CIME-EMS Summer School in Applied Mathematics

Splines and PDEs: Recent Advances
from Approximation Theory to Structured Numerical Linear Algebra

July 3 - July 7, 2017 - Cetraro

Introduction to Isogeometric Analysis

Giancarlo Sangalli

...with results from many colleagues: P. Antolín, L. Beirao da Veiga, A. Buffa, G. Elber, M. Martinelli, F. Massarwi, R. Vázquez,...

Outline

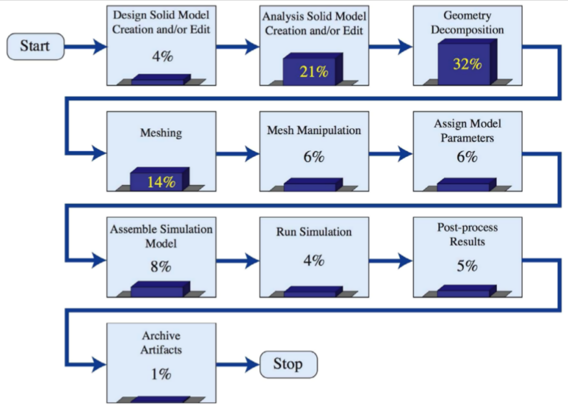
- introduction to IGA: motivation, appearance, developments...
- approximation properties: smoothness, multipatch, local refinement and trimming
- volumetring locking, stability and LBB (inf-sup) condition
- "structure preserving" within IGA
- implementation of IGA: fast formation/assembling ~~and preconditioning~~ of isogeometric matrices

Main motivation for IGA

Engineering designs are created in CAD systems.

Models for the Finite Element Method (FEM) are created from CAD representations.

Fixing CAD geometry and creating the FEM mesh takes more than 70% of the whole process.



M. Hardwick, R. Clay (Sandia National Laboratories, 2005)

The origin and motivation of IGA...

ELSEVIER

Available online at www.sciencedirect.com

SCIENCE DIRECT®

Comput. Methods Appl. Mech. Engrg. 194 (2005) 4135–4195

Computer methods in applied mechanics and engineering

www.elsevier.com/locate/cma

Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement

T.J.R. Hughes *, J.A. Cottrell, Y. Bazilevs

Institute for Computational Engineering and Sciences, The University of Texas at Austin, 201 East 24th Street, 1 University Station C0200, Austin, TX 78712-0027, United States

Received 28 September 2004; accepted 20 October 2004

- to be geometrically exact no matter how coarse the discretization;
- to simplify mesh refinement by eliminating the need for communication with the CAD geometry once the initial mesh is constructed;
- to more tightly weave the mesh generation process within CAD.

The origin and motivation of IGA...

ELSEVIER

Available online at www.sciencedirect.com

SCIENCE DIRECT®

Comput. Methods Appl. Mech. Engrg. 194 (2005) 4135–4195

Computer methods in applied mechanics and engineering

www.elsevier.com/locate/cma

Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement

T.J.R. Hughes *, J.A. Cottrell, Y. Bazilevs

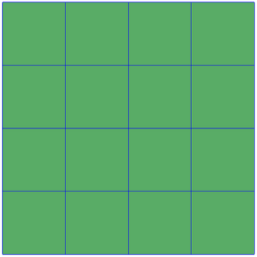
Institute for Computational Engineering and Sciences, The University of Texas at Austin, 201 East 24th Street, 1 University Station C0200, Austin, TX 78712-0027, United States

Received 28 September 2004; accepted 20 October 2004

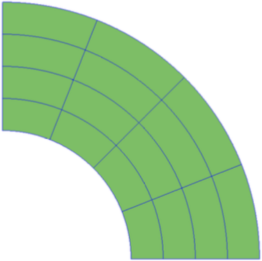
"the solution space for dependent variables is represented in terms of the same functions which represent the geometry".

IGA main concepts

The computational domain Ω is parametrized by spline/NURBS:
$\mathbf{F} : \widehat{\Omega} \to \Omega$, where $\mathbf{F}(\xi, \eta) = \sum_i \mathbf{C}_i B_i(\xi, \eta)$, and $\mathbf{C}_i$ are the control points

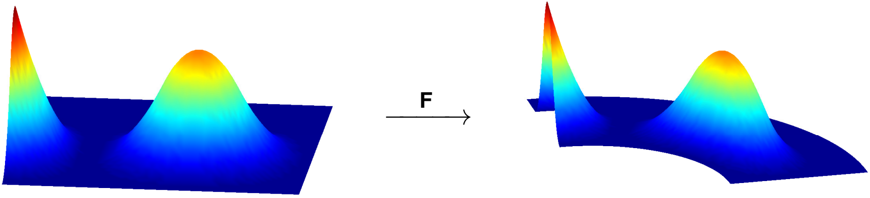


$\xrightarrow{\mathbf{F}}$



IGA main concepts

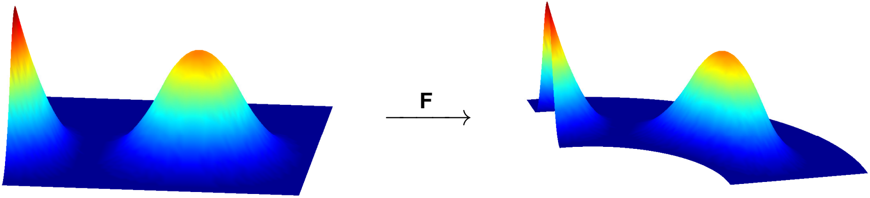
The computational domain Ω is parametrized by spline/NURBS:
$\mathbf{F} : \widehat{\Omega} \rightarrow \Omega$, where $\mathbf{F}(\xi, \eta) = \sum_i \mathbf{C}_i B_i(\xi, \eta)$, and $\mathbf{C}_i$ are the control points



$B_i(\xi, \eta)$ $\qquad\qquad$ $B_i \cdot \mathbf{F}(\mathbf{x}, \mathbf{y})$

IGA main concepts

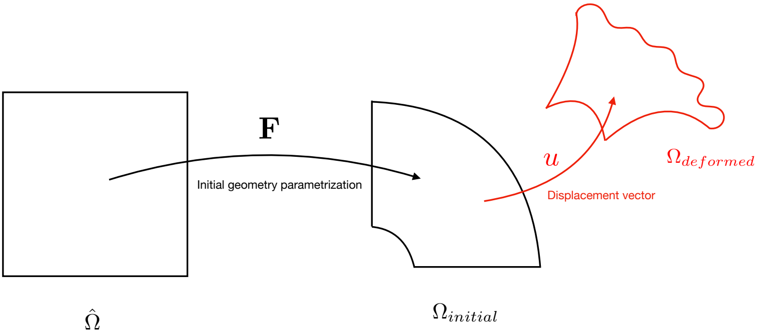
The computational domain Ω is parametrized by spline/NURBS:
$\mathbf{F} : \widehat{\Omega} \to \Omega$, where $\mathbf{F}(\xi, \eta) = \sum_i \mathbf{C}_i B_i(\xi, \eta)$, and $\mathbf{C}_i$ are the control points



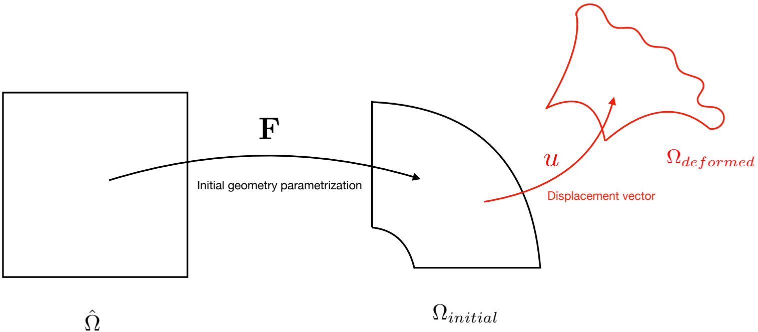
IGA has been proposed by structural engineers:

- **Isoparametric paradigm**: push-forward of splines/NURBS basis functions on the computational domain Ω to approximate the solution of the PDE.
- **Finite Element** kind method (splines etc. replace piecewise polynomials)

Isoparametric concept I



Isoparametric concept I

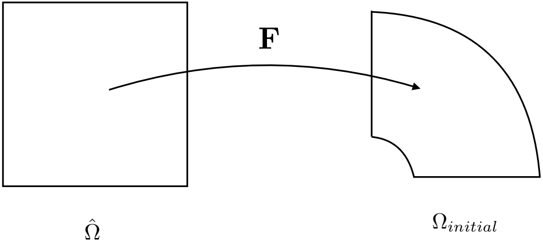


$\Omega_{deformed}$ is parametrized by $(\mathbf{F} + \underbrace{\mathbf{u} \cdot \mathbf{F}}_{\widehat{\mathbf{u}}})$

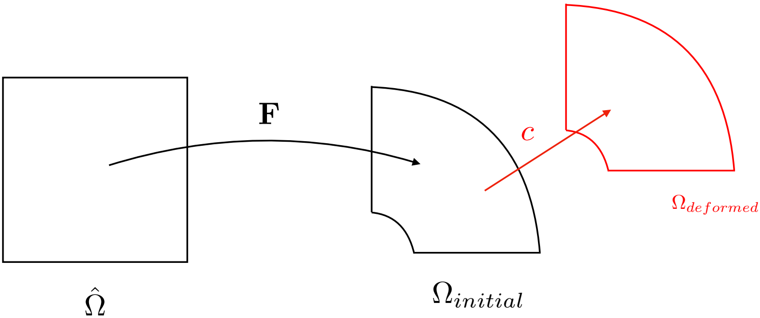
Isoparametric construction

The unknown written w.r.t. the parametric variables ($\widehat{\mathbf{u}} : \widehat{\Omega} \to \Omega_{deformed}$) is represented by the same fuctions that are used for the geometry parametrization $\mathbf{F} : \widehat{\Omega} \to \Omega_{initial}$

Isoparametric concept II

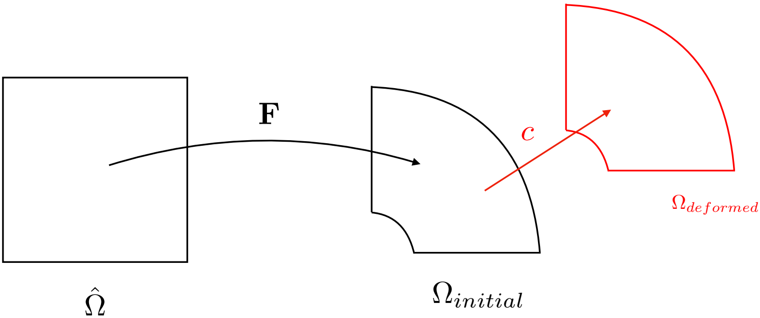


Isoparametric concept II



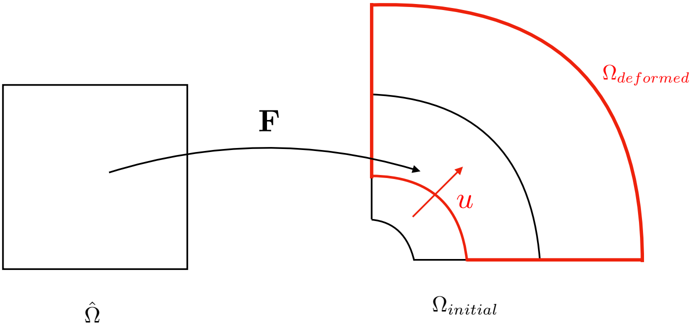
- a constant $\mathbf{c}$ belongs to the isoparametric space if the same constant $\widehat{\mathbf{c}} = \mathbf{c} \cdot \mathbf{F}$ is represented in the basis over $\widehat{\Omega}$...

Isoparametric concept II



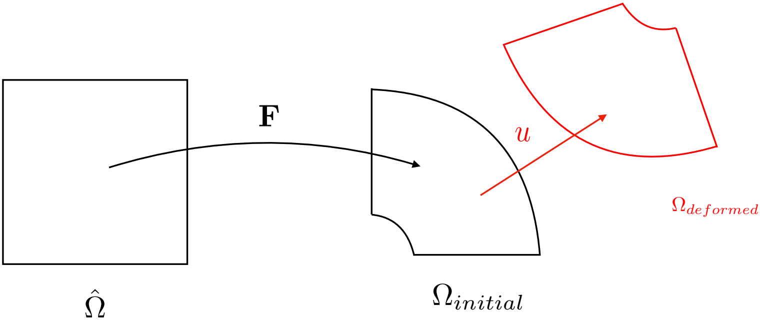
- a constant $\mathbf{c}$ belongs to the isoparametric space if the same constant $\widehat{\mathbf{c}} = \mathbf{c} \cdot \mathbf{F}$ is represented in the basis over $\widehat{\Omega}$... ok for splines and NURBS (partition of the unity)

Isoparametric concept II



- a constant $\mathbf{c}$ belongs to the isoparametric space if the same constant $\widehat{\mathbf{c}} = \mathbf{c} \cdot \mathbf{F}$ is represented in the basis over $\widehat{\Omega}$... ok for splines and NURBS (partition of the unity)
- $\mathbf{u}(x, y) = \mathbf{A} \begin{bmatrix} x \\ y \end{bmatrix}$ is in any isoparametric space since $\widehat{\mathbf{u}}(\xi, \eta) = \mathbf{A}\,\mathbf{F} \begin{bmatrix} \xi \\ \eta \end{bmatrix} = \mathbf{A} \begin{bmatrix} F_1(\xi,\eta) \\ F_2(\xi,\eta) \end{bmatrix}$ is a vector field whose components are linear combination of the parametrization components $F_1(\xi, \eta)$ and $F_2(\xi, \eta)$, and then is represented in the same basis

Isoparametric concept II



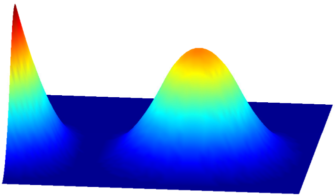
The isoparametric space contains any rigid body motion

$$\mathbf{u}(x, y) = \mathbf{c} + \mathbf{R} \begin{bmatrix} x \\ y \end{bmatrix}$$

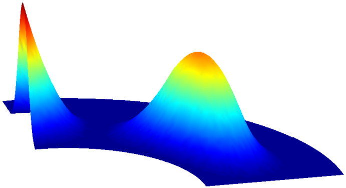
that form the kernel of the internal elastic energy

IGA main side-effect

The computational domain Ω is parametrized by spline/NURBS:
$\mathbf{F} : \widehat{\Omega} \to \Omega$, where $\mathbf{F}(\xi, \eta) = \sum_i \mathbf{C}_i B_i(\xi, \eta)$, and $\mathbf{C}_i$ are the control points



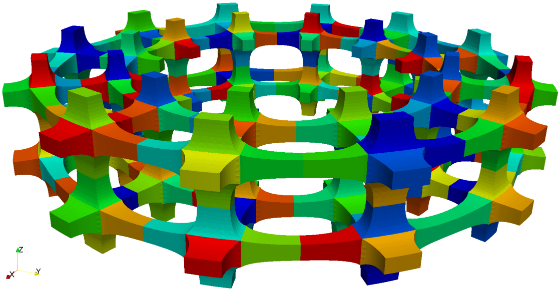
$\xrightarrow{\mathbf{F}}$



Isogeometric spaces are smooth

Example of IGA with analysis-aware geometric design

- Microstructure parametrization by IRIT, G. Elber, Technion.
- Elastic simulation by IGATOOLS-ISOLDE, P. Antolin, EPFL



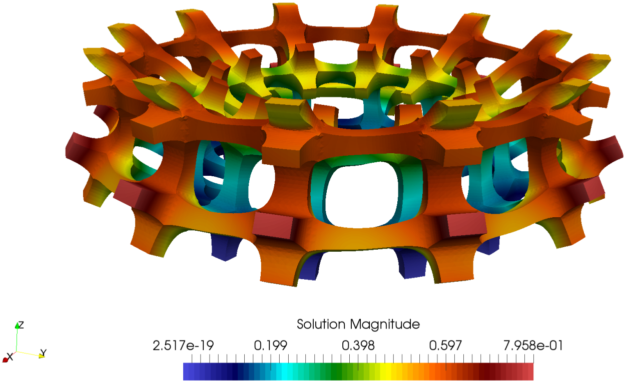
Example of IGA with analysis-aware geometric design

- Microstructure parametrization by IRIT, G. Elber, Technion.
- Elastic simulation by IGATOOLS-ISOLDE, P. Antolin, EPFL

- IRIT V-rep geometry formed by 336 trivariate Bézier elements of degree $(4, 4, 8)$.
- linear elasticity analysis
- isotropic material: Young mod. $E = 1$ and Poisson coeff. $\nu = 0.3$
- boundary conditions: a lower internal ring that is blocked and a Dirichlet boundary condition is applied for the top faces of the upper external ring, that is moved vertically by 0.5.

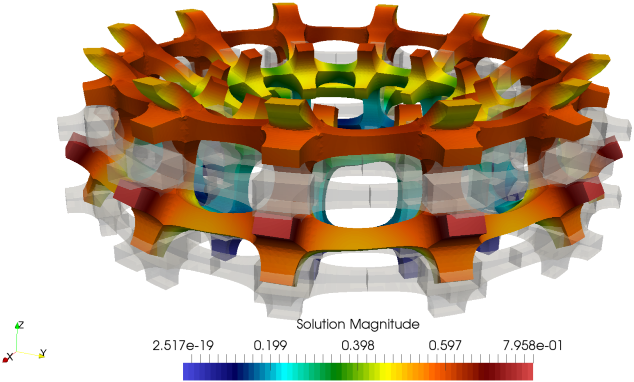
Example of IGA with analysis-aware geometric design

- Microstructure parametrization by IRIT, G. Elber, Technion.
- Elastic simulation by IGATOOLS-ISOLDE, P. Antolin, EPFL



Example of IGA with analysis-aware geometric design

- Microstructure parametrization by IRIT, G. Elber, Technion.
- Elastic simulation by IGATOOLS-ISOLDE, P. Antolin, EPFL



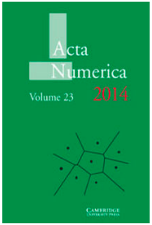
Further reading...

Acta Numerica

http://journals.cambridge.org/ANU

Additional services for ***Acta Numerica:***

Email alerts: Click here
Subscriptions: Click here
Commercial reprints: Click here
Terms of use : Click here



Mathematical analysis of variational isogeometric methods

L. Beirão da Veiga, A. Buffa, G. Sangalli and R. Vázquez

Acta Numerica / Volume 23 / May 2014, pp 157 - 287
DOI: 10.1017/S096249291400004X, Published online: 12 May 2014

Link to this article: http://journals.cambridge.org/abstract_S096249291400004X

How to cite this article:
L. Beirão da Veiga, A. Buffa, G. Sangalli and R. Vázquez (2014). Mathematical analysis of variational isogeometric methods . Acta Numerica, 23, pp 157-287 doi:10.1017/S096249291400004X