Name: Matriculation number:

February 6, 2020, L.A.G. exam

Solve the following exercises, explaining clearly each passage:

1) Consider the matrix $M = kI + A \cdot A$

where k is a real parameter, I is the 2×2 identity matrix and $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

(a) Compute the rank of M depending on the parameter k.

(b) For which values of k is M invertible?

(c) Compute the inverse of M for the values of k in (b).

2) Consider the linear transformation $f : \mathbb{R}^3 \to \mathbb{R}^2$ defined by

$$f(\begin{bmatrix} x \\ y \\ z \end{bmatrix}) = x \begin{bmatrix} 1 \\ 1 \end{bmatrix} + y \begin{bmatrix} 1 \\ -1 \end{bmatrix} - z \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

(a) Determine the dimension and a basis of $\mathrm{Ker}(f)$ and of $\mathrm{Im}(f)$.

(b) Say if f is injective, if it is surjective and if it is bijective.

(c) Determine all the vectors $v \in \mathbb{R}^3$, if they exist, such that $f(v) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

3) In the 3-dim. euclidean space consider the line r passing through the origin, and perpendicular to the plane $x - z = 400$, and the line s passing through the point $(1, 0, 1)$ and parallel to the vector $(0, 500, -500)$.

(a) Find parametric equations of r and of s.

(b) Determine the relative position of r and s (parallel, incident or skew)

(c) Find a vector of length 1 perpendicular to both r and s.

4) Consider the matrix $M = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

(a) Calculate the real eigenvalues of M, and their algebraic and geometric multiplicities.

(b) Determine whether M is diagonalizable.

(c) For each eigenvalue find an orthonormal basis of its eigenspace.

Name: Matriculation number:

February 6, 2020, L.A.G. exam

Solve the following exercises, explaining clearly each passage:

1) Consider the matrix $M = kI + A \cdot A$

where k is a real parameter, I is the 2×2 identity matrix and $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(a) Compute the rank of M depending on the parameter k.

(b) For which values of k is M invertible?

(c) Compute the inverse of M for the values of k in (b).

2) Consider the linear transformation $f : \mathbb{R}^2 \to \mathbb{R}^3$ defined by

$$f(\begin{bmatrix} x \\ y \end{bmatrix}) = x \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} - y \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}$$

(a) Determine the dimension and a basis of $\mathrm{Ker}(f)$ and of $\mathrm{Im}(f)$.

(b) Say if f is injective, if it is surjective and if it is bijective.

(c) Determine all the vectors $v \in \mathbb{R}^2$, if they exist, such that $f(v) = \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}$

3) In the 3-dim. euclidean space consider the line r passing through the origin, and perpendicular to the plane $x + z = 400$, and the line s passing through the point $(1, 0, 1)$ and parallel to the vector $(0, 500, 500)$.

(a) Find parametric equations of r and of s.

(b) Determine the relative position of r and s (parallel, incident or skew)

(c) Find a vector of length 1 perpendicular to both r and s.

4) Consider the matrix $M = \begin{bmatrix} -1 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(a) Calculate the real eigenvalues of M, and their algebraic and geometric multiplicities.

(b) Determine whether M is diagonalizable.

(c) For each eigenvalue find an orthonormal basis of its eigenspace.