Proposed solution to problem U686

Let $f: [0,1] \to \mathbf{R}$ be a convex function such that $f(0) = 0$. Prove that

$$\left(\int_0^1 f(x)dx \right)^2 \leq \frac{3}{4} \int_0^1 (f(x))^2 dx$$