Proposed solution of problem U664

For any integer $n \geq 0$ define

$$I_n = \int \frac{x^n e^{\arctan x}}{\sqrt{1+x^2}} dx$$

Prove that

$$(n+1)I_{n+1} + I_n + nI_{n-1} = x^n \sqrt{1+x^2} e^{\arctan x}$$

for all $n > 0$ and then evaluate I_2.