Proposed solution to problem 155, Mathproblems, vol.6 issue 2

Let $a \in \mathbb{R}^*_+$, and let $(L_n)_{n\geq 0}$ be Lucas sequence and a positive real sequence such that $\lim_{n\to\infty} \frac{a_{n+1}}{n^2 a_n} = a$. Find

$$\lim_{n \to \infty} \left(\sqrt[n+1]{\frac{a_{n+1}L_{n+1}}{(2n+1)!!}} - \sqrt[n]{\frac{a_nL_n}{(2n-1)!!}} \right)$$