AFFINE ALGEBRAIC SURFACES
and
the ZARISKI CANCELLATION PROBLEM

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This course aims to give an introduction into an active research domain of Affine Algebraic Geometry, with its important open problems and diverse techniques. We will focus on the most studied case of surfaces, their geometry, elements of classification, and algebraic group actions on surfaces. The second half of the course is devoted to the Zariski Cancellation Problem for surfaces, tending to introduce to very recent achievements.

PROGRAM

Part 1: Geometry of affine surfaces

(1) Iitaka-Kodaira logarithmic dimension.
(2) Ruled affine surfaces.
(3) Geometry of the affine plane. Abhyankar-Moh-Suzuki Embedding theorem.
(4) Characterization of the affine plane (after Ramanujam and Miyanishi-Sugie-Fujita).

Part 2: Group actions on affine surfaces

(1) Jung-van der Kulk Theorem for the automorphism group of the affine plane.
(2) Toric affine surfaces: combinatorial description. Cyclic quotients of the affine plane.
(3) $\mathbb{G}_m$-actions on affine surfaces and Dolgachev-Pinkham-Demazure presentation.
(4) Locally nilpotent derivations and $\mathbb{G}_a$-actions. Free $\mathbb{G}_a$-actions.
(5) Makar-Limanov invariant and Gizatullin surfaces.

Part 3: Zariski Cancellation Problem

(1) Cancellation for the affine plane.
(2) Danielewski examples of non-cancellation. Danielewski-Fieseler factors.
(3) Strong cancellation and $\mathbb{G}_a$-actions. Bandman-Makar-Limanov-Crachiola Theorem.
(4) Affine modifications and Asanuma trick.
(5) Cancellation and non-cancellation for $\mathbb{G}_a$-surfaces.

Literature

M. Miyanishi. Open algebraic surfaces. CRM monograph series 12, American Mathematical Society, Providence, RI, 2001.