Prof. Harold Reiter
Dept.of Math. Univ. of North Carolina Charlotte
9021 University City Boulevard, Charlotte NC 28223–0001

Proposed solution of prob. 1211

Let $\sum_{n=1}^{\infty} a_n$ be a convergent series with positive terms. Prove that $\sum_{n=1}^{\infty} \frac{\sqrt{a_n}}{n}$ converges.
Note from the poser: Since this problem is well-known, the challenge is to provide an elementary solution, one that does not invoke the Cauchy-Schwartz Inequality

Proof By $(a-b)^2 \geq 0$ it follows $a^2+b^2 \geq 2ab$ hence

$$0 \leq \sum_{n=1}^{\infty} \frac{\sqrt{a_n}}{n} \leq \frac{1}{2}\sum_{n=1}^{\infty}\left(a_n + \frac{1}{n^2}\right) = \frac{1}{2}\sum_{n=1}^{\infty} a_n + \frac{\pi^2}{12} < \infty$$