Name: Matriculation number:

September 2, 2019, L.A.G. exam

Solve the following exercises, explaining clearly each passage:

1) Compute the rank of the matrix $\begin{bmatrix} 1 & 1 & k \\ 0 & k-1 & 0 \\ k-1 & 0 & 0 \end{bmatrix}$ depending on the parameter k. For which values of k is the matrix invertible?

2) Consider the linear transformation $f : \mathbb{R}^3 \to \mathbb{R}^2$ defined by multiplication by the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \end{bmatrix}$

(a) Determine the dimension and a basis of $\mathrm{Ker}(f)$ and of $\mathrm{Im}(f)$.

(b) Say if f is injective and if it is surjective.

(c) Determine all the vectors $v \in \mathbb{R}^3$, if they exist, such that $f(v) = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$

3) In the euclidean plane consider the line r passing through the point $P = (1, -1)$, parallel to the vector $v = (1, 2)$, and the line s, with cartesian equation $x + y = 1$

(a) Find parametric equations of r and of s.

(b) Determine the intersection point of r and s.

(c) Determine the angle between r and s.

(d) Determine the distance between the line r and the origin.

4) Consider the matrix $M = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$

(a) Calculate the real eigenvalues of M, and their algebraic and geometric multiplicities.

(b) Determine whether M is diagonalizable.

(c) For each eigenvalue find an orthonormal basis of its eigenspace.

(d) Is there an orthogonal matrix C such that $C^T M C$ is a diagonal matrix?

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Solve the following exercises, explaining clearly each passage:

1) Compute the rank of the matrix

$$\begin{bmatrix} 1 & k & 1 \\ 1 & 0 & k-1 \\ k-2 & 0 & 0 \end{bmatrix}$$

depending on the parameter k. For which values of k is the matrix invertible?

2) Consider the linear transformation $f : \mathbb{R}^2 \to \mathbb{R}^3$ defined by multiplication by the matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ -1 & 0 \end{bmatrix}$

(a) Determine the dimension and a basis of $\mathrm{Ker}(f)$ and of $\mathrm{Im}(f)$.

(b) Say if f is injective and if it is surjective.

(c) Determine all the vectors $v \in \mathbb{R}^2$, if they exist, such that $f(v) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

3) In the euclidean plane consider the line r passing through the point $P = (1, -1)$, parallel to the vector $v = (2, 1)$, and the line s, with cartesian equation $x - y = 1$

(a) Find parametric equations of r and of s.

(b) Determine the intersection point of r and s.

(c) Determine the angle between r and s.

(d) Determine the distance between the line r and the origin.

4) Consider the matrix $M = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

(a) Calculate the real eigenvalues of M, and their algebraic and geometric multiplicities.

(b) Determine whether M is diagonalizable.

(c) For each eigenvalue find an orthonormal basis of its eigenspace.

(d) Is there an orthogonal matrix C such that $C^T M C$ is a diagonal matrix?