Proposed solution of problem O646

Let x, y, z be non–zero real numbers such that $x + y + z = xyz$. Prove that

$$\left| x + y + z - \frac{1}{x} - \frac{1}{y} - \frac{1}{z} \right| \geq 2\sqrt{3}$$

When does equality hold?