AFFINE ALGEBRAIC SURFACES and the ZARISKI CANCELLATION PROBLEM

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1. Introduction (29.09-06.10.2015)

1. Affine varieties (generalities).
2. Elements of classification: log-Kodaira dimension.
3. Embedding Problem.
4. Automorphism groups. Linearization of group actions.
5. Zariski Cancellation Problem.

Literature

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2. Epimorphism Theorem (06-13-20.10.2015)

1. Gutwirth-Nagata Theorem.
2. Pseudominimal completions of $\mathbb{A}^1$-fibrations.
3. A formula for Euler characteristic.
4. Epimorphism Theorem over $\mathbb{C}$ via Suzuki's approach.
5. Abhyankar-Moh Epimorphism Theorem.

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3. Simply connected affine plane curves (20-27.10–03.11.2015)

1. Simply connected curves. Quasihomogeneous curves.
2. Suzuki's Formula for Euler characteristic.
3. Teichmüller spaces. Ahlfors-Bers complex structure. Grothendieck-Earle-Engber Universality Theorem.
4. Milnor fibration of an isolated singularity.
5. Extending and linearizing a regular $\mathbb{G}_m$-action.

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4. Locally nilpotent derivations and cancellation (03-10-17.11.2015)

1. LND's: first properties. Associated degree functions.
2. Local slice construction.
3. Associated affine rulings.
4. Gradings and homogeneous LND's.
5. Descent of LND's and cancellation.

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5. Zariski Cancellation Problem for surfaces (17-24.11.2015)

1. Strong cancellation property. Zariski factors.
2. Danielewski-Fieseler quotient; Danielewski's example.
3. Cancellation invariants.
4. Generalized Danielewski surfaces.
5. Affine modifications and cancellation.

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