Name: Matriculation number:

September 12, 2019, L.A.G. exam

Solve the following exercises, explaining clearly each passage:

1) Consider the matrix $A = \begin{bmatrix} 1 & -k \\ 2-k & -1 \end{bmatrix}$

(1a) Compute the rank of $A$ depending on the parameter $k$.

(1b) For which values of $k$ is the matrix invertible?

(1c) Compute the inverse of $A$ for the values of $k$ from the previous point

2) Consider the linear transformation $f : \mathbb{R}^3 \to \mathbb{R}^2$ defined by

$$f \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 + x_3 \\ x_2 \end{bmatrix}$$

(2a) Determine the dimension and a basis of $\mathrm{Ker}(f)$ and of $\mathrm{Im}(f)$.

(2b) Say if $f$ is injective and if it is surjective.

(2c) Determine all the vectors $v \in \mathbb{R}^3$, if they exist, such that $f(v) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

3) Consider in the 3-dimensional space the line $r$ of cartesian equation $\begin{cases} x + 2z - 1 = 0 \\ y - z - 1 = 0 \end{cases}$

(3a) Find a parametric equation of $r$.

(3b) Determine the plane $\pi$ perpendicular to $r$ passing through the point $(1, 2, 1)$.

(3c) Find the distance between $\pi$ and the origin.

(3d) Find the angle between $r$ and the $x$ axis.

4) Consider in the euclidean plane the conic

$$x^2 + 2y^2 - 4x - 12y + 21 = 0$$

(4a) Find a translation $X = x + a, Y = y + b$ that puts the conic in canonical form

(4b) Recognize the type of conic

(4c) Find the coordinates of the centre, if it exists

(4d) Find the coordinates of the vertices.

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September 12, 2019, L.A.G. exam

Solve the following exercises, explaining clearly each passage:

1) Consider the matrix $A = \begin{bmatrix} k & k \\ 1 & 2-k \end{bmatrix}$

(1a) Compute the rank of $A$ depending on the parameter $k$.

(1b) For which values of $k$ is the matrix invertible?

(1c) Compute the inverse of $A$ for the values of $k$ from the previous point

2) Consider the linear transformation $f : \mathbb{R}^2 \to \mathbb{R}^3$ defined by

$$f \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_1 + x_2 \\ x_1 \end{bmatrix}$$

(2a) Determine the dimension and a basis of $\mathrm{Ker}(f)$ and of $\mathrm{Im}(f)$.

(2b) Say if $f$ is injective and if it is surjective.

(2c) Determine all the vectors $v \in \mathbb{R}^2$, if they exist, such that $f(v) = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

3) Consider in the 3-dimensional space the line $r$ of cartesian equation $\begin{cases} x - 2z - 1 = 0 \\ y + z - 1 = 0 \end{cases}$

(3a) Find a parametric equation of $r$.

(3b) Determine the cartesian equation of the plane $\pi$ perpendicular to $r$ passing through the point $(1, 1, 1)$

(3c) Find the distance between $\pi$ and the origin.

(3d) Find the angle between $r$ and the $x$ axis.

4) Consider in the euclidean plane the conic

$$x - 2y^2 + 8y - 9 = 0$$

(4a) Find a translation $X = x + a, Y = y + b$ that puts the conic in canonical form

(4b) Recognize the type of conic

(4c) Find the coordinates of the centre, if it exists

(4d) Find the coordinates of the vertices.