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POLITECNICO
MILANO 1863

DIPARTIMENTO DI MATEMATICA

Ivan Fumagalli

**Special Course "*Mathematical modeling of the human brain:
from physiology to neurodegenerative disorders*"**

Università degli Studi di Roma Tor Vergata

22 January 2025



The BrainNum project

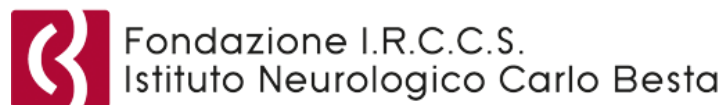


brainum.mox.polimi.it



lymph.bitbucket.io

Clinical collaborations with:



Paola F. Antonietti
(P.I.)



Mattia Corti



Ivan Fumagalli



Caterina Leimer Saglio



Niteen Kumar



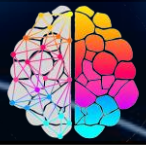
Stefano Pagani



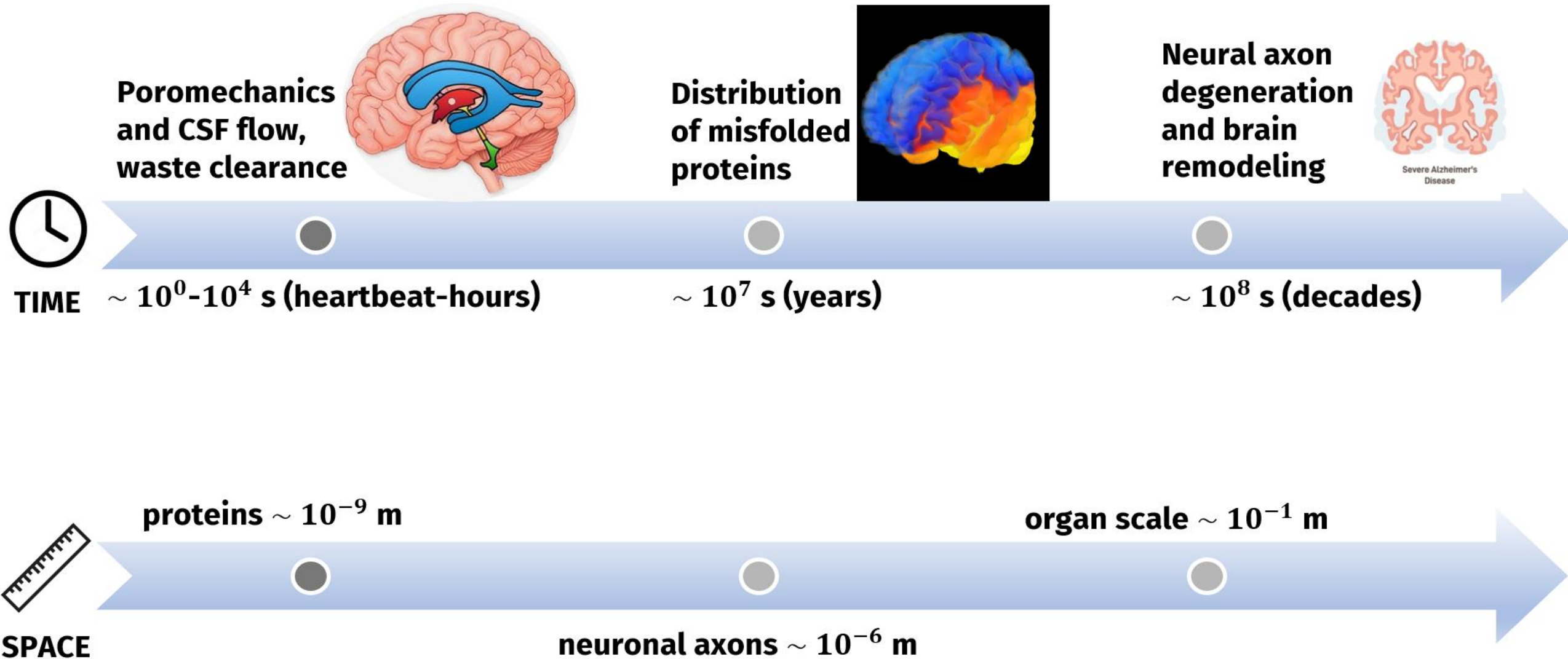
Francesca Bonizzoni

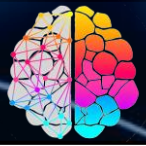


Nicola Parolini



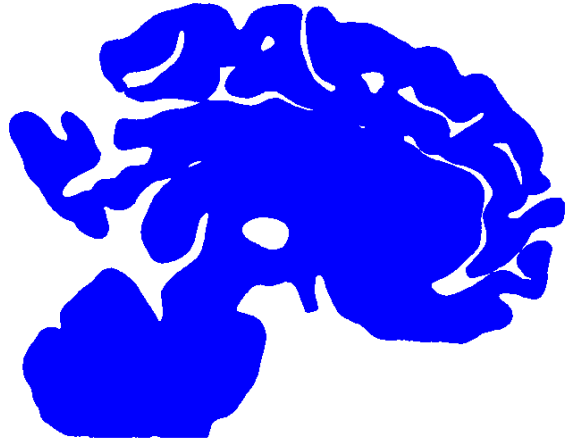
Recap – numerical modelling of the brain function



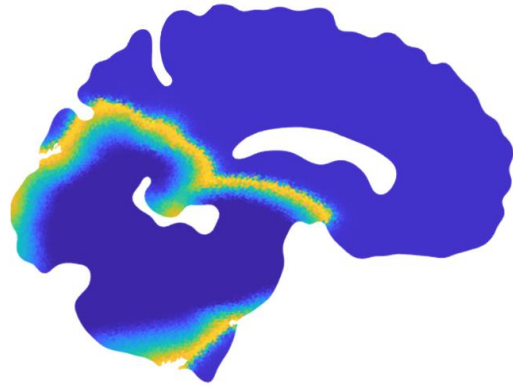


Recap – numerical modelling of the brain function

CSF flow

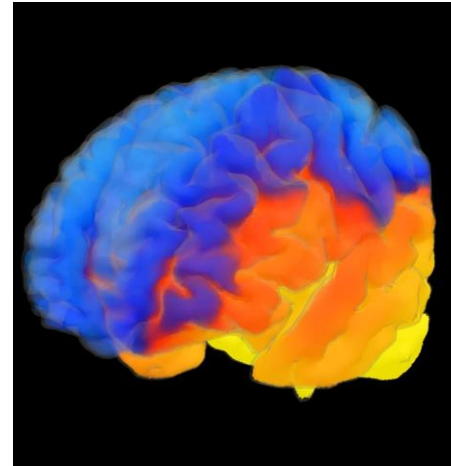


Signal propagation



courtesy: C.B.Leimer Saglio

Protein spreading



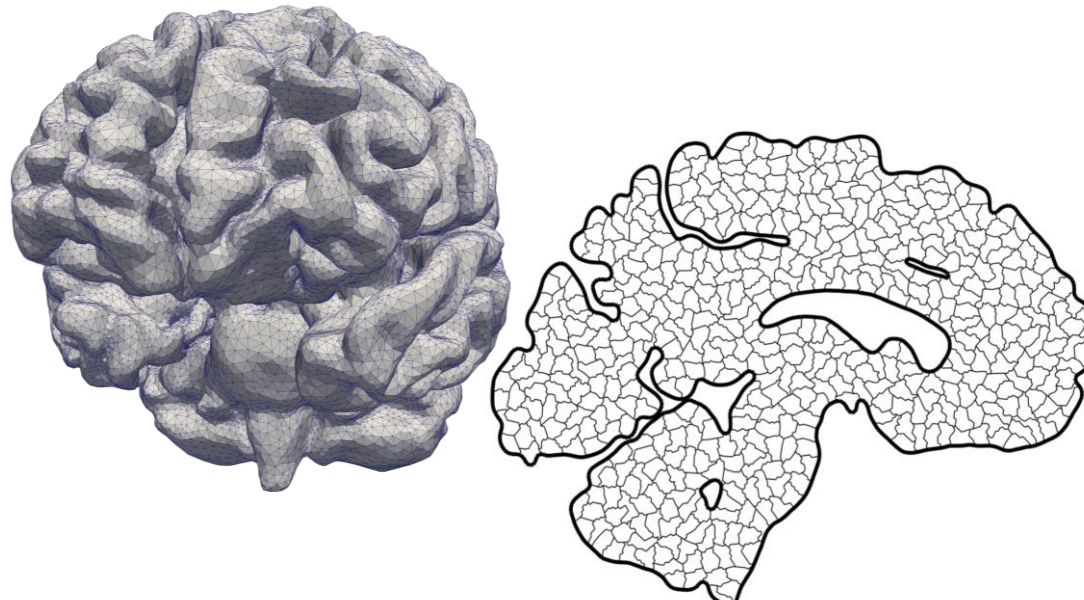
courtesy: M. Corti

Hemodynamics



courtesy: E. Irali

...all need a
computational
domain (mesh)



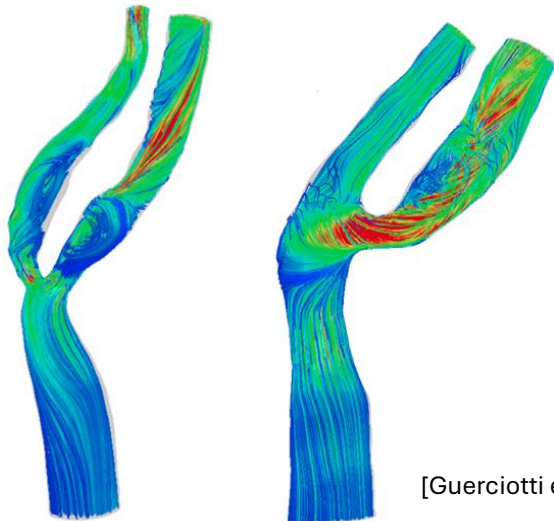


Geometric domain and data

Computational studies can be performed in

Idealized and/or average models

- ✓ Parametric studies
- ✓ Benchmarking and verification
- ✓ No image segmentation required
- Real geometry not available
- ✗ **Geometrical features** may have major impact on results
- ✗ Following **time evolution** of pathology not easy

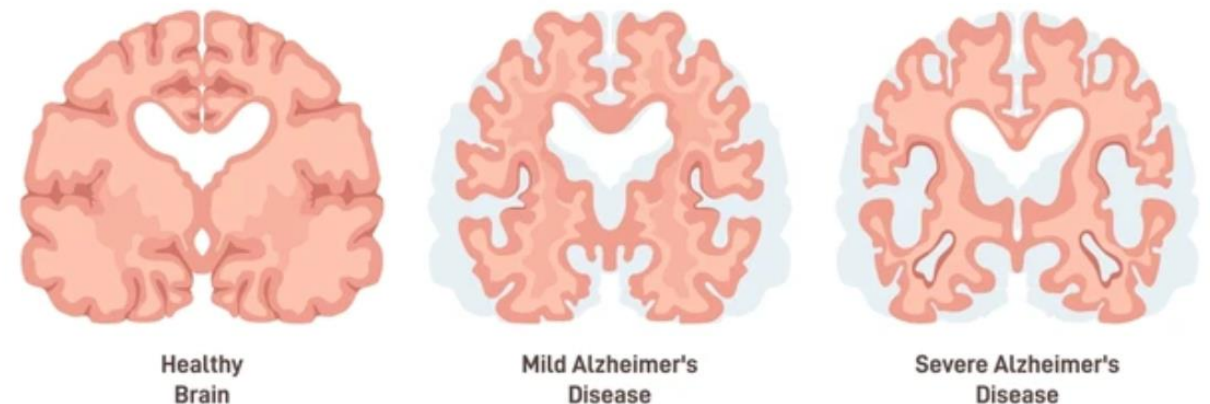


[Guerciotti et al. 2017]

Patient-specific models

- ✓ **Actual** patient's condition (e.g. atrophy)
- ✓ Can follow **pathology development**
- ✓ Include non-pathological **anomalies**
- Real geometry available as imaging data
- ✗ Need of accurate/appropriate data
- ✗ Reconstruction/calibration may be **time expensive**

Stages of Alzheimer's Disease

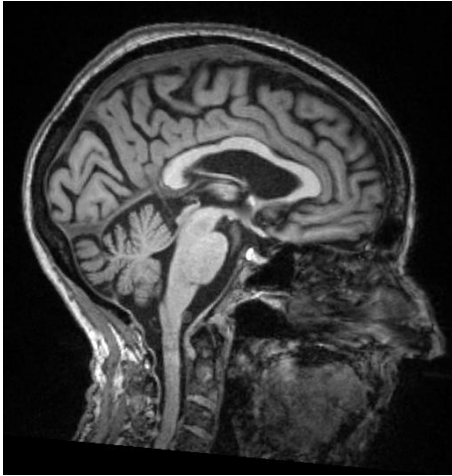




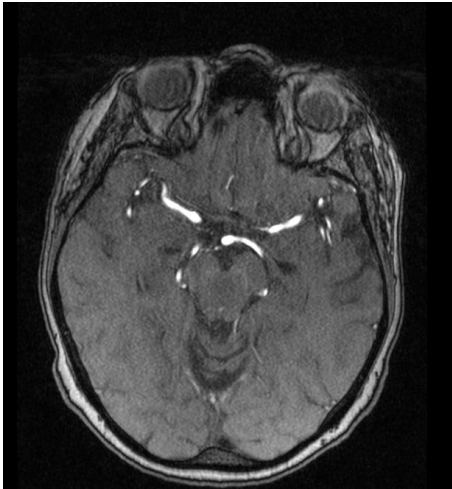
Workflow: from images to computational mesh

Imaging data

MRI



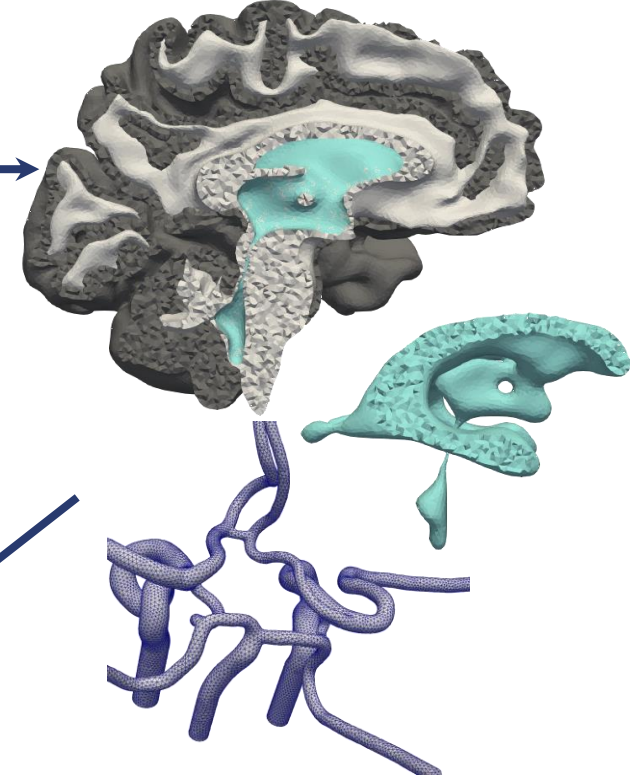
angiography



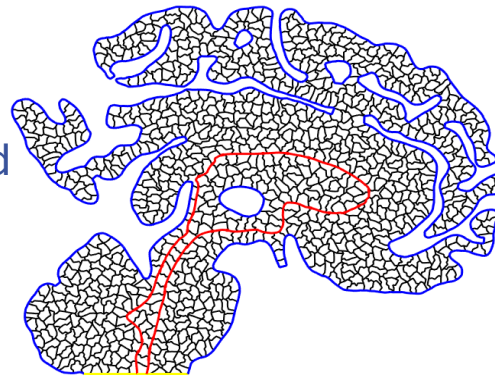
Domain reconstruction



Mesh generation



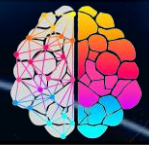
Agglomerated
poly grid





Contents

1. Imaging data in a nutshell
2. From clinical images to a computational mesh: a reconstruction pipeline
 1. Segmentation of brain MRI: identifying brain regions
 2. From the segmentation to the computational mesh
 1. Full pipeline for the gray matter
 2. Complex geometries: surface processing
 3. Combining surfaces: ventricles, grey matter and the whole parenchyma
3. Computational modelling of brain hemodynamics: reconstruction and simulation



Shared OneDrive folder:

<https://tinyurl.com/LabBrain2025>

You can find data and results of the reconstruction part of this lecture.


Docker image with all the requirements for brain mesh generation:

<https://hub.docker.com/repository/docker/ivanfumagalli/my-svmtk>




About the data

The T1-weighted MRI data uploaded in <https://tinyurl.com/LabBrain2025> is taken from the OpenNeuro database [<http://openneuro.org>], in particular from subject 087 of project *ds004471* [M. Abbass, G. Gilmore, A. Taha, R. Chevalier, M. Jach, T. M. Peters, A. R. Khan, J. C. Lau (2023). London Heath Sciences Center Parkinson's Disease Dataset (LHSCPD). OpenNeuro. [Dataset] doi: [10.18112/openneuro.ds004471.v1.0.1](https://doi.org/10.18112/openneuro.ds004471.v1.0.1)



 **OpenNEURO**




SEARCH SUPPORT DOCUMENTATION

Sign in

 **London Heath Sciences Center Parkinson's Disease Dataset (LHSCPD)**

★ Follow 3 📖 Bookmark 8

BIDS Validation  1 WARNING  Valid Clone

 Files  Download  Metadata

README

The London Health Sciences Center Parkinson's disease (LHSCPD) dataset consists of 40 subjects diagnosed with Parkinson's Disease (age: 60.2 ± 6.8 , sex: 13 female and 27 male) and have undergone deep brain stimulation (DBS) surgery.

This release currently contains pre-DBS gadolinium-enhanced imaging collected at our institution (University Hospital in London, ON, Canada) with a 1.5 T scanner (Signa, General Electric, Milwaukee, Wisconsin, USA).

OpenNeuro Accession Number

ds004471

Authors

Mohamad Abbass, Greydon Gilmore, Alaa Taha, Ryan Chevalier, Magdalena Jach, Terry M. Peters, Ali R. Khan, Jonathan C. Lau

Available Modalities

MRI

Versions

1 – Imaging data in a nutshell



Computer Tomography (CT)

Diagnostic imaging procedure that combines multiple cross-sectional (*tomographic*) X-ray acquisitions and digital geometry processing to generate a **3D volume** of the inside of the body, made of **pixels**.

Grey values represent the attenuation coefficient (strictly related to **density**) of the tissues, expressed in Hounsfield units (HU)

- Air: -1000
- Fat: -100
- Water: 0
- **CSF: 15**
- **White matter: 20-30**
- **Blood: 30-45**
- **Grey matter: 37-45**
- Muscle: 40
- Bone: 700-3000



Wikimedia Commons – lic. CC0-1.0 Mikael Häggström, M.D.

- ✓ Great **space resolution**: ~1 mm (even ~100 μm for high-res. machines)
- ✓ Relatively quick acquisition (few minutes)
- ✗ Radiations can be damaging
- ✗ Very limited time series

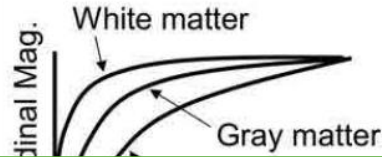
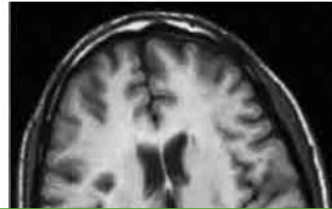


Magnetic Resonance Imaging (MRI)

Diagnostic imaging procedure based on the application of heterogeneous and time-varying magnetic field: based on magnetic relaxation properties of the hydrogen nuclei contained in the tissue, construct a “**3D**” **volume** (actually an array of “**2D**” **slabs**) of the inside of the body, made of **voxels**.

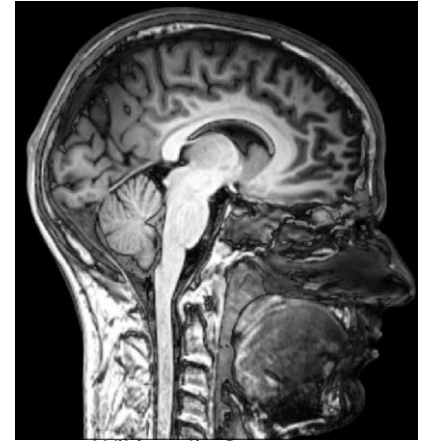
Grey values represent the **magnetic relaxation time** of the tissues:

T1-weighted MRI
relaxation time of
longitudinal mag



T2-weighted MRI
relaxation time
transverse mag

MRI is the gold standard in clinical practice, routinely employed in diagnosis and follow-up



Wikimedia Commons
lic. CC-BY:SA 2.0 Chiswick Chap

- ✓ **Non-invasive** (no radiations – except if contrast agent is used)
- ✓ **Highly parametrizable:**
 - allows different orientations
 - different acquisition protocols yield **different contrast**
- Longer scan time w.r.t. CT
- Resolution: **1-3 mm**

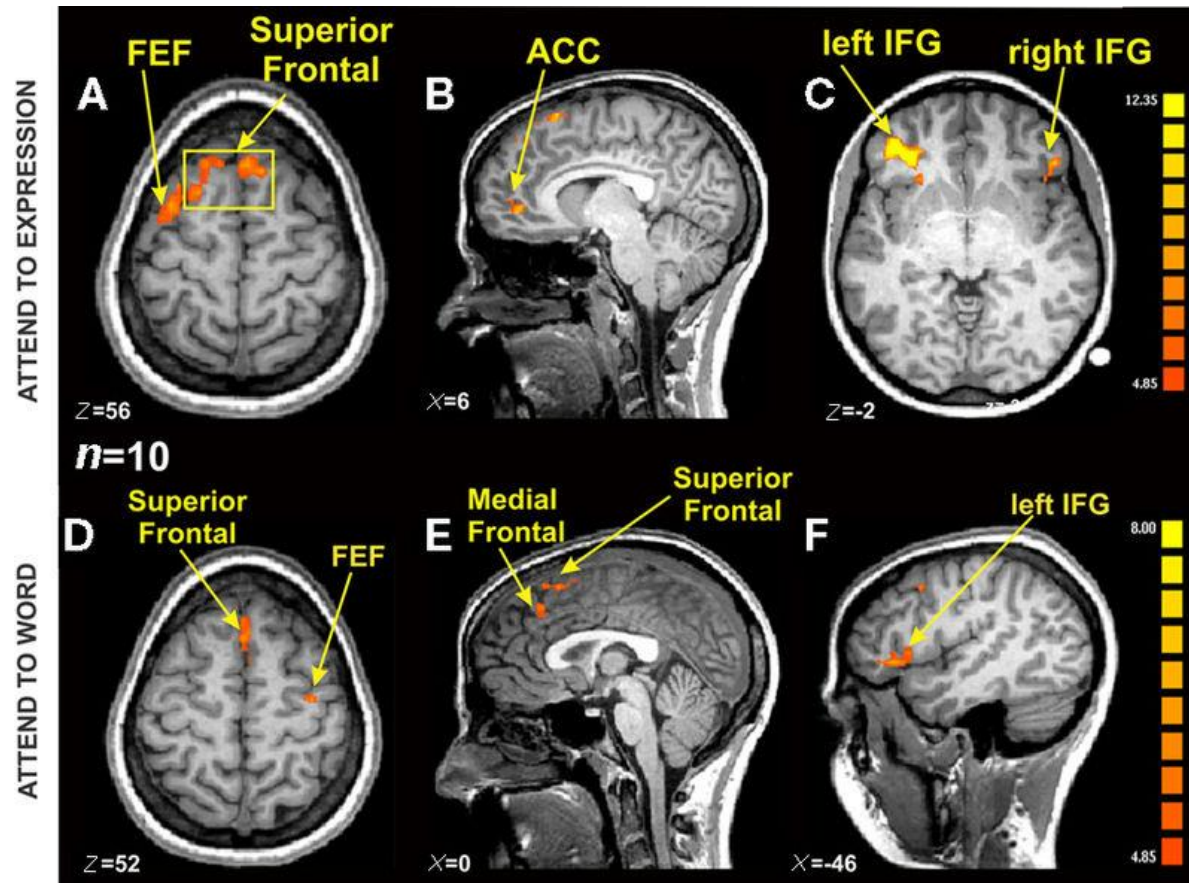


Functional Magnetic Resonance Imaging (fMRI)

Employs the techniques of MRI to measure **oxygen levels** in the tissue.

Function is measured relying on the Blood Oxygenation Level Dependent (**BOLD**) phenomenon:

neuronal activation \Rightarrow increased blood flow \Rightarrow increased oxygenation



Wikimedia Commons - lic. CC-BY 2.5 Shima Ovaysikia et al (2010)

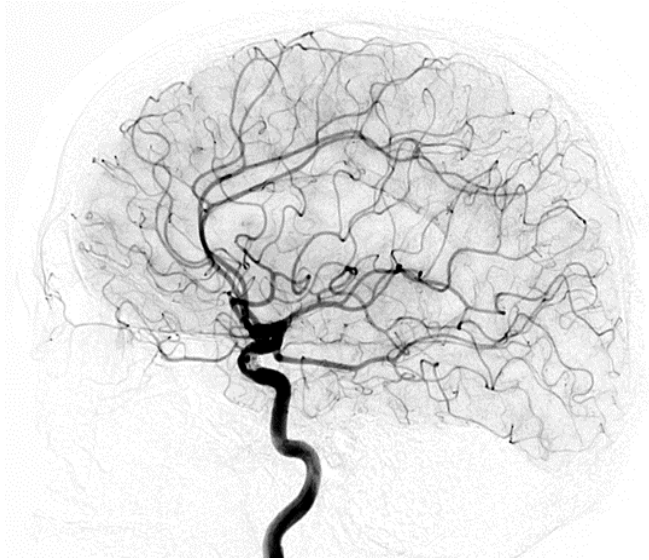
- Measures **function**, not anatomy
- Clinical applications:
 - mapping motor and language areas, and their impairment
 - fMRI-guided neurosurgery



Angiography

General naming of a technique used to visualize the inside (***lumen***) of **blood vessels** and cavities.

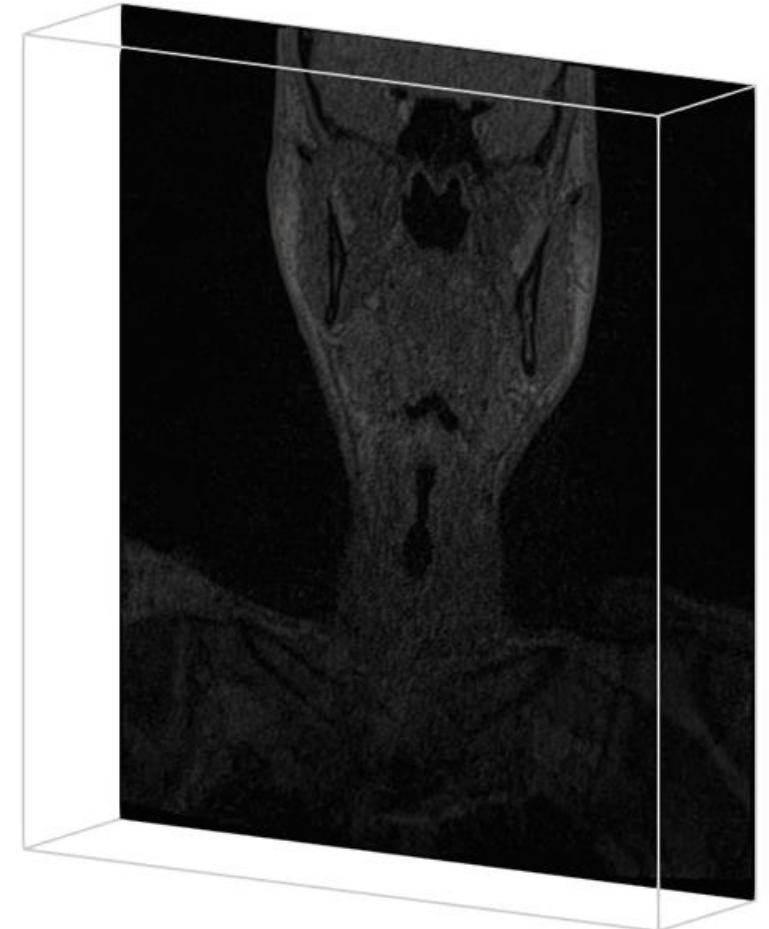
- Traditional angiography: X-ray based techniques in combination with radio-opaque contrast agent
- Now, typically obtained from CT or MRI



Traditional cerebral angiogram



CT coronary angiogram



MR angiogram of neck and basal cranial vessels

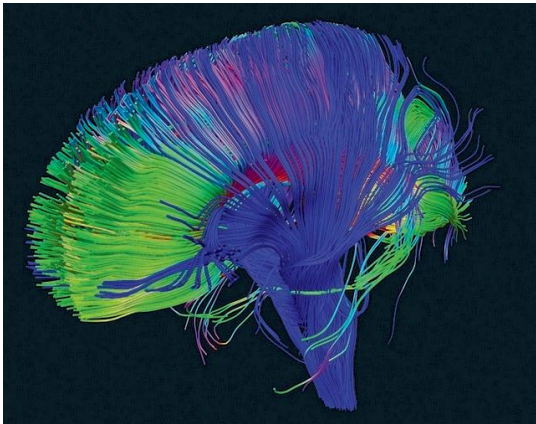


Other acquisitions for specific tasks

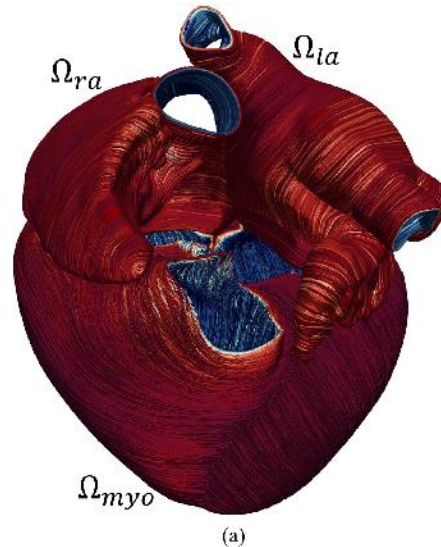
Diffusion-tensor MRI (dt-MRI)

Used to measuring the diffusion of water molecules through living tissues

- ⇒ highlight areas of activity
- ⇒ allow reconstruction of **neuronal fibers**
- ⇒ used also for myocardial fibers (ex vivo)



Credit: NICHD/P. Basser

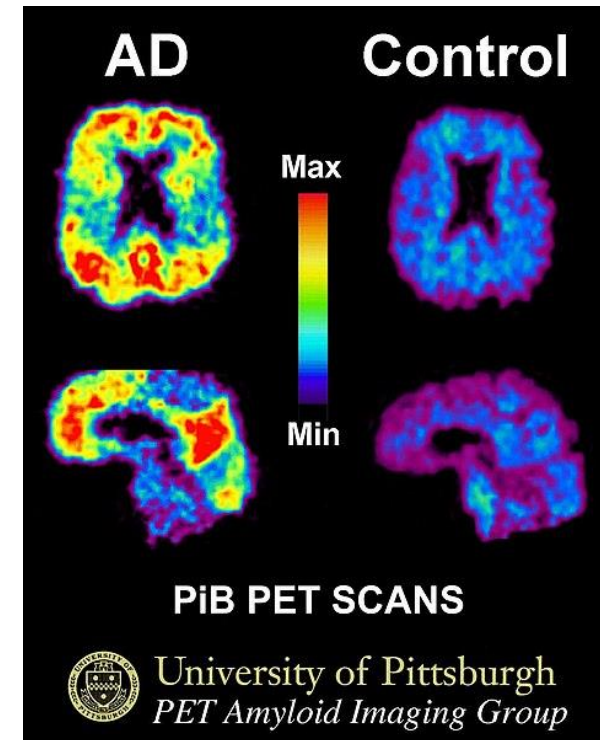


(a)
[Piersanti et al. CMAME 373 (2011)]

Positron Emission Tomography (PET)

A positron-emitting agent is injected in bloodstream. Radioactivity is affected by biochemical processes: it can identify functional anomalies:

- ⇒ used to detect tumors and amyloid deposits
- ⇒ limited use due to radiations



Wikimedia Commons - lic. CC-BY-SA 3.0 Klunkwe (2008)

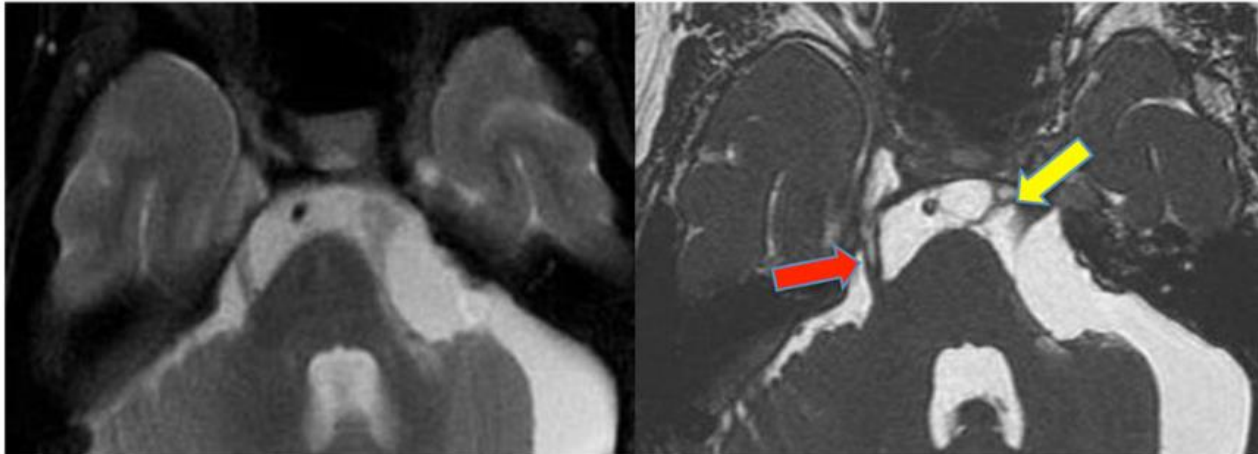
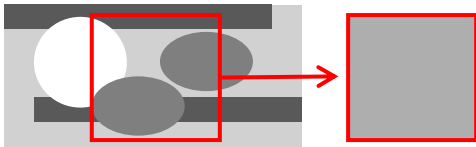


Possible issues - MRI

Partial Volume Effect (PVE)

More than one tissue type is comprised in a voxel

⇒ voxel intensity = (weighted) average



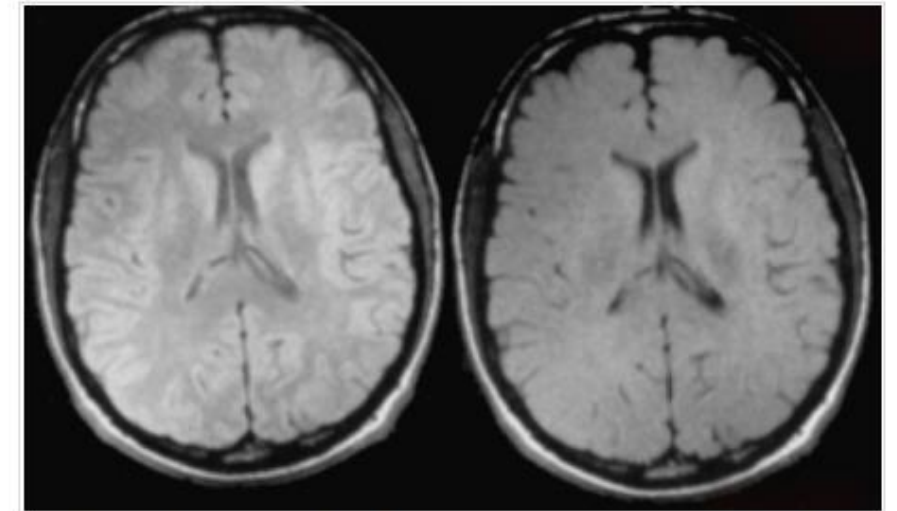
Partial volume averaging. In 5-mm-thick section (left) delicate nerves and scar in the subarachnoid space cannot be resolved as their signals are mixed/averaged with CSF and other tissues. Thin-section (1-mm-thick) image (right) displays the detailed anatomy to good advantage.

<https://mriquestions.com/>

Cross-talk artifacts

Due to slab **overlap** (actual or due to no-gap)

⇒ contrast is impaired



Effect of cross-talk on image contrast. On left is a SE 2000/20 image with 50% gap showing expected spin-density contrast. On right the same sequence with 0% gap demonstrating impaired contrast.

Many image-processing tools are available (even integrated in scanning machines) to avoid/correct these and other issues. Yet, being aware of them allows to improve certified accuracy of the solutions.

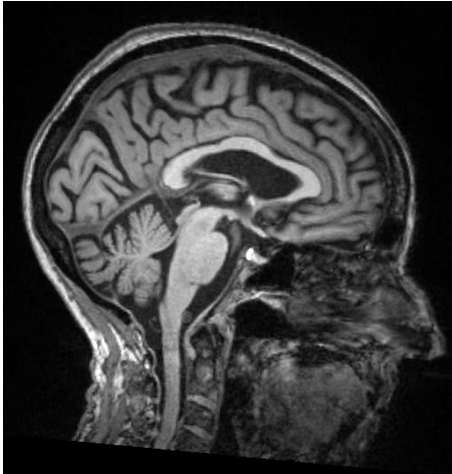
2 –From images to computational mesh: a reconstruction pipeline



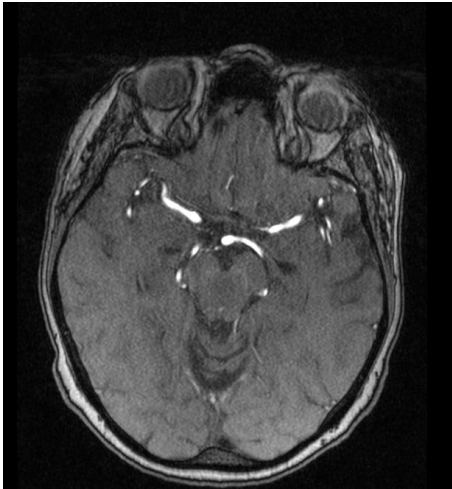
Overall workflow and tools

Imaging data

MRI



angiography



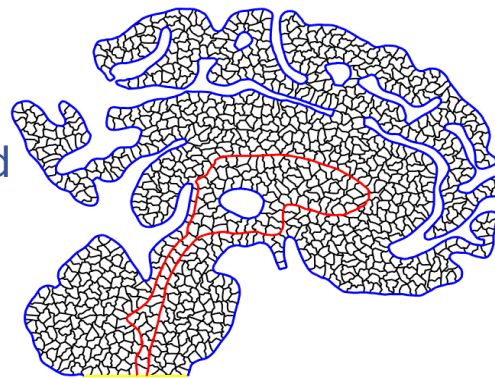
Domain reconstruction



FreeSurfer

<https://surfer.nmr.mgh.harvard.edu/>

Agglomerated
poly grid



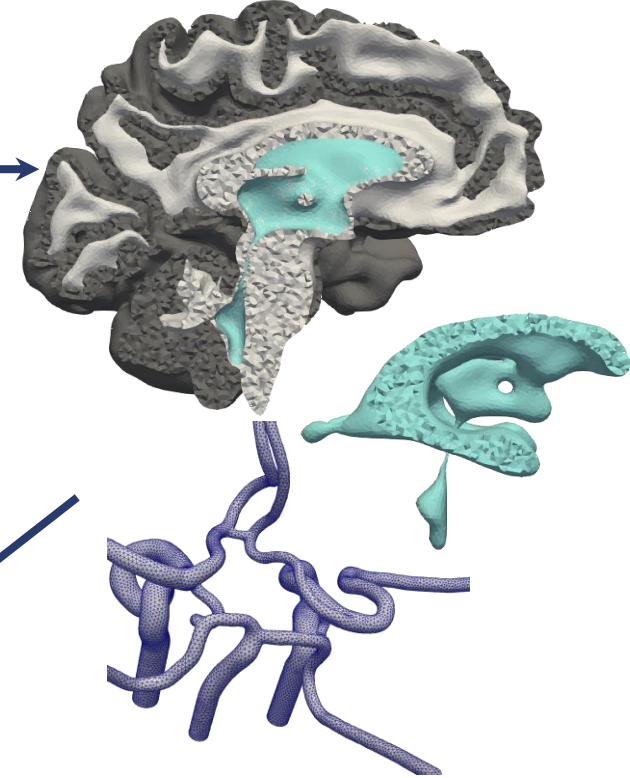
SVMTK

github.com/SVMTK/SVMTK
[Mardal, Rognes, Thompson, Valnes (2022)]

vmtk

vmtk.org
github.com/checkrenzi/vmtk

Mesh generation



ParMETIS

NB: Many other tools
are available!

2.1 – Segmentation of brain MRI



Segmentation techniques

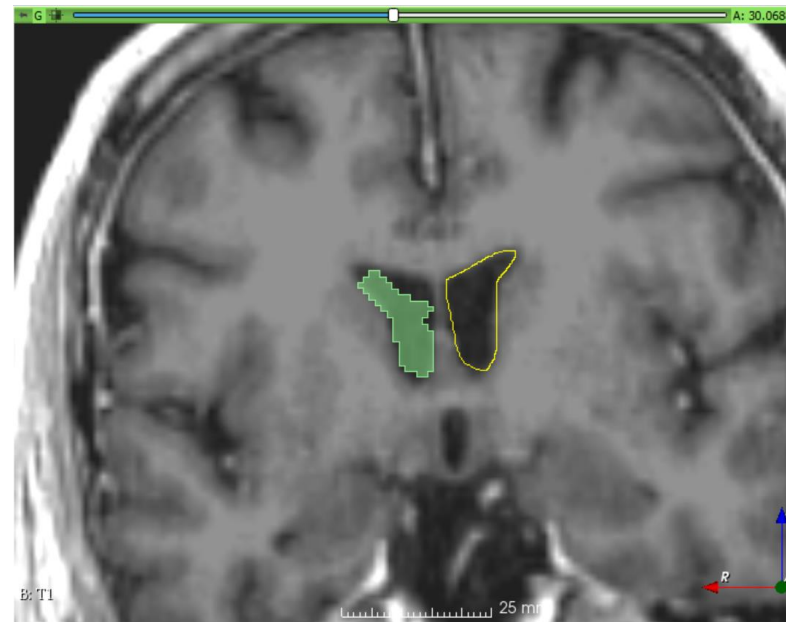
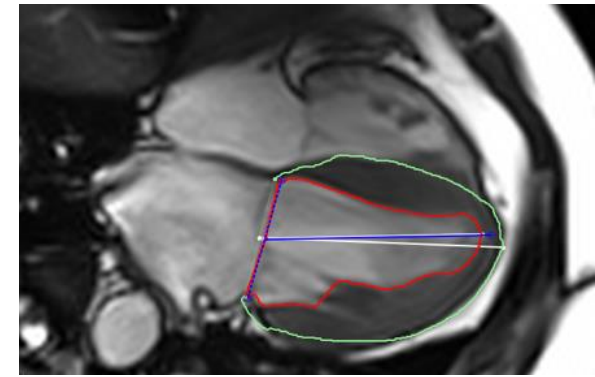
- **Manual segmentation**
- Thresholding / isosurface
- Front propagation
 - Fast marching
 - Colliding fronts
- AI/ML/DL techniques
- Deformable models / registration
 - Level set method

Widely employed in clinical practice for **cardiac** imaging.

Little use for brain images: complex geometry and structures.

Only for:

- geometry sizing (e.g. tumors) of typically pre-processed images
- creating benchmark/**ground truth** templates for other techniques



- Results (can be) accurate
- ✗ **Time** consuming
- ✗ Intra- and inter-patient **variability**
- ✗ Requires **expert** knowledge of the district of interest



Segmentation techniques

- Manual segmentation
- **Thresholding / isosurface**
- Front propagation
 - Fast marching
 - Colliding fronts
- AI/ML/DL techniques
- Deformable models / registration
 - Level set method

Identification of a suitable gray **level** (isosurface) or **range** of values (region thresholding) to identify the region of interest:

- by relying on Hounsfield units (for CT) or other normalized ranges
- via suitable algorithms (e.g. Otsu's method based on intensity histograms)



- ✓ **Fast**
- ✓ Good **initialization** for other techniques
- ✗ No control on shape
- ✗ Not robust w.r.t. patient's characteristics, anatomical features, scan protocol, contrast agent...



Segmentation techniques

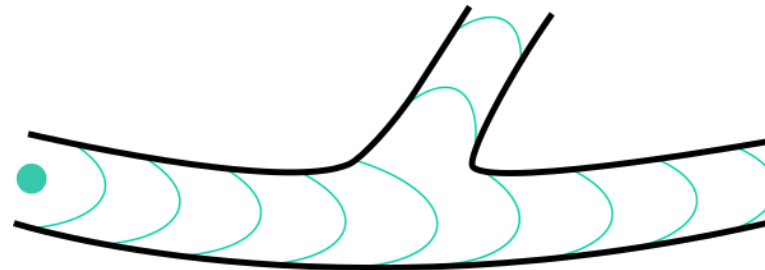
- Manual segmentation
- Thresholding / isosurface
- **Front propagation**
 - **Fast marching**
 - **Colliding fronts**
- AI/ML/DL techniques
- Deformable models / registration
 - Level set method

Starting from an initial condition (e.g. obtained by thresholding), a wavefront is tracked (e.g. solution of a PDE or a graph-based advancing scheme).

Wavefront velocity typically **decreasing with the gradient of the gray level**:

(i.e. slows down close to the boundary of a uniform region)

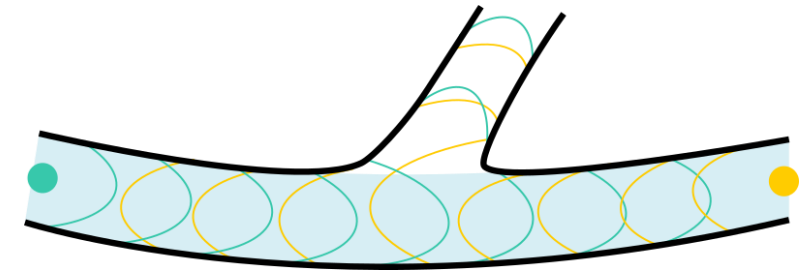
- **Fast marching**: Front propagates from a source, stops when reaching a target



courtesy: L. Bennati

- ✓ Suitable for **round** objects / intricate **vessel network**

- **Colliding fronts**: Front propagates from multiple seeds



courtesy: L. Bennati

- ✓ Suitable for **single vessels**

✗ Not reliable for **complex bulk geometries** (e.g. grey/white matter)



Segmentation techniques

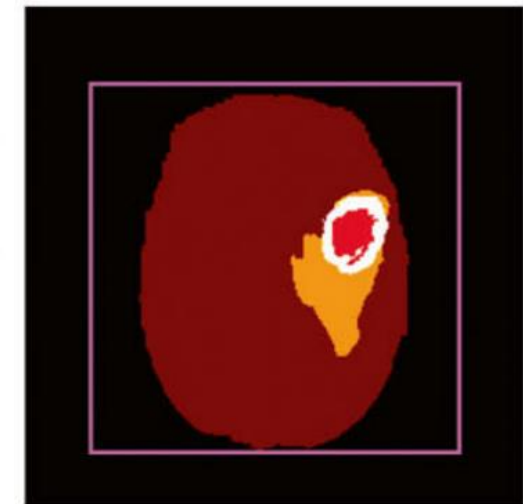
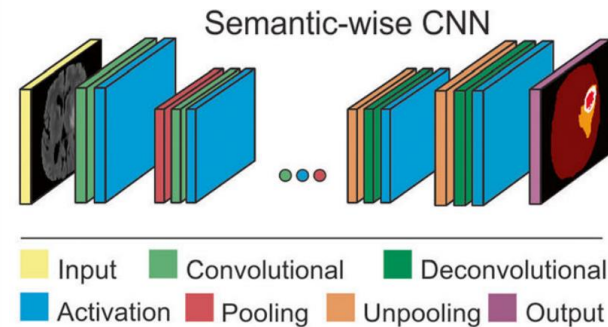
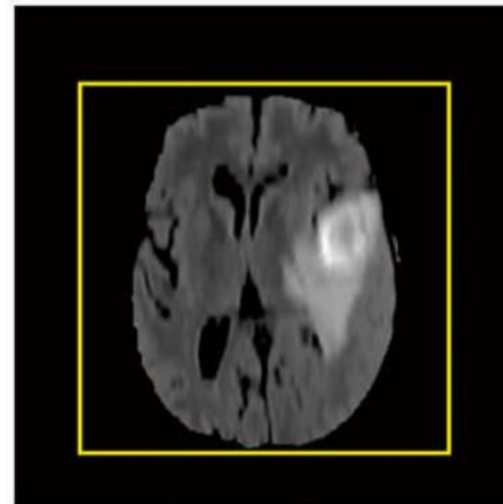
- Manual segmentation
- Thresholding / isosurface
- Front propagation
 - Fast marching
 - Colliding fronts
- **AI/ML/DL techniques**
- Deformable models / registration
 - Level set method

Based on the training of a neural network (or other filtering systems) mapping:

DICOM images



segmentation masks



[Akkus et al. J Digit Imaging 30:449-459 (2017)]

Once trained, the model can automatically detect structures and segment.

- ✗ Training requires **large datasets** and significant time
- ✓ **Pre-trained and verified models available** on the market

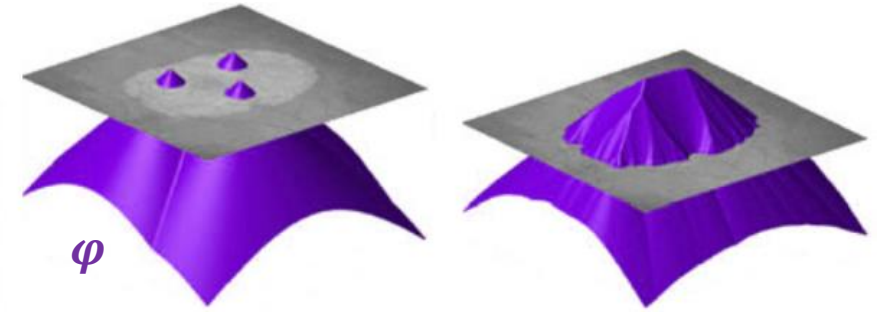


Segmentation techniques

- Manual segmentation
- Thresholding / isosurface
- Front propagation
 - Fast marching
 - Colliding fronts
- AI/ML/DL techniques
- **Deformable models / registration**
 - **Level set method**

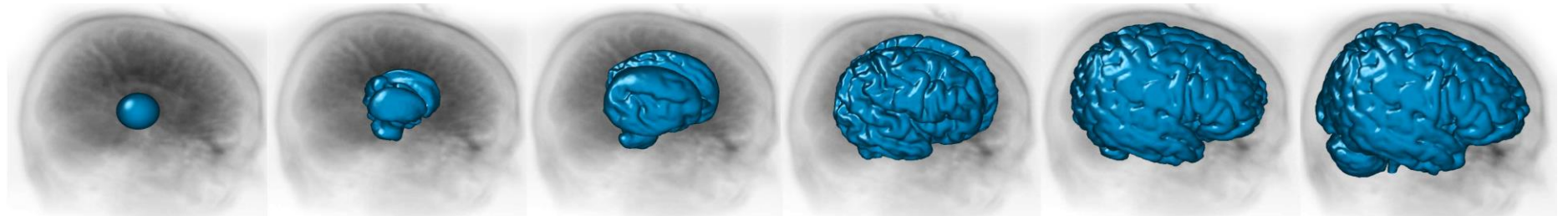
Surface is represented as the **zero-level set** of a function $\varphi : \Sigma = \{x \in \text{image} : \varphi(x) = 0\}$

- ✓ Can use any other technique as initialization
- ✓ Tunable parameters for the trade-off between **smoothness** and **accuracy**
- ✗ Calibration can require multiple attempts, to avoid **over-smoothing**



Given an initialization, the level-set function evolves as result of the balance between:

- internal forces: preserve the **smoothness** of the surface
- external forces: **gray level gradient** (slow down when approaching boundaries)



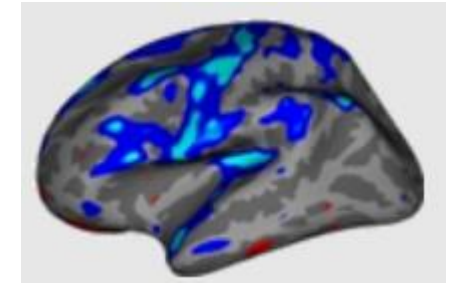


Automatic (pre-)segmentation of brain regions

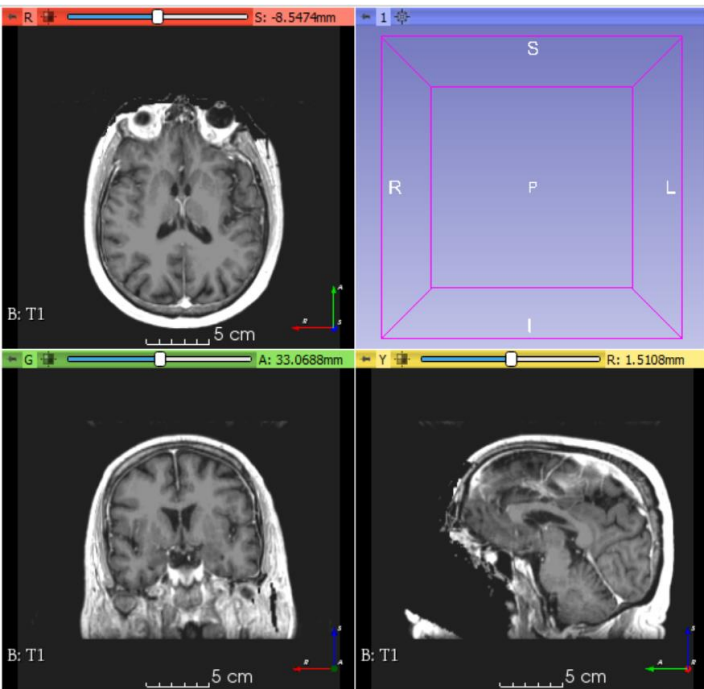
“FreeSurfer is a software package for the **analysis** and **visualization** of structural and functional neuroimaging data from cross-sectional or longitudinal studies. It is developed by the Laboratory for Computational Neuroimaging at the Athinoula A. Martinos Center for Biomedical Imaging. FreeSurfer is the **structural MRI analysis** software of choice for the Human Connectome Project.” [FreeSurfer wiki]

It is released under an open source license.

FreeSurfer



<https://surfer.nmr.mgh.harvard.edu/>



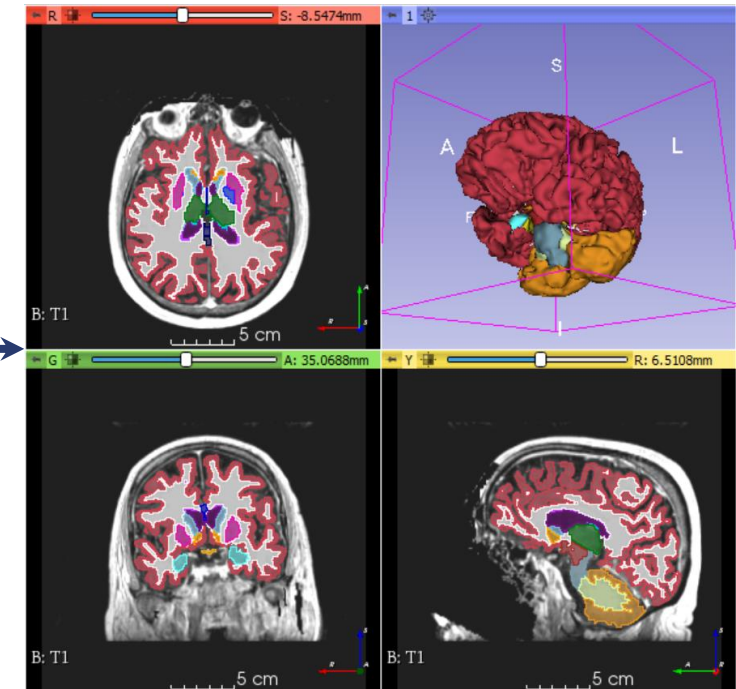
Automatic subcortical segmentation

Automatic procedure that classifies the voxels of a given brain MRI scan in ~40 labels corresponding to regions of the brain. It is based on:

- a probabilistic atlas of structure locations
- anatomically-robust registration

Further details in:

Fischl et al., Neuron, 33:341-355 (2002).



FreeSurfer for automatic region reconstruction

- Given an MRI, we can identify different brain regions automatically with the following command (running takes 12-24 hours)

```
recon-all -subjid out-directory -i T1.mgz -all
```

- The output to this command contains a lot of information and statistics: we are using the tags contained in

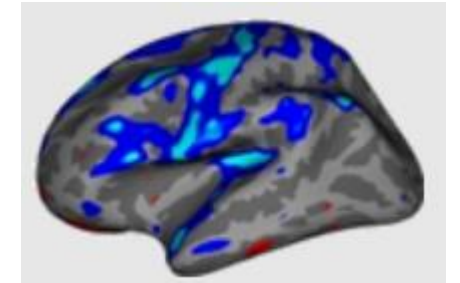
```
out-directory/mri/aseg.mgz
```

(available in the shared folder)

- We can visualize and further process the result in 3DSlicer, enabling the SlicerFreeSurfer extension from 3DSlicer menu View – Extension Manager



FreeSurfer



<https://surfer.nmr.mgh.harvard.edu/>



FreeSurfer for automatic region reconstruction – results

3D Slicer 4.11.20200930

File Edit View Help



Modules:

Segment Editor



3DSlicer

Help & Acknowledgement

Segmentation: aseg

Master volume: T1



Remove



Show 3D



Segmentations...

	Name	
<input checked="" type="checkbox"/>	Left-Cerebral-White-Matter	<input checked="" type="checkbox"/>
<input checked="" type="checkbox"/>	Left-Cerebral-Cortex	<input checked="" type="checkbox"/>
<input checked="" type="checkbox"/>	Left-Lateral-Ventricle	<input checked="" type="checkbox"/>
<input checked="" type="checkbox"/>	Left-Inf-Lat-Vent	<input checked="" type="checkbox"/>
<input checked="" type="checkbox"/>	Left-Cerebellum-White-Matter	<input checked="" type="checkbox"/>
<input checked="" type="checkbox"/>	Left-Cerebellum-Cortex	<input checked="" type="checkbox"/>
<input checked="" type="checkbox"/>	Left-Thalamus	<input checked="" type="checkbox"/>
<input checked="" type="checkbox"/>	Left-Caudate	<input checked="" type="checkbox"/>
<input checked="" type="checkbox"/>	Left-Putamen	<input checked="" type="checkbox"/>

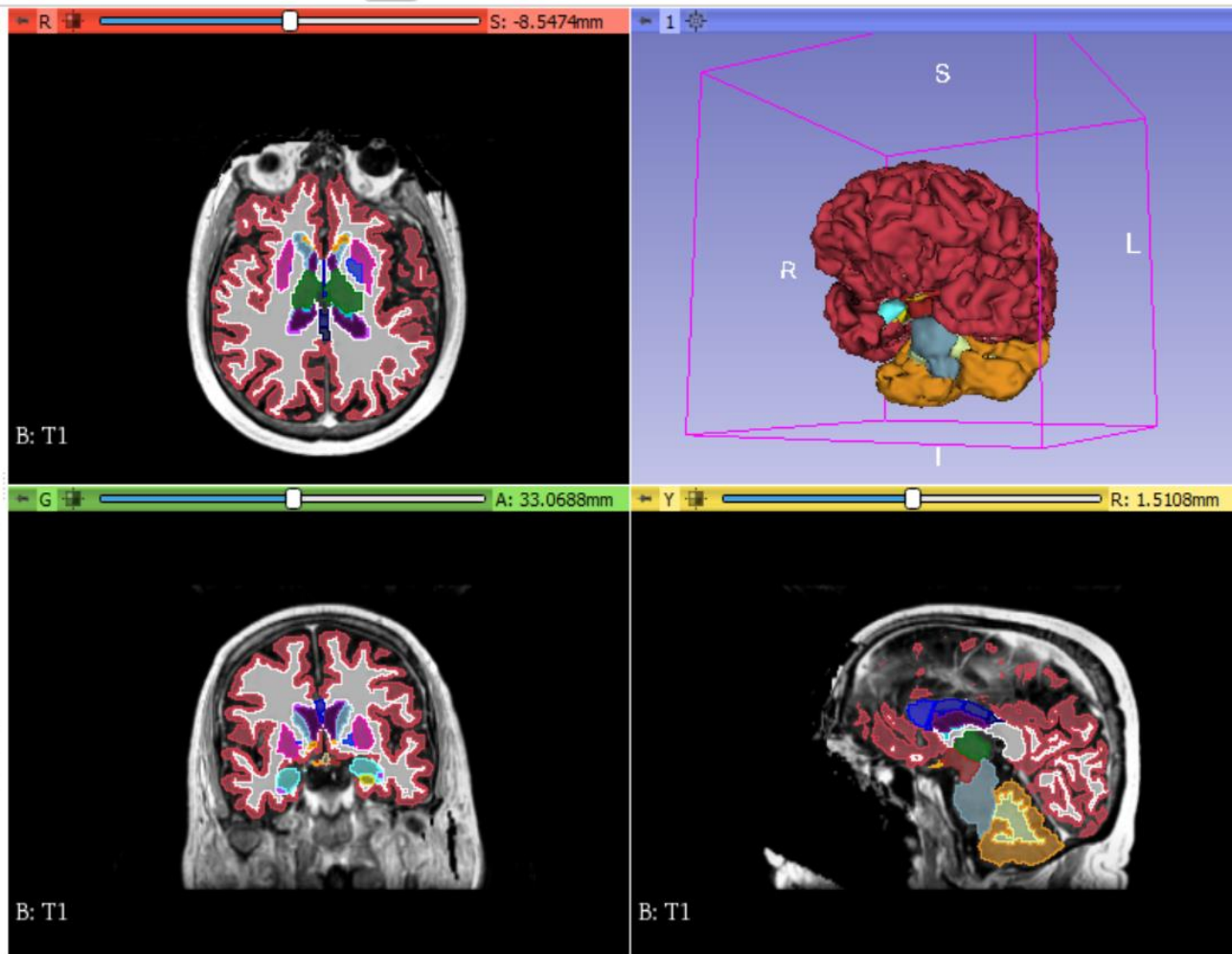
Data Probe

☐ Show Zoomed Slice

L

F

B

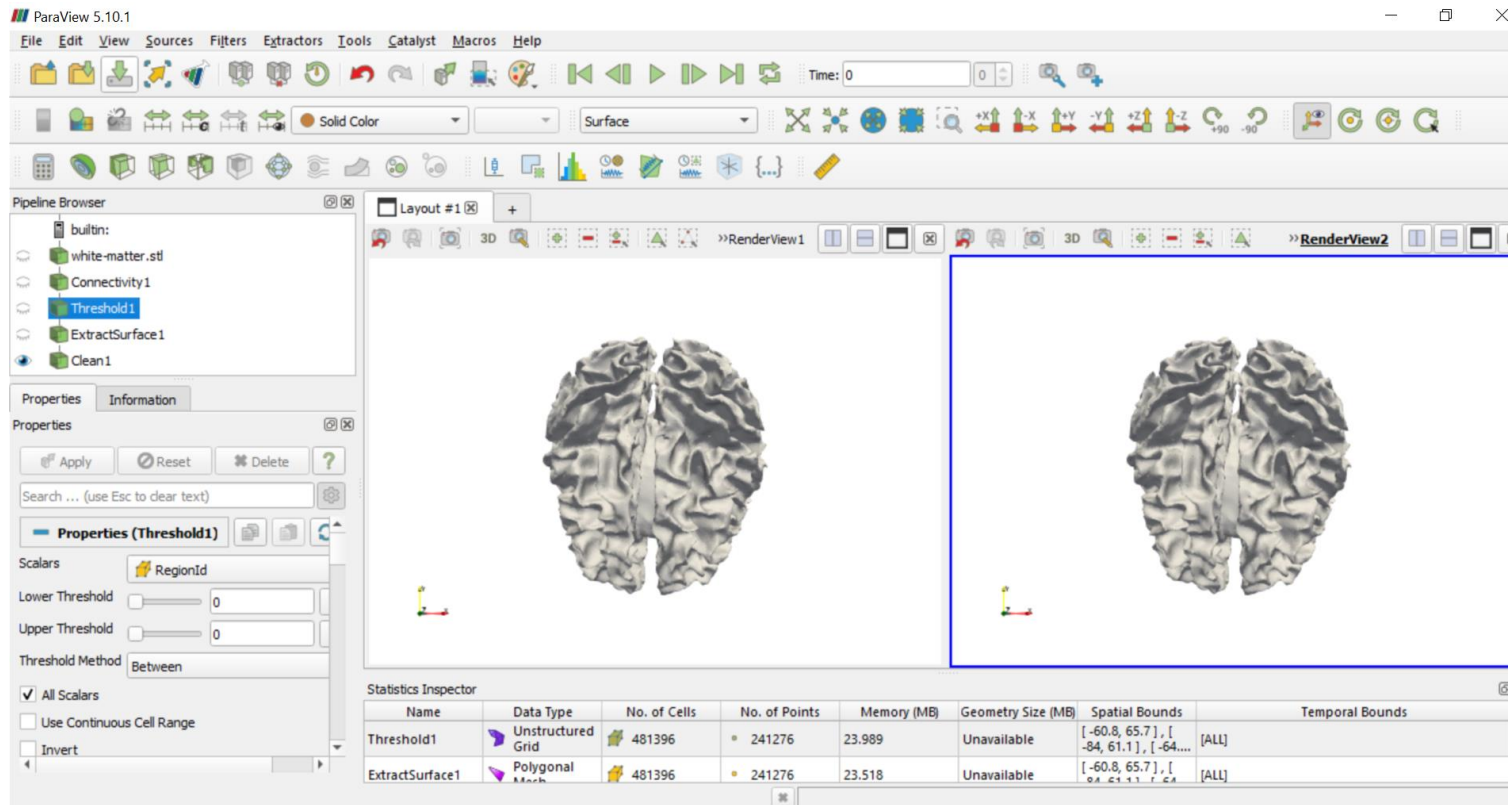


2.2 – Surface processing and mesh generation



From segmentations to a computational mesh

- In 3DSlicer, the content of aseg.mgz is called *Segmentation* and the single labeled regions are *Segments*
- We can combine and modify segments to create surfaces containing the complete grey matter, white matter, and brain ventricles, and then export them as STL surfaces (see shared folder)
- These surfaces can contain defects: a first, rough cleanup can be done in Paraview



Note: the segmentation of the **ventricles** in 3DSlicer requires **significant manual work**. A full ventricle segmentation is provided in the shared folder.



The Surface Volume Meshing Toolkit (SVMTK)

SIMULA SPRINGER BRIEFS ON COMPUTING 10

Kent-André Mardal
Marie E. Rognes
Travis B. Thompson
Lars Magnus Valnes

Mathematical Modeling of the Human Brain From Magnetic Resonance Images to Finite Element Simulation

simula


OPEN ACCESS

 Springer

<https://github.com/SVMTK/SVMTK>

SVMTK is a Python library for **surface processing and mesh generation** based on the [Computational Geometry Algorithms Library \(CGAL\)](#). It is designed to create volume meshes of soft organic tissue surfaces, like the **human brain**, with the option to **repair unphysical errors** on the surfaces.

Files and formats

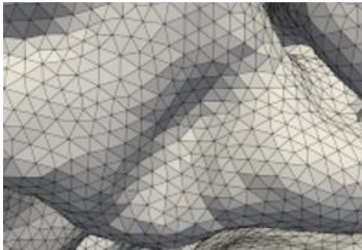
SVMTK	• .stl	broadly employed format for triangulated surface
	• .mesh	(2D/3D mesh in the format used by FEniCS https://fenicsproject.org/ 
ParaView	• .vtp	VTK (not SVMTK) format for triangulated surface
	• .vtu	VTK (not SVMTK) format for tessellated mesh

Format conversion by [meshio](#)

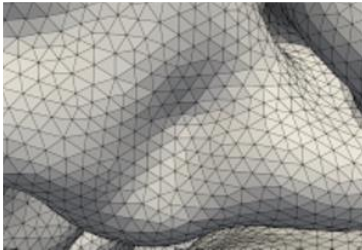


Surface processing – common issues

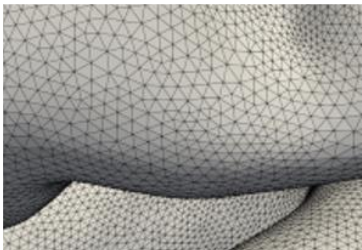
Surface smoothing



original

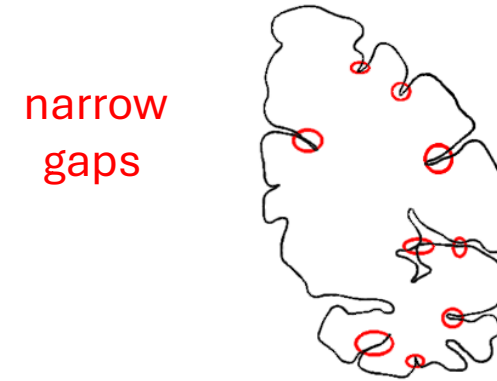


Taubin
smoothing



Laplace
oversmoot'g

Prevent self-intersection



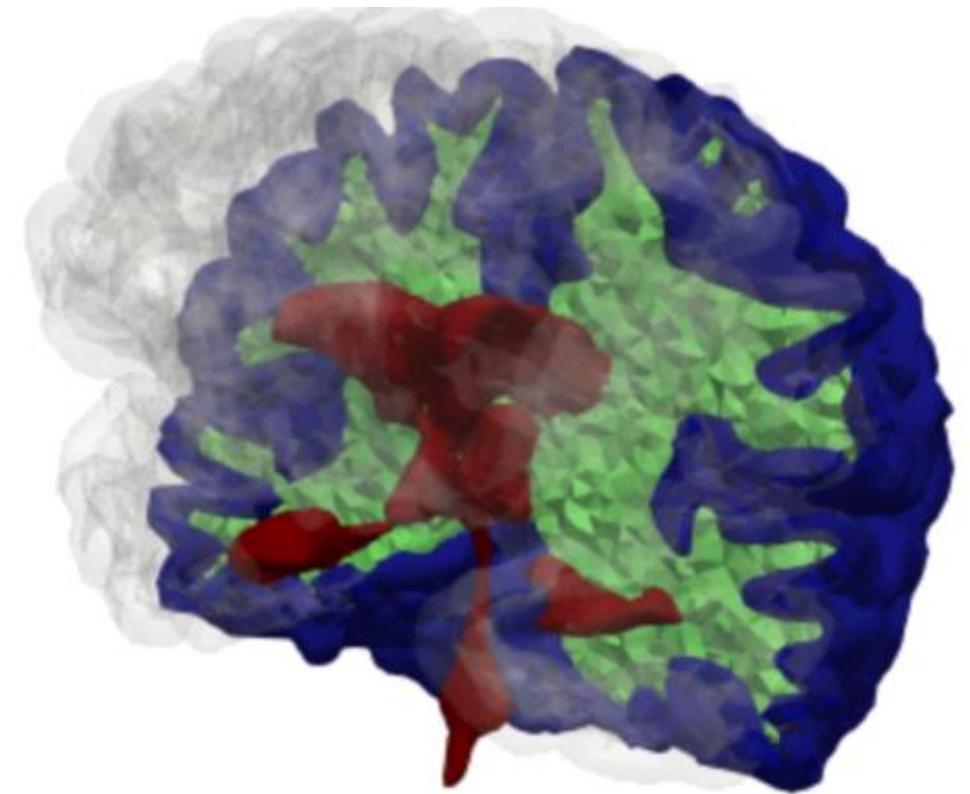
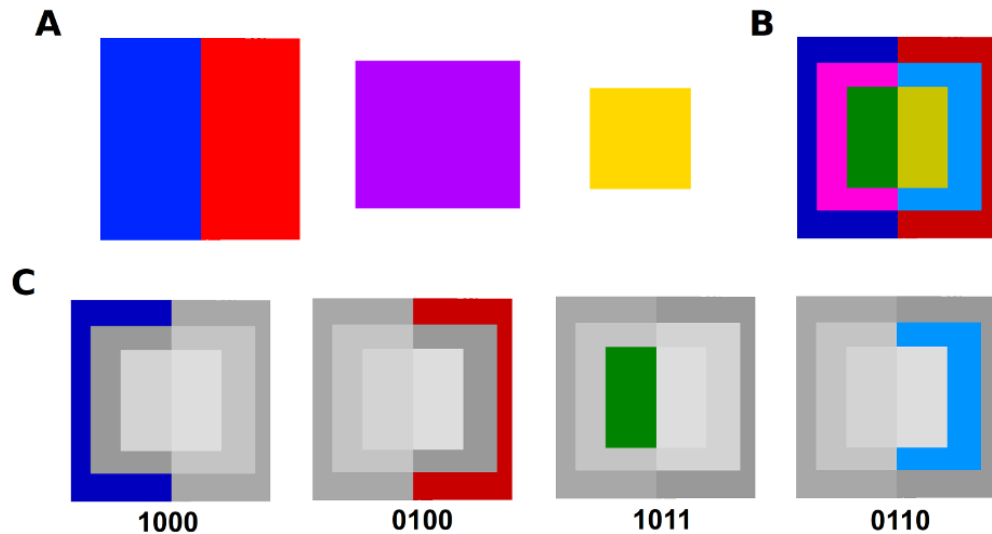
- **fill** holes (useful to remove **internal** artifacts)
- **separate** narrow gaps (useful for **sulci**)



Multi-domain geometry

Combining multiple surfaces

- Surface processing (smoothing/holes) should also account for intersections **across surfaces**
- Volumes are defined as inside (1) or outside (0) a given surface





Computational results: brain perfusion and CSF flow

- displacement and Stokes source terms

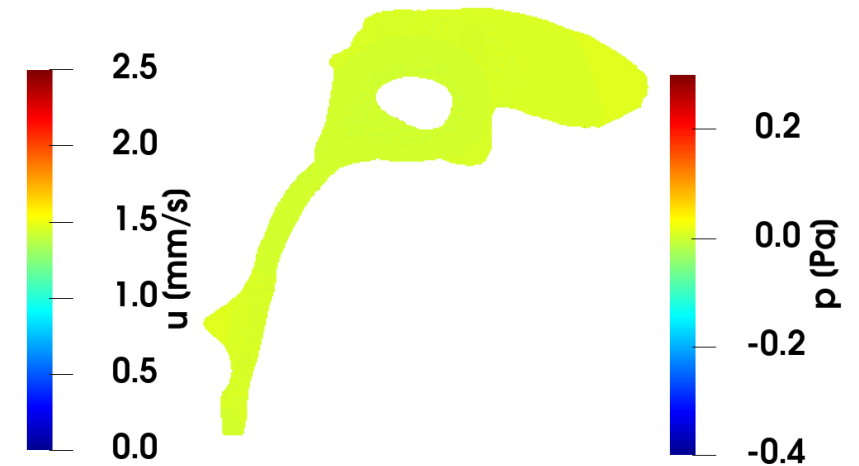
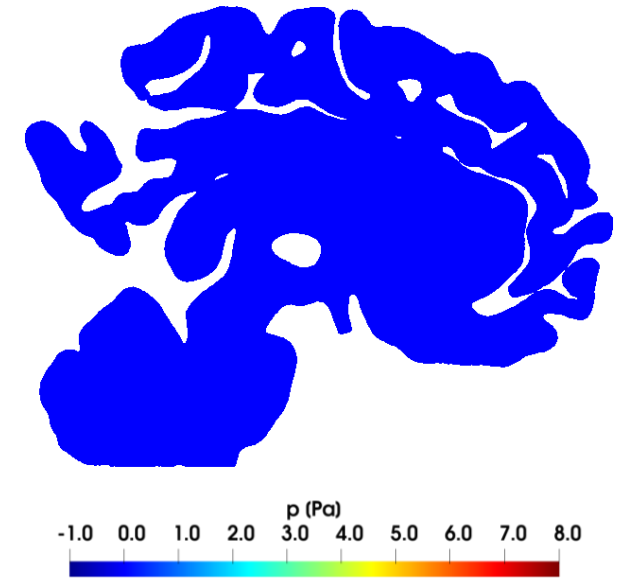
$$\mathbf{f}_{\text{el}} = 0, \mathbf{f}_{\text{f}} = 0$$

- **distributed CSF source:**

(interstitial) CSF pressure source term

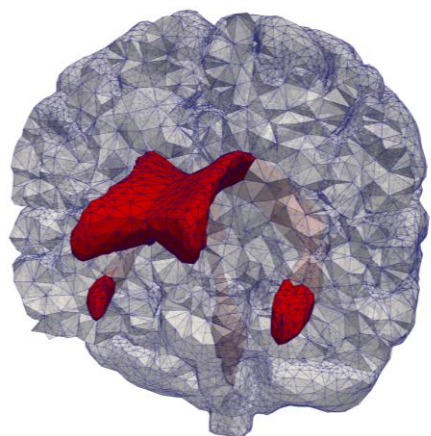
$$\mathbf{g}_{\text{el}} = c_p 2\pi \cdot 10^3 \sin(2\pi t) \text{ [s}^{-1}\text{]}$$

- displacement ~ 0.01 mm
- (-) pressure gradient **towards brain ventricles** until $t \sim 0.7$ s
- **outbound flow**
- \sim **continuous pressure** at interface





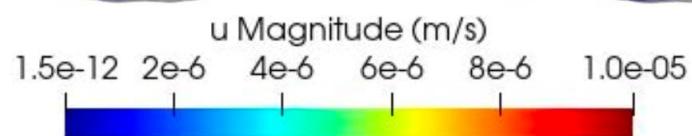
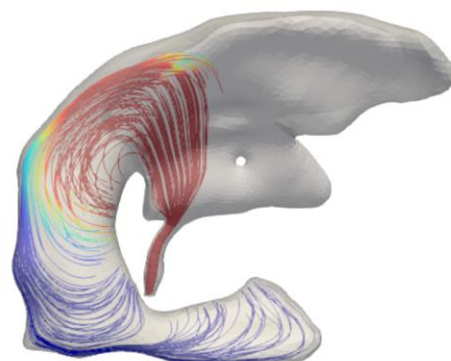
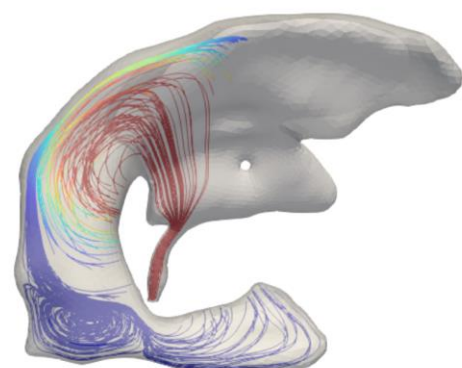
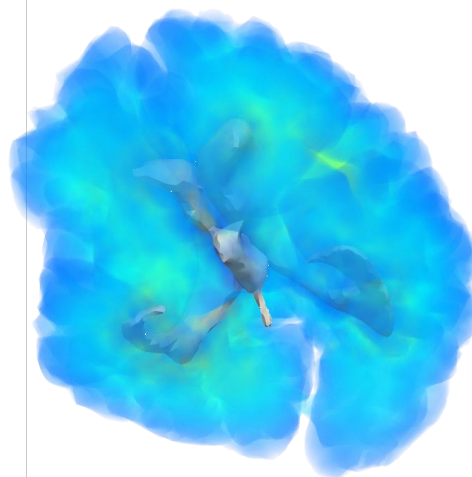
Computational results: brain perfusion and CSF flow



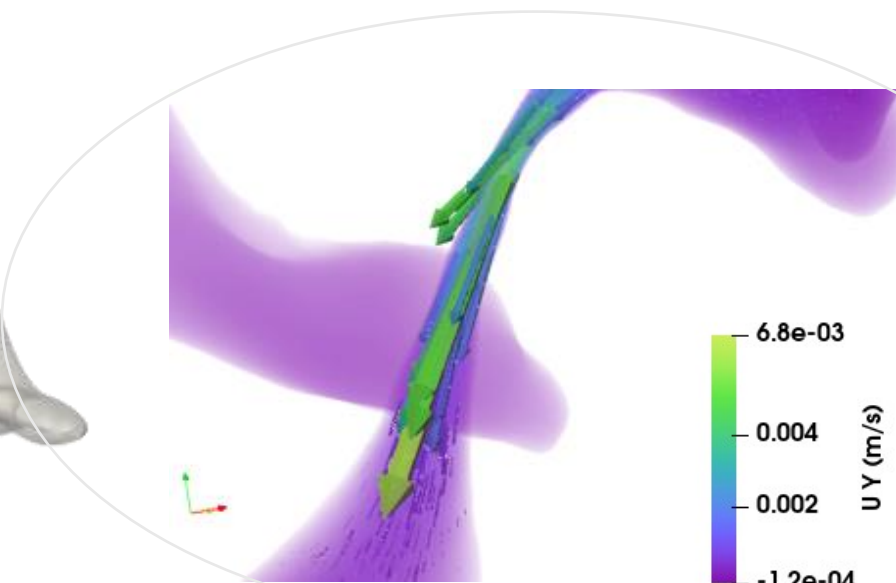
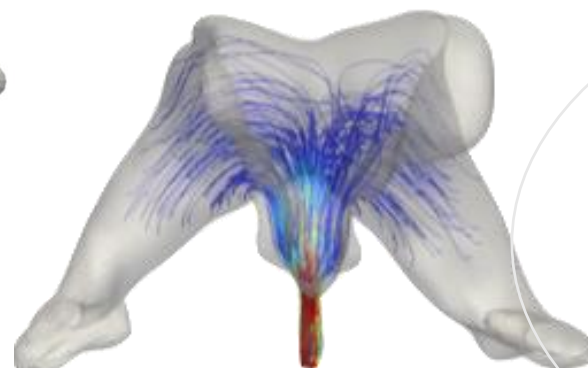
Time: 0.250 s



Time: 0.350 s



Credits: I. De Vittori



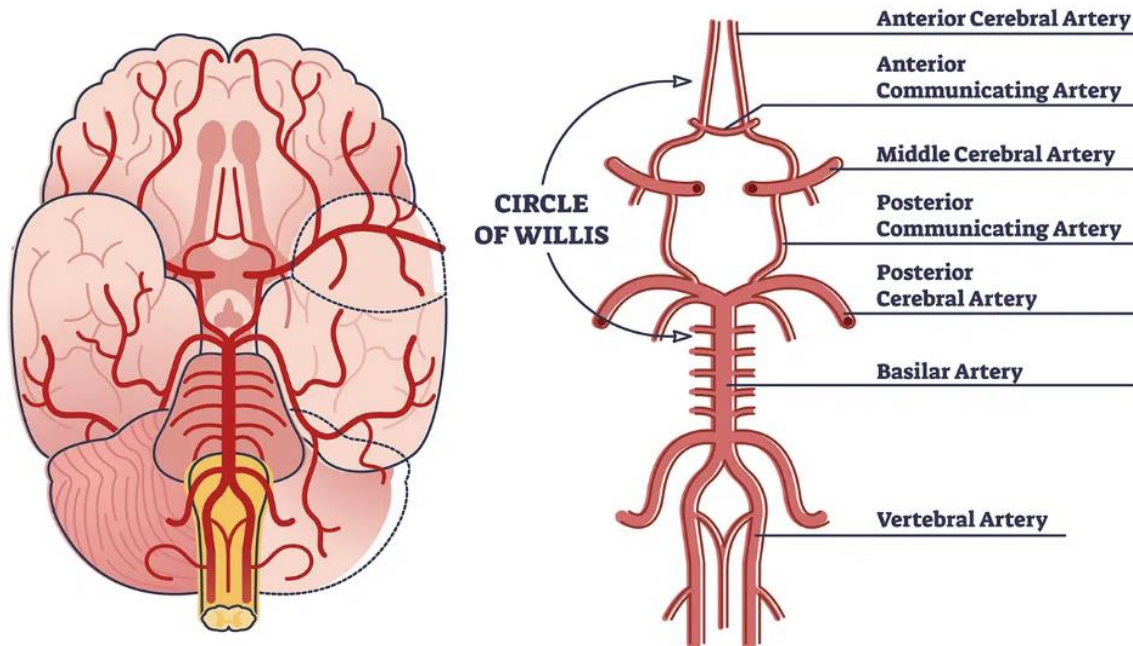
Zoom in the aqueduct of Sylvius. Credits: E. Iràli

3 – Computational modelling of brain hemodynamics



Blood flow in the brain

- Blood flow supplied by (left/right) **carotid and basilar arteries**
- Medium-sized vessels: **Circle of Willis** (CoW)
 - Arterial **network** at the base of the brain
 - Circular structure: **redundancy** to preserve perfusion despite vessel **occlusion**
 - Incomplete CoW in >50% adults (also in the physiological case)



Benchmark geometry (\varnothing 0.8-2.0 mm)

[Figueroa (2020) <https://doi.org/10.7302/xx1r-zg70>]



Blood flow in the brain – 1D

[R.Petrucci, MSc Math.Eng., PoliMI (2024)

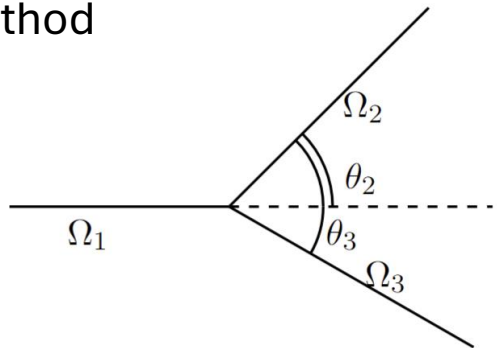
L.Mueller, E.Toro, IJNMBE (2014)

G.Bertaglia, V.Caleffi, L.Pareschi, A.Valiani, JCP (2021)]

1D fluid-structure interaction model:

hyperbolic conservation laws for Q and A (Taylor-Galerkin disc.)

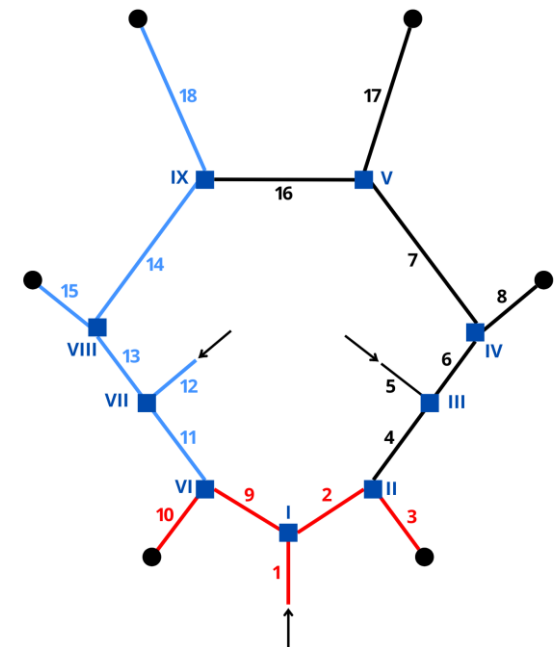
- assumptions: **linear elastic** walls, power-law profile
- discretization: 2nd-order **Taylor-Galerkin**
- BC: physiological Q (inflow), P (outflow) + compat. conditions
- Branching**: nonlinear system (treated semi-implicitly, solved with Newton method)



$$Q_1 = Q_2 + Q_3$$

$$P_{tot,1} = P_{tot,i} + \gamma_i \frac{Q_i^3}{Q_i A_i^2} \sqrt{2(1 - \cos \theta_i)}, \quad i = 2, 3$$

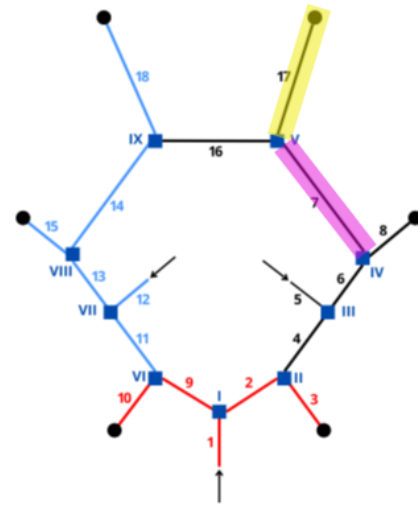
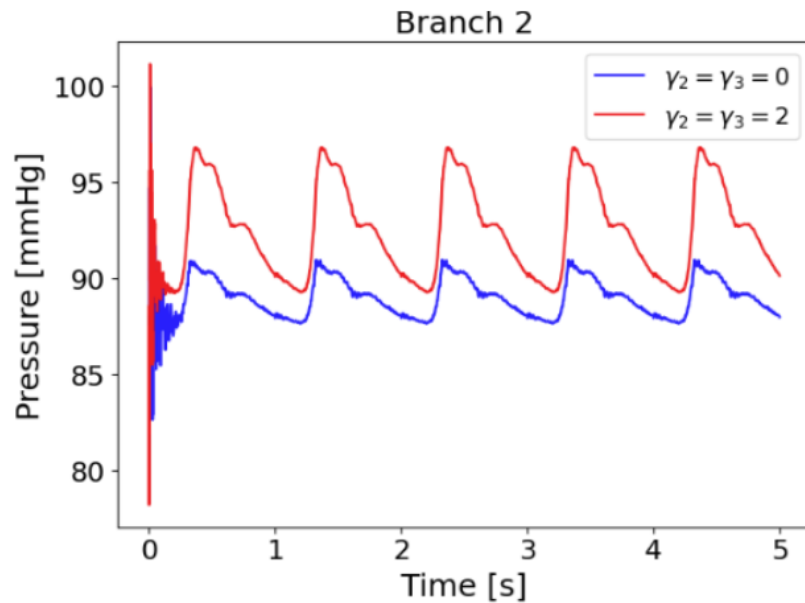
$$\begin{cases} \partial_t A + \partial_z Q = 0, \\ \partial_t Q + \alpha \partial_z \left(\frac{Q^2}{A} \right) + \frac{A}{\rho_f} \partial_z P + K_r \frac{Q}{A} = 0 \\ P - P_{\text{ref}} = \beta \frac{\sqrt{A} - \sqrt{A_0}}{A_0} \end{cases}$$





Blood flow in the brain – 1D – (preliminary) results

Modeling: dissipative effects of bifurcation angles

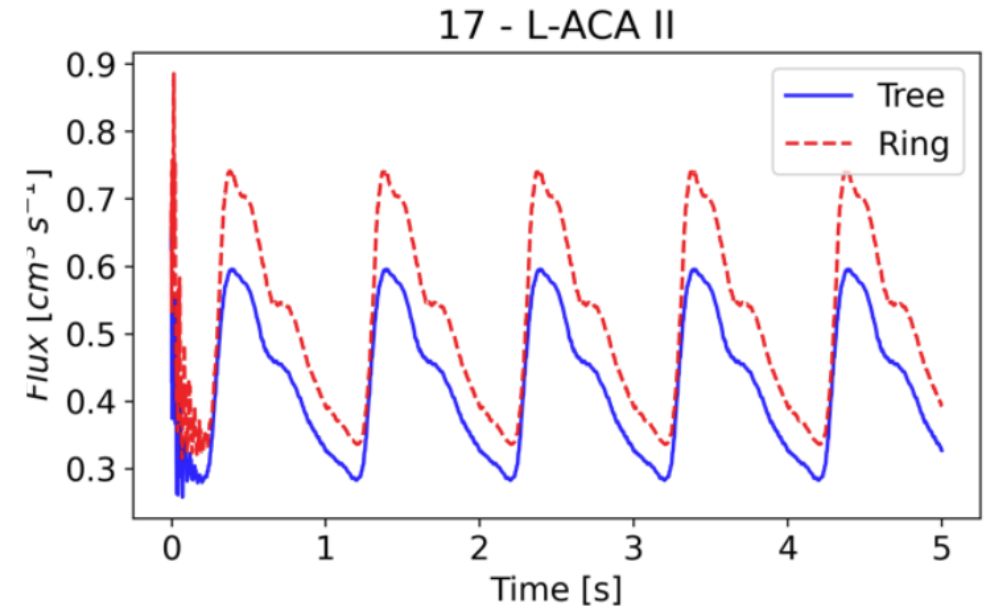


- including angles: reduced amplitude + better agreement with experimental measures [DeVault et al (2008)]

Open issues

- surrounding cerebral tissue (viscoelastic effects)
- improve numerical method (order of accuracy, dispersion)
- calibration of branching model

Application: effects of arterial obstruction

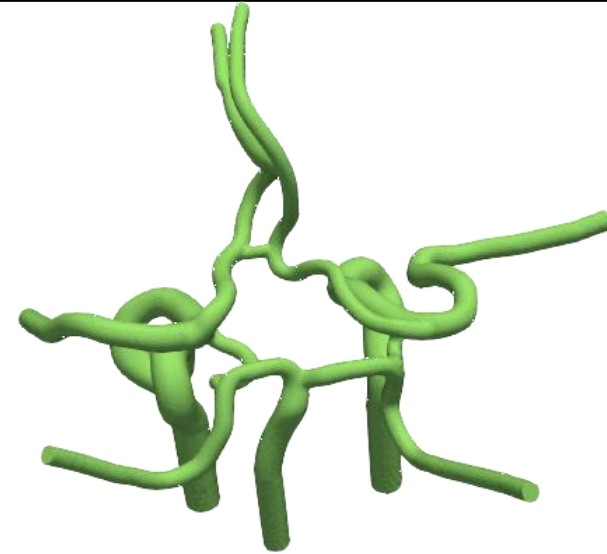


- incomplete CoW: >50% adults
- increased risk of neurological events
- reduced perfusion

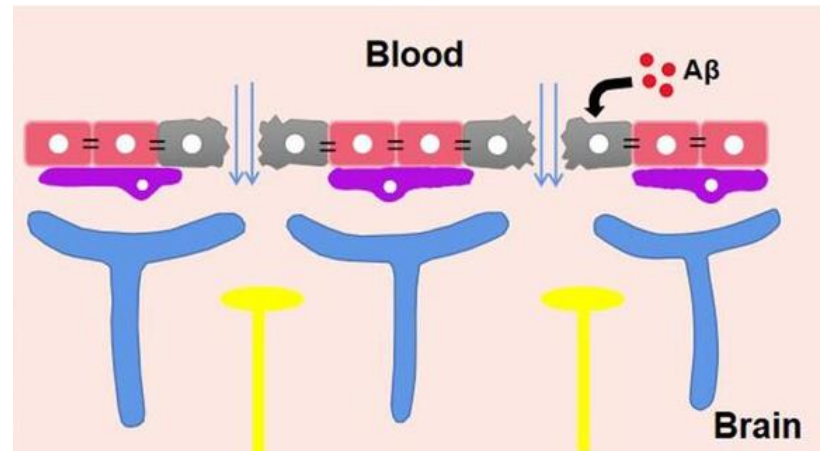


Blood flow in the brain – 3D

- 1D model is computationally inexpensive, but makes assumptions on
 - velocity profile
 - branching flowrate/pressure repartition
- **3D model** needed when **detailed flow** is of interest, e.g. for
 - cerebral aneurysm
 - amyloid plaques on vessel walls



[Figuerola (2020) <https://doi.org/10.7302/xx1r-zg70>]





Vessel segmentation from clinical imaging data

1. Marching cubes algorithm (level-set front propagation): extracts the **surface polygonal mesh** from the isosurface at value zero of the *level set* image.



vmtk.org
github.com/checkrenzi/vmtk



1

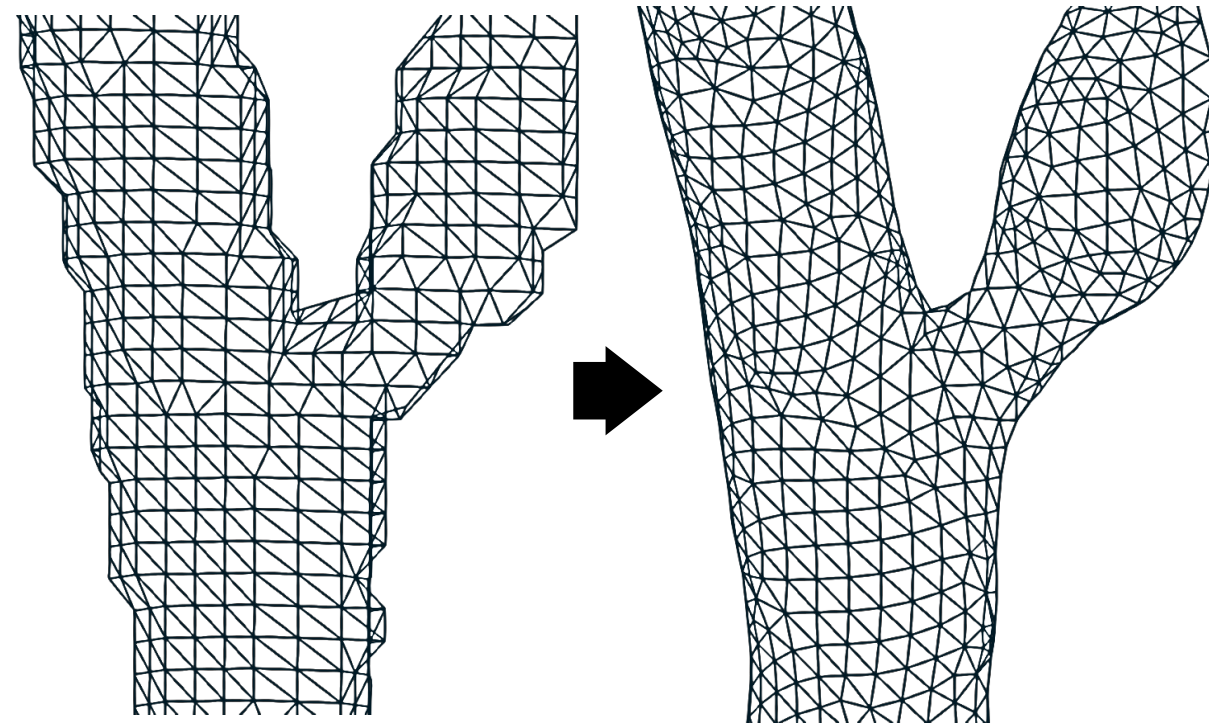
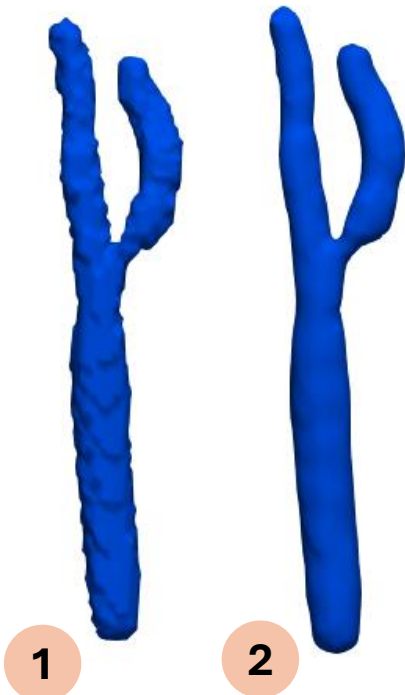


Vessel segmentation from clinical imaging data

1. Marching cubes algorithm (level-set front propagation): extracts the **surface polygonal mesh** from the isosurface at value zero of the *level set* image.
2. Smoothing: surface is artificially rough



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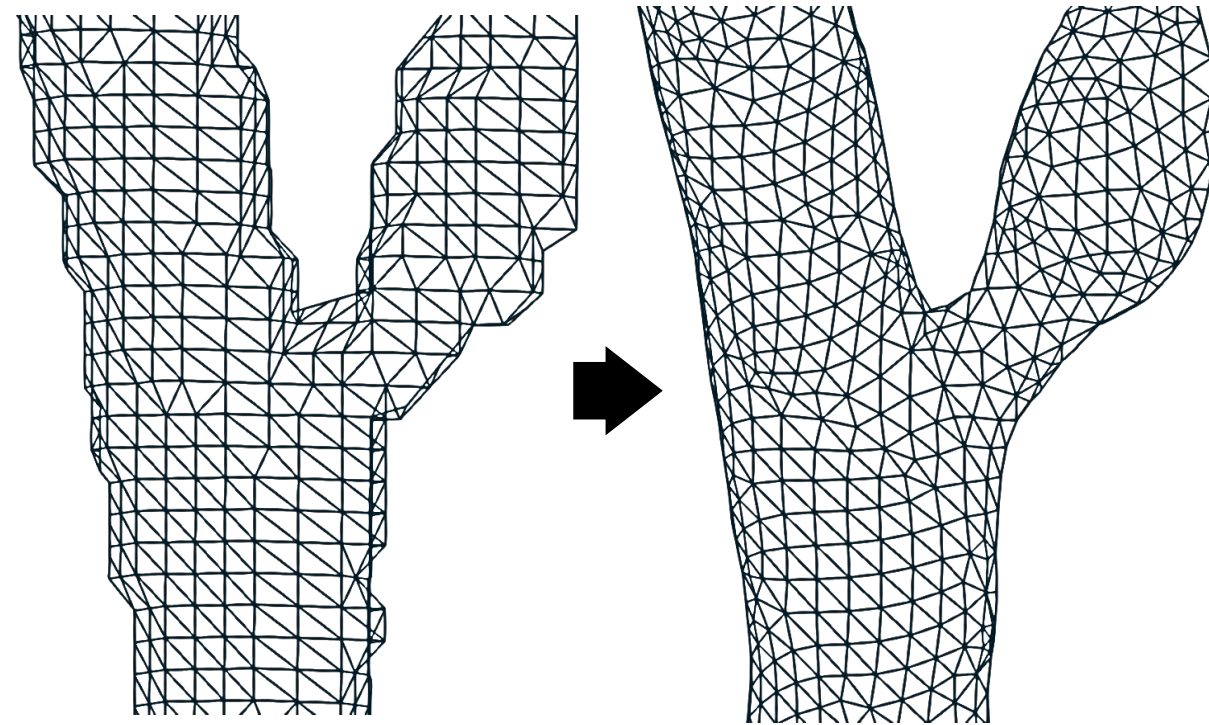
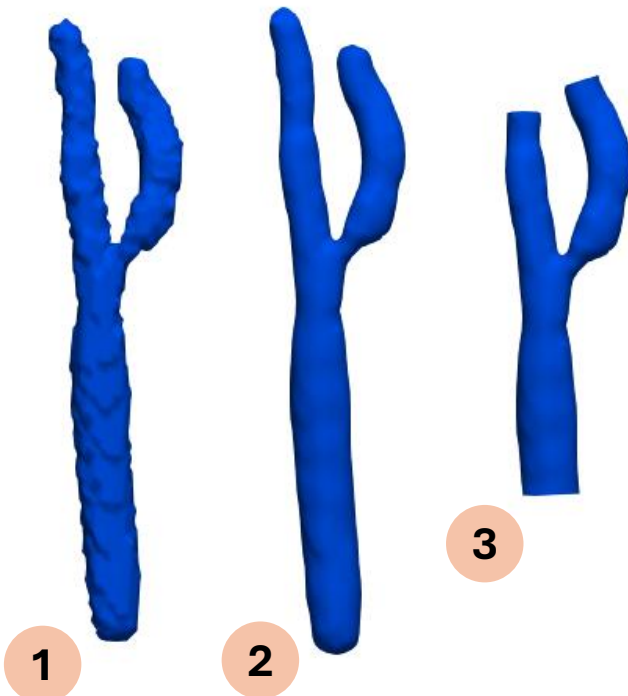


Vessel segmentation from clinical imaging data

1. Marching cubes algorithm (level-set front propagation): extracts the **surface polygonal mesh** from the isosurface at value zero of the *level set* image.
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3. Endpoint clipping



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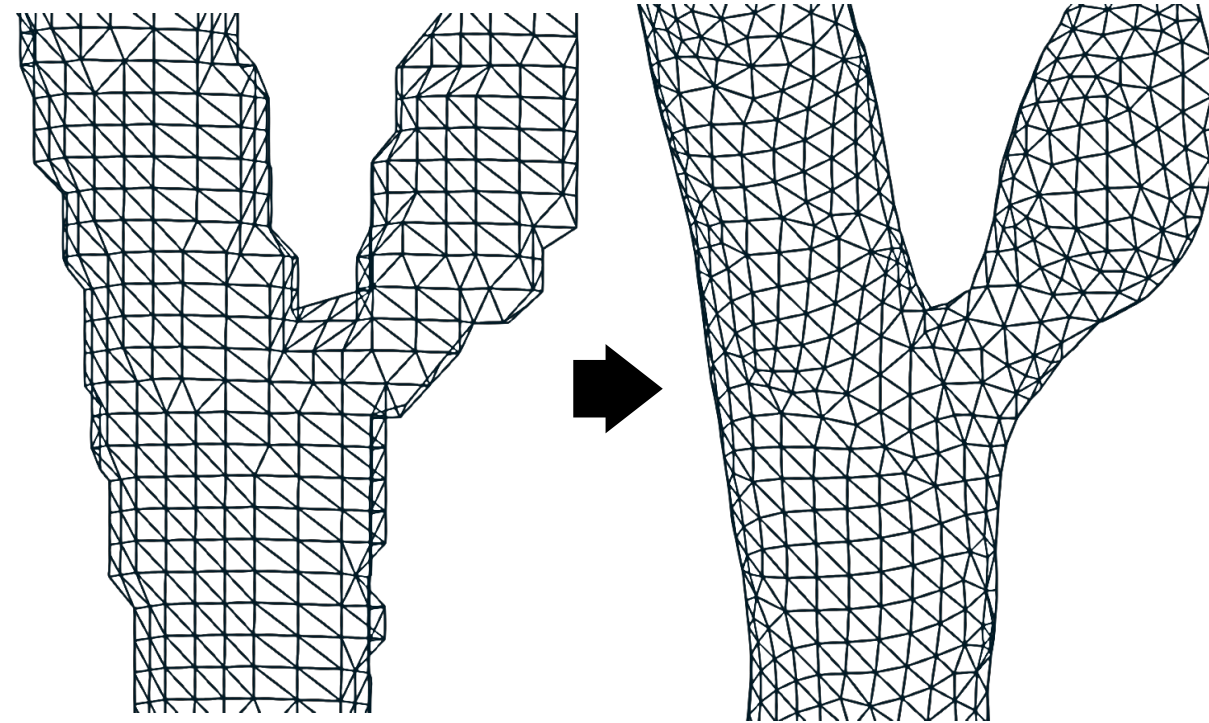
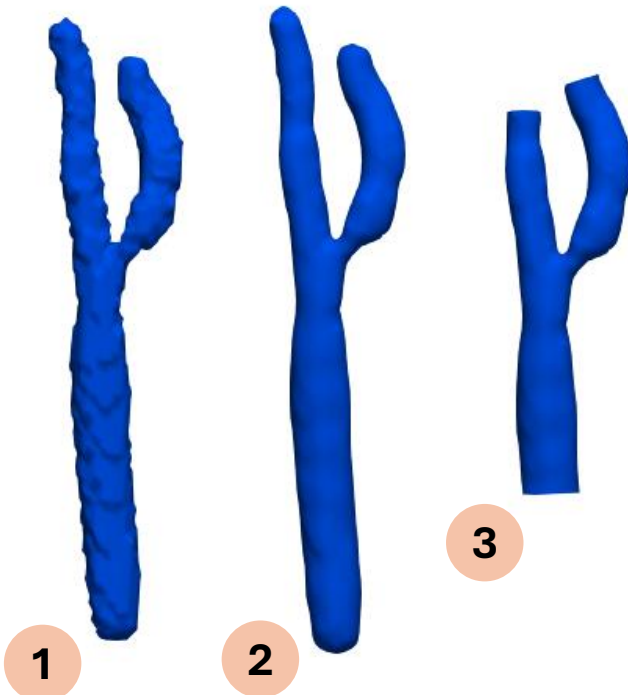


Vessel segmentation from clinical imaging data

1. Marching cubes algorithm (level-set front propagation): extracts the **surface polygonal mesh** from the isosurface at value zero of the *level set* image.
2. Smoothing: surface is artificially rough
3. Endpoint clipping
4. Flow extensions and/or centerlines
5. Surface remeshing



vmtk.org
github.com/checkrenzi/vmtk



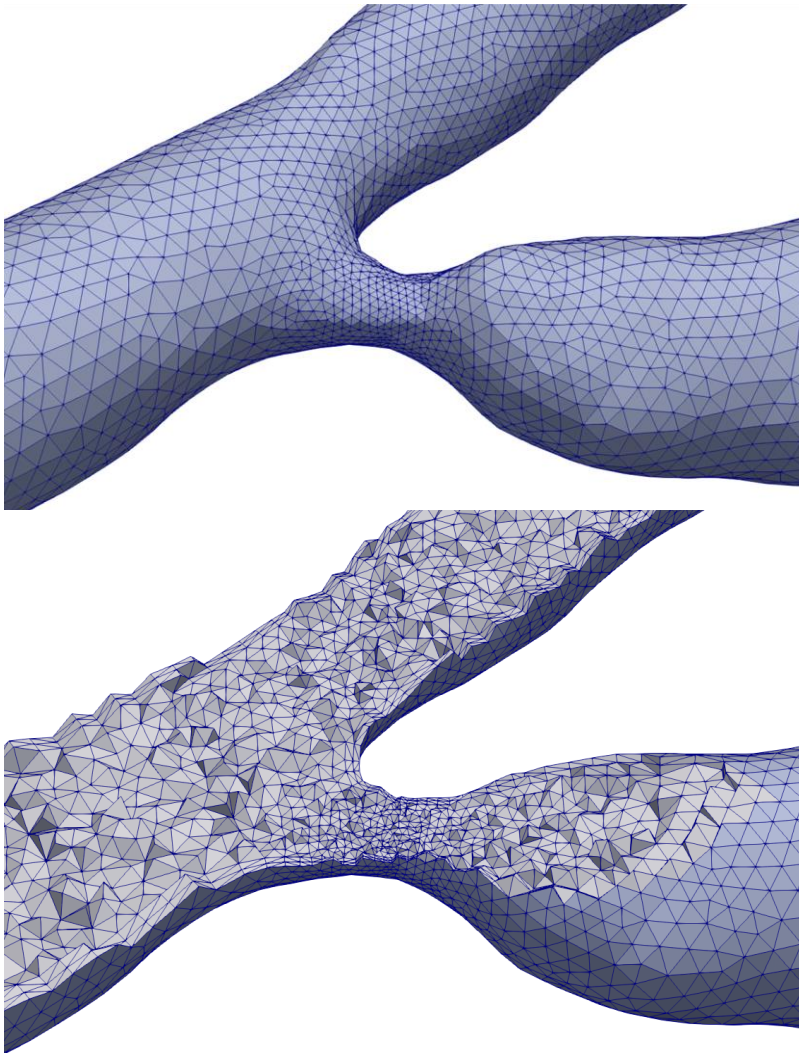


Mesh generation

- Automatic tetrahedralization of triangulated surface
⇒ many tools available on the market (e.g. tetgen)

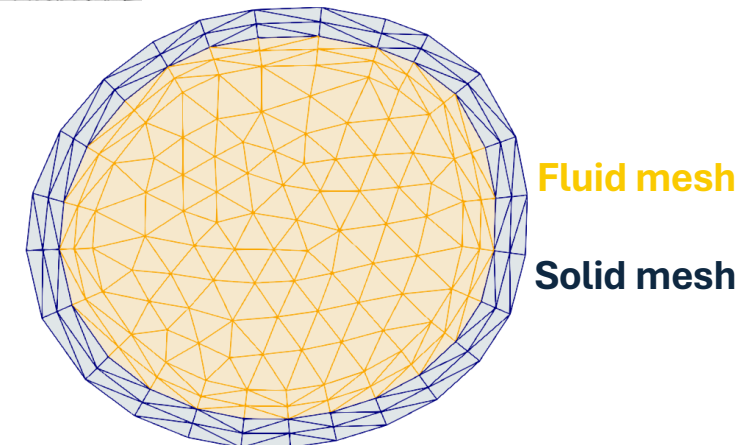
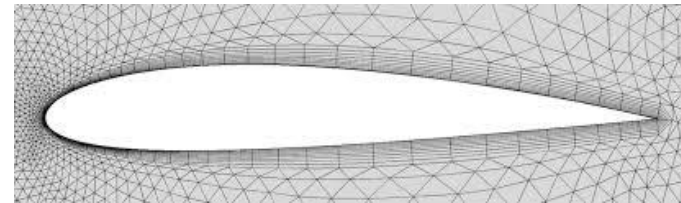


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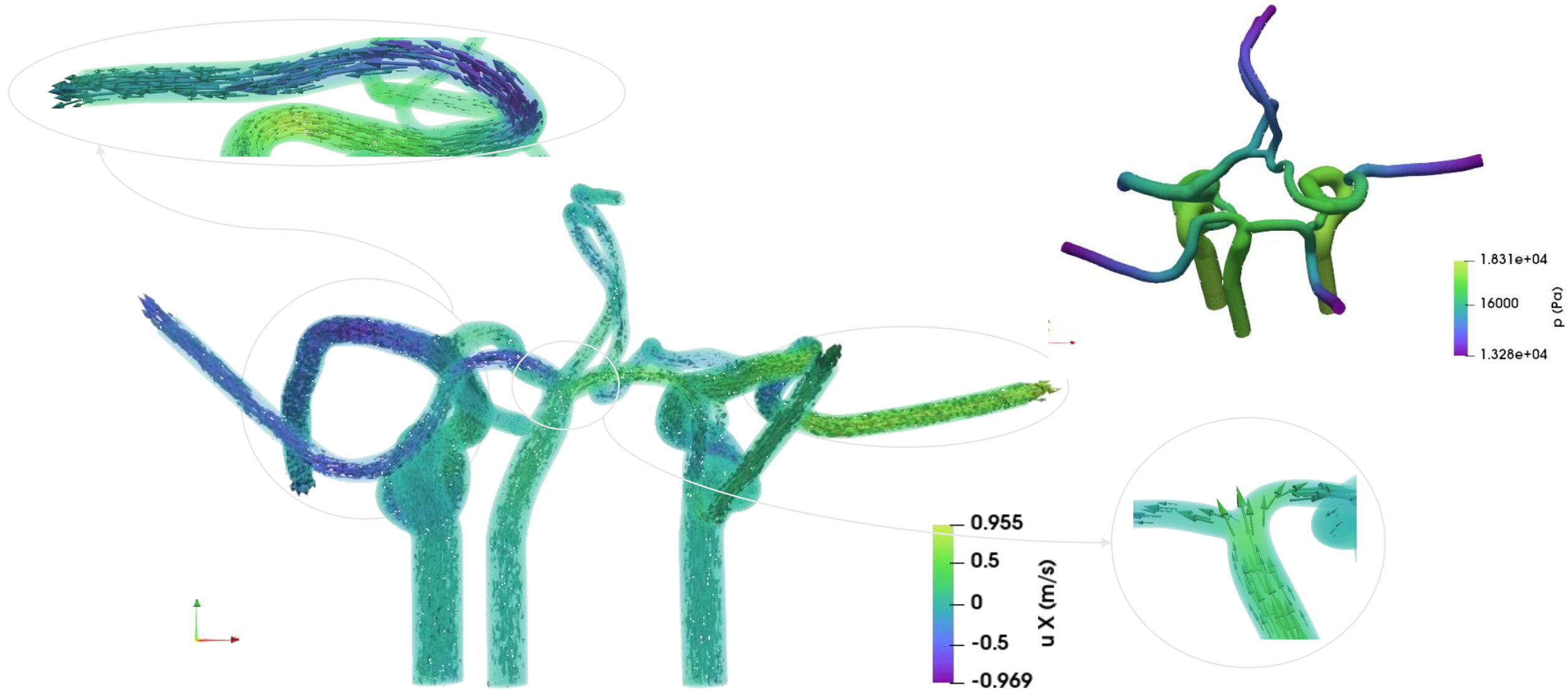
More complex meshing, involving a boundary layer, is required in some applications, e.g.

- CFD with accurate resolution of the flow boundary layer
- Fluid-structure interaction





Blood flow in the brain – 3D – results



Horizontal component of the velocity in the cerebral arteries at $t = 0.005$ s.

Main references

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lymph.bitbucket.io

(open-source)



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Acknowledgments

We thank L. Bennati, M. Corti, E. Irali, R. Petrucci, F. Renzi, I. De Vittori for their contributions to the graphics of this presentation and to the development of the presented tools. We acknowledge the use of open access imaging data by A. Taha et al. [OpenNeuro project ds004471]. This work has been partially funded by the Italian Research Center on High Performance Computing, Big Data and Quantum Computing (ICSC), under the NextGenerationEU project. The present research is part of the activities of "Dipartimento di Eccellenza 2023-2027", Dipartimento di Matematica, Politecnico di Milano. All the authors are members of INdAM-GNCS.