sarov and J. Peters. Refinable polycube G-splines. *Comput. Graph.*, 58:92–101, 2016.

[5] J. Wallner and H. Pottmann. Spline orbifolds. In: A. Le Méhauté, C. Rabut, and L.L. Schumaker (eds.) *Curves and Surfaces with Applications in CAGD*, pp. 445–464, 1997.

Towards an automated mesh generation framework for tokamaks

Ahmed Ratnani (Mohammed VI Polytechnic University, Morocco)

Due to the very large anisotropic character of strongly magnetized plasma, the use of flux aligned grid is generally believed to be highly useful (or even mandatory) to obtain accurate and reliable simulations for fusion applications. For real geometries, the magnetic topology can only be computed by the use of specialized equilibrium solvers solving the non-linear Grad-Shafranov equation. The output of these solvers then have to be used as input to construct flux aligned meshes that respect the magnetic topology. This process usually requires some manual input and expertise from the final users to identify the relevant features of the magnetic topology (X points, magnetic axis).

In the first part of this talk, we will describe an original method for the automated construction of flux aligned grids. This method assumes that the magnetic flux is a Morse function and consequently that the results of Morse theory can be applied: the topological set of the iso-contours of the flux function consists of finite connected components that are either (a) circle cells which are homeomorphic to open disks, (b) circle bands which are homeomorphic to open annulus, or (c) saddle connections.

The construction of flux aligned grid relies then on the analysis of the singularities of the magnetic flux function and the construction of a graph known as the Reeb graph that encodes the segmentation of the physical domain into sub-domains that can be mapped to a reference square domain. We will present several examples taken from existing tokamaks to illustrate this grid generation process.

Then we will describe a new method for constructing adaptive and anisotropic mappings by solving an optimal transport problem. This method leads to equidistributed meshes and ensures the one-to-one constraint.

Joint work with Yaman Güçlü, Hervé Guillard, Jalal Lakhlili, Adrien Loseille, Alexis Loyer, Eric Sonnendrücker, and Hendrik Speleers.

Meshing

Malcolm A. Sabin (Numerical Geometry Ltd, UK)

Meshing for finite element analysis was historically thought of as partitioning the domain (the shape of the object to be analysed) into simple pieces. It is much more useful to think of it as choosing a set of basis functions, with the partitioning merely emerging as a side-effect. A good choice of basis (typically but not necessarily tensor product B-splines) can give good accuracy with low computing cost and turnaround times compatible with putting the analysis inside an optimisation loop. The ideas are relevant to all linear elliptic PDEs (i.e. elastostatics, electrostatics, thermal, and subsonic aerodynamics as well as to approximation), and to both finite elements and cartesian grids – maybe even to boundary elements.

Quadrature rules for singular integrals arising in IgA-BEM

M. Lucia Sampoli (University of Siena, Italy)

We propose a new class of quadrature rules based on spline quasi-interpolation for the approximation of singular integrals (weakly, strongly and hypersingular) occurring as entries of the stiffness matrix associated with Isogeometric Boundary Element Methods (IgA-BEMs). These formulas are efficient, since they combine the locality of any spline quasi-interpolation scheme with the capability to compute the modified moments for B-splines. The peculiarity of the considered singular integrals is given by the fact that the regular part of the integrand is defined as the product of a B-spline times a general function. Then exploiting a recurrence relation for B-spline products an efficient formulation can be obtained. Convergence results of the proposed quadrature rules are given, with respect to both smooth and non smooth integrands.

Joint work with Alessandra Aimi, Francesco Calabrò, Antonella Falini, and Alessandra Sestini.

The role of smoothness in spline approximation

Espen Sande (University of Rome Tor Vergata, Italy)

Classical error estimates for spline approximation are expressed in terms of (a) a certain power of the maximal grid spacing, (b) an appropriate derivative of the function to be approximated, and (c) a "constant" which is independent of the previous quantities but usually depends on the degree and smoothness of the spline.

An explicit expression of the constant in (c) is not always available in the literature, because it is a minor issue in the most standard approximation analysis. They are mainly interested in the approximation power of spline spaces of a fixed degree. However, one of the most interesting features in Isogeometric Analysis is k-refinement, which denotes degree elevation with increasing interelement smoothness. The above mentioned error estimates are not sufficient to explain the benefits of approximation under k-refinement as long as it is not well understood how the constant in (c) behaves.

In this talk we provide a priori error estimates for spline approximation with explicit upper bounds on the constant in (c). The presented error estimates indicate that the approximation properties of spline spaces improve per degree of freedom as their smoothness increases. Moreover, by comparing the constant for spline approximation of maximal smoothness with a lower bound on the constant for continuous and discontinuous spline approximation, we prove that approximation with maximally smooth splines provide better approximation per degree of freedom in almost all cases of practical interest when compared to (dis-)continuous splines.

Joint work with Andrea Bressan, Carla Manni, and Hendrik Speleers.

Cubature rules based on quasi-interpolation for B-spline weighted weakly singular integrals

Alessandra Sestini (University of Florence, Italy)

B-spline weighted singular integrals appear in the context of IgA-BEM. Here we focus on the weakly singular ones derived when the 3D Laplace problem on bounded or unbounded domains is considered. For their numerical approximation we propose a new class of cubature rules based on spline quasi-interpolation, since this allows us a specific treatment of the tensor product B-spline weight and integration on its whole support. Our method has two key points. First, a suited quasi-interpolant is used to approximate the non singular part of the integrand excluding the B-spline factor. Then an algorithm performing the spline product is used. The integral of the so approximated integrand can be exactly computed, since the so called *double modified moments* (double integrals of products between the singular kernel and B-spline functions) can be computed by using the tensor product generalization of the well known B-spline recursion. Examples will be discussed.

Joint work with Antonella Falini, Tadej Kanduč, and Maria Lucia Sampoli.

Approximate C^1 bases on planar domains and their application to IGA

Thomas Takacs (Johannes Kepler University, Austria)

A key element of isogeometric analysis is that it allows high order smoothness within one patch. However, to represent complex geometries, a discretization comprised of multiple patches is needed. In this case, global continuity of order C^0 is feasible, whereas higher order continuity is hard to achieve. For C^1 isogeometric functions, a special construction for the basis is needed. Such spaces are of interest when solving numerically fourth-order PDE problems, such as the biharmonic equation or Kirchhoff-Love plate/shell formulations, using an isogeometric Galerkin method.

With the construction of so-called analysis-suitable G^1 (in short, AS-G^1) parametrizations it is possible to have C^1 isogeometric spaces with optimal approximation properties. These geometries satisfy certain constraints along the interfaces. A drawback of the construction is that most complex geometries are not AS-G^1 geometries. Therefore we define basis functions for isogeometric spaces by enforcing approximate C^1 conditions. For this reason the defined function spaces are not exactly C^1 but only approximately. We study the convergence behavior on domains with non-trivial interfaces and define function spaces that behave optimally under h-refinement, by locally introducing functions of higher polynomial degree and/or lower regularity.

Smooth splines on planar meshes of arbitrary topologies

Deepesh Toshniwal (Delft University of Technology, the Netherlands)

Modelling and numerically solving PDEs on complex geometries require the use of splines on unstructured meshes. For quadrilateral meshes, this means the introduction of parameterization singularities (extraordinary points and polar points). In this talk, I will discuss smooth splines on a special class of unstructured quadrilateral meshes: planar T-meshes with holes. These are meshes obtained by deleting some quadrilaterals from a T-mesh of the unit square. I will discuss two topics: (a) how to compute the dimension of splines on such meshes, and (b) how to build splines on them for isogeometric analysis.

Spline spaces on polyhedral cells

Nelly Villamizar (Swansea University, UK)

In this talk we shall consider the space of spline functions defined on polyhedral cells. These cells are the union of 3-dimensional polytopes sharing a common vertex, so that the intersection of any two of the polytopes is a face of both. In the talk, we will present new bounds on the dimension of this spline space. We provide a bound on the contribution of the homology term to the dimension count, and prove upper and lower bounds on the ideal of the interior vertex which depend only on combinatorial (or matroidal) information of the cell. We use inverse systems to convert the problem of finding the dimension of ideals generated by powers of linear forms to a computation of dimensions of so-called fat point ideals. The fat point schemes that comes from dualizing polyhedral cells is particularly well-suited and leads to the exact dimension in many cases of interest that will also be presented in the talk.

Joint work with Michael DiPasquale.

Overlapping multi-patch isogeometric method with minimal stabilization

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We present a novel method for isogeometric analysis (IGA) to directly work on geometries constructed by Boolean operations including difference (i.e., trimming), union and intersection. Particularly, this work focuses on the union operation, which involves multiple independent, generally non-conforming and trimmed spline patches. Given a series of patches, we overlay one on top of another in a certain order. While the invisible part of each patch is trimmed away, the visible parts of all the patches constitute the entire computational domain. We employ the Nitsche's method to weakly couple independent patches through visible interfaces. Moreover, we propose a minimal stabilization method to address the instability issue that arises on the interfaces shared by small trimmed elements. We show in theory that our proposed method recovers stability and guarantees well-posedness of the problem as well as optimal error estimates. In the end, we numerically verify the theory by solving the Poisson's equation on various geometries that are obtained by the union operation.