Solution 1, by Oliver Geupel.

First, applying the law of tangents

$$\tan\frac{\angle B-\angle C}{2}=\frac{b-c}{b+c}\cot\frac{\angle A}{2},$$

we find $\frac{\angle B-\angle C}{2}$. Then, adding and subtracting

$$\frac{\angle B-\angle C}{2}\qquad\text{and}\qquad\frac{\angle B+\angle C}{2}=90^\circ-\frac{\angle A}{2},$$

we compute $\angle B$ and $\angle C$.

Solution 2, by Roy Barbara.

To calculate the angle B (angle C can be calculated similarly), let O denote the projection of C onto the line AB. Consider the coordinate system with origin O that contains the triangle ABC with coordinates $A(\alpha,0)$, $B(\beta,0)$, and $C(0,h)$ with $h>0$. Then $h=b\sin A$. From $\alpha=-b\cos A$ we obtain $\beta=\alpha+c=c-b\cos A$. Finally,

$$\cot B=\frac{\beta}{h}=\frac{c-b\cos A}{b\sin A}.$$

3889. *Proposed by Cristinel Mortici.*

Prove that

$$e^\pi>\left(\frac{e^2+\pi^2}{2e}\right)^e.$$

We received ten correct solutions, and a Mathematica verification. We present two solutions which are representative of all solutions.

Solution 1, by Paolo Perfetti.

More generally we prove that for $x\in\mathbb{R}$,

$$e^{|x|}>\left(\frac{e^2+x^2}{2e}\right)^e\quad\Longleftrightarrow\quad|x|\ge e.$$

The inequality is equivalent to

$$|x|\ge e\ln(e^2+x^2)-e\ln 2-e.$$

Let

$$f(x)=|x|-e\ln(e^2+x^2)+e\ln 2+e.$$

The function f is even, ie. $f(x)=f(-x)$, so it suffices to consider $x\ge 0$. Clearly $f(0)=e(\ln 2-1)<0$, $f(e)=0$ and

$$f'(x) = 1 - \frac{2xe}{e^2 + x^2} = \frac{(e - x)^2}{e^2 + x^2} \geq 0.$$

The result follows.

Solution 2, by C.R. Pranesachar.

We shall prove the more general inequality

$$e^{e+x} > \left(\frac{e^2 + (e + x)^2}{2e}\right)^e \qquad (1)$$

for all $x > 0$. The desired inequality follows by substituting $x = \pi - e$, which is positive. To prove (1), we replace x by et, where $t > 0$. Then (1) reduces to

$$e^{e(1+t)} > \left(\frac{2e^2 + 2e^2 t + e^2 t^2}{2e}\right)^e,$$

for $t > 0$, which is equivalent to $e^t > 1 + t + \frac{t^2}{2}$. This is true for $t > 0$, since the right-hand side is the degree two Maclaurin expansion of e^t, and all of the terms in the Maclaurin series for e^t are positive. This proves (1). Note that (1) becomes equality for $x = 0$, and the inequality sign flips for $x < 0$, since for $t < 0$, we have $e^t < 1 + t + \frac{t^2}{2}$, which may be proven by some simple derivative arguments. We are done.

Editor's Comments. The main ideas of Solution 1, i.e. taking a logarithm, defining a function, and taking its derivative to prove the inequality, were utilized by a large majority of the solvers, though the specific function used varied. The Taylor approximation inequality in Solution 2 is also the main step in the proposer's solution, which looks slightly different and is used as the starting point, as opposed to arising at the end of the solution. Paolo Perfetti commented that this problem showed up in *Mathematical Reflections*, 2015–2, as problem U334.

3890*. *Proposed by Šefket Arslanagić.*

Let $\alpha, \beta, \gamma \in \mathbb{R}$. Prove or disprove that

$$|\sin\alpha| + |\sin\beta| + |\sin\gamma| + |\cos(\alpha + \beta + \gamma)| \leq 1 + \frac{3\sqrt{3}}{2}.$$

We received nine correct solutions. The proposed inequality is false in general, which was pointed out by all the solvers who gave various counterexamples as listed below. We also present a correct version with proof.

Solution 1, various counterexamples.

Let S denote the left side of the given inequality.