USA; ROY BARBARA, Lebanese University, Fanar, Lebanon; MICHEL BATAILLE, Rouen, France; PRITHWIJIT DE, Homi Bhabha Centre for Science Education, Mumbai, India; PAUL DEIERMANN, Southeast Missouri State University, Cape Girardeau, MO, USA; OLIVER GEUPEL, Brühl, NRW, Germany; KEE-WAI LAU, Hong Kong, China; PAOLO PERFETTI, Dipartimento di Matematica, Università degli studi di Tor Vergata Roma, Rome, Italy; DIGBY SMITH, Mount Royal University, Calgary, AB; ALBERT STADLER, Herrliberg, Switzerland; TITU ZVONARU, Comăneşti, Romania; AN-ANDUUD Problem Solving Group, Ulaanbaatar, Mongolia; and the proposer.

It is easy to determine the situation for other values of k. When $k = 945/16$, then $x = 7/2$ is an additional root of $P(x)$. When $k > 945/16$, then the final factor vanishes for one positive and one negative value of $(2x-7)^2$. The positive value gives rise to two real roots of the final factor. However, these two roots are 0 and 7 when $k = 252$. Thus, $P(x)$ has two roots if and only if $k < 945/16$ or $k = 252$, three roots when $k = 945/16$ and four roots when $k \neq 252$ and $k > 945/16$.

About half of the solvers identified values of k less than 945/16 as yielding two solutions, but only four picked up $k = 252$. Nine solvers employed the substitution $y = x^2 - 7x$ which led to their analyzing the equation $y(y^2 + 28y + 252 - k) = 0$, while Geupel and Stadler let $u = \frac{x+7}{2}$ and analyzed the equation $(u^2 - 49)(u^4 + 14u^2 + 945 - 16k) = 0$.

3633. [2011 : 171, 173] *Proposed by Razvan Tudoran, Universitatea de Vest din Timisoara, Timisoara, Romania; and Ovidiu Furdui, Campia Turzii, Cluj, Romania.*

Let $g_1(x) = x$ and for natural numbers $n > 1$ define $g_n(x) = x^{g_{n-1}(x)}$. Let $f : (0,1) \to \mathbb{R}$ be the function defined by $f(x) = g_n(x)$, where $n = \left\lfloor \frac{1}{x} \right\rfloor$. For example, $f\left(\frac{1}{3}\right) = \frac{1}{3}^{\frac{1}{3}^{\frac{1}{3}}}$. Here $\lfloor a \rfloor$ denotes the floor of a. Determine $\lim\limits_{x\to 0^+} f(x)$ or prove it does not exist.

[*Ed.: Note when the problem was published, Razvan Turdoran's name was mistakenly omitted from the problem. Our apologies to Razvan.*]

Solution by Paolo Perfetti, Dipartimento di Matematica, Università degli studi di Tor Vergata Roma, Rome, Italy, modified slightly by the editor.

The limit does not exist. We consider $\lim\limits_{x\to 0^+} f(x)$ when $x = \dfrac{1}{2n+1}$ and $x = \dfrac{1}{2n}$ separately where $n \in \mathbb{N}$.

In the published solution to problem #922 of the *College Mathematics Journal* (Vol. 42, No. 2, March 2011; pp. 152-155) the following results were obtained:

(a) $$\frac{1}{2n+1} < f\left(\frac{1}{2n+1}\right) < \frac{1}{\ln(2n+1)},$$

(b) $$\left(\frac{1}{2n}\right)^{\frac{1}{\ln(2n)}} < f\left(\frac{1}{2n}\right) < \left(\frac{1}{2n}\right)^{\frac{1}{2n}}.$$

From (a) $\lim\limits_{n\to\infty} f\left(\dfrac{1}{2n+1}\right) = 0$ follows immediately. Hence, if $\lim\limits_{x\to 0^+} f(x)$ exists, then letting $n = \left\lfloor \dfrac{1}{x} \right\rfloor$ we must have $\lim\limits_{n\to\infty} f\left(\dfrac{1}{2n}\right) = 0$ which is impossible in view of (b) since $\lim_{n\to\infty}\left(\frac{1}{2n}\right)^{\frac{1}{2n}} = 1$ [*Ed: This can be shown easily by using the l'Hôpital's rule.*] and $\left(\frac{1}{2n}\right)^{\frac{1}{\ln(2n)}} = e^{-1} > 0$.

Also solved by RICHARD I. HESS, Rancho Palos Verdes, CA, USA; and the proposers.

The proposers of the current problem are the same as the problem in the College Mathematics Journal. They simply combine the results in (a) and (b) and came to the immediate conclusion about $\lim_{x\to 0^+} f(x)$.

3634. [2011: 171, 174] *Proposed by Michel Bataille, Rouen, France.*

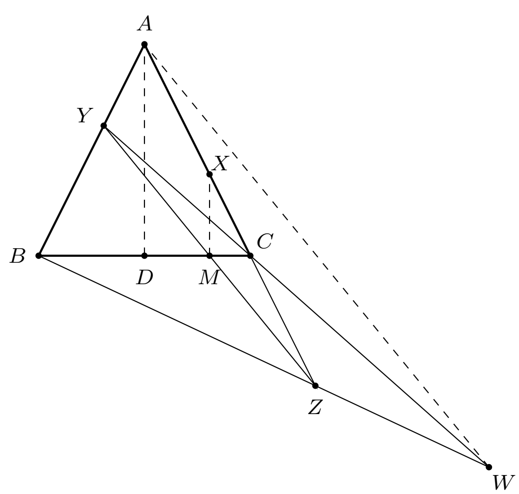
ABC is an isosceles triangle with $AB = AC$. Points X, Y and Z are on rays $\overrightarrow{AC}$, $\overrightarrow{BA}$ and $\overrightarrow{AC}$ respectively with $AZ > AC$ and $AX = BY = CZ$.

(a) Show that the orthogonal projection of X onto BC is the midpoint of YZ.

(b) If BZ and YC intersect in W, show that the triangles CYA and CWZ have the same area.

Composition of solutions by Šefket Arslanagić, University of Sarajevo, Sarajevo, Bosnia and Herzegovina; and Titu Zvonaru, Cománeşti, Romania.

First, we will assume that $X \neq C$, as otherwise the problem becomes trivial. Let D be the midpoint of BC, and let YZ and BC intersect in M.



a) By Menelaus' Theorem applied to $\triangle AYZ$ and the transversal $B - M - C$, we have: $\frac{AB}{BY} \cdot \frac{YM}{MZ} \cdot \frac{ZC}{CA} = 1$ so $\frac{YM}{MZ} = 1$; hence M is the midpoint of YZ. Similarly,