

Name:

Matriculation number:

June 14, 2019

2nd Midterm L.A.G. exam

Solve the following exercises, explaining clearly each passage:

1) Consider in the 3-dimensional space the line l with cartesian equation

$$\begin{cases} x + 2y = 0 \\ x - 2z = -2 \end{cases}$$

and the line r passing through the point $P = (2, 3, 4)$ and the point $Q = (1, 4, 3)$.

- Find a parametric equation of the line r .
- Determine the mutual position of the two lines l and r .
- Find the angle between the lines l and r .
- Find the area of the triangle with vertices the origin, P and Q .

2) Consider the matrix

$$A = \begin{bmatrix} -1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

- Find the eigenvalues of A .
- Determine if A is diagonalizable.
- For each eigenvalue compute an orthonormal basis of its eigenspace.

3) Consider the conic

$$xy + x + y = 0$$

- Determine a rotation and a translation that puts it in canonical form
- Identify the type of conic
- Find, if they exist, vertices, axis and centre of symmetry.

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Solve the following exercises, explaining clearly each passage:

1) Consider in the 3-dimensional space the line l with cartesian equation

$$\begin{cases} x + 2y = 0 \\ x - 2z = 2 \end{cases}$$

and the line r passing through the point $P = (2, 3, 2)$ and the point $Q = (1, 4, 3)$.

- Find a parametric equation of the line r .
- Determine the mutual position of the two lines l and r .
- Find the angle between the lines l and r .
- Find the area of the triangle with vertices the origin, P and Q .

2) Consider the matrix

$$A = \begin{bmatrix} -1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

- Find the eigenvalues of A .
- Determine if A is diagonalizable.
- For each eigenvalue compute an orthonormal basis of its eigenspace.

3) Consider the conic

$$xy + x + y + 1 = 0$$

- Determine a rotation and a translation that puts it in canonical form
- Identify the type of conic
- Find, if they exist, vertices, axis and centre of symmetry.