Name: Matriculation number:

February 19, 2020, L.A.G. exam

Solve the following exercises, explaining clearly each passage:

1) Consider the matrix $M = \begin{bmatrix} -1 & k \\ k & -1 \end{bmatrix}$

(a) Compute the determinant of M

(b) For which values of k does the inverse of M exist ?

(c) Compute the rank of M for all values of k

2) Consider the linear transformation $f : \mathbb{R}^3 \to \mathbb{R}^2$ defined by

$$f(\begin{bmatrix} x \\ y \\ z \end{bmatrix}) = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} y \\ z \end{bmatrix} + \begin{bmatrix} z \\ x \end{bmatrix}$$

(a) Find a matrix A such that $f(\begin{bmatrix} x \\ y \\ z \end{bmatrix}) = A \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

(b) Find a basis of the kernel of f and its dimension

(c) Find a vector $w \in \mathbb{R}^2$ that is not in the image of f

3) In the euclidean 2-dim. plane consider the line r passing through the points $P = (6, 2)$ and $Q = (5, 1)$; consider also the line s defined by the cartesian equation $x + y - 1 = 0$

(a) Find parametric equations of r and of s.

(b) Determine the intersection point of r and s, if it exists.

(c) Determine the angle between r and s

4) Consider the matrix $M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(a) Calculate the real eigenvalues of M, and their respective algebraic and geometric multiplicities.

(b) Determine whether M is diagonalizable.

(c) Find an orthonormal basis of eigenvectors of M, if it exists

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1) Consider the matrix $M = \begin{bmatrix} 1 & 1 \\ k & -k \end{bmatrix}$

(a) Compute the determinant of M

(b) For which values of k does the inverse of M exist ?

(c) Compute the rank of M for all values of k

2) Consider the linear transformation $f : \mathbb{R}^2 \to \mathbb{R}^3$ defined by

$$f(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} x \\ y \\ x \end{bmatrix} + 2 \begin{bmatrix} y \\ x \\ y \end{bmatrix}$$

(a) Find a matrix A such that $f(\begin{bmatrix} x \\ y \end{bmatrix}) = A \cdot \begin{bmatrix} x \\ y \end{bmatrix}$

(b) Find a basis of the image of f and its dimension

(c) Find a vector $w \in \mathbb{R}^3$ that is not in the image of f

3) In the euclidean 2-dim. plane consider the line r passing through the points $P = (1, 2)$ and $Q = (2, 1)$; consider also the line s defined by the cartesian equation $2x + y = 0$

(a) Find parametric equations of r and of s.

(b) Determine the intersection point of r and s, if it exists.

(c) Determine the angle between r and s

4) Consider the matrix $M = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(a) Calculate the real eigenvalues of M, and their respective algebraic and geometric multiplicities.

(b) Determine whether M is diagonalizable.

(c) Find an orthonormal basis of eigenvectors of M, if it exists