

Proposed problem 3557 sept.2010
Crux Mathematicorum with mathematical Mayhem

Let $\{a_k\}_{k=1}^{\infty} > 0$ be a sequence of positive real numbers with $\sum_{k=1}^{\infty} a_k = 1$
and $a_{k+1} \leq a_k/(1 - a_k)$. Let $S_n^{(p)} \doteq \left(\sum_{k=1}^n a_k^p\right)^{1/p}$ and for $p \geq 1$ prove that

$$\lim_{n \rightarrow +\infty} \sum_{k=n+1}^{2n} \frac{k}{n} \left(\prod_{j=1}^n \frac{j^{\frac{1}{p}} a_{k+j} a_j}{S_{k+j}^{(p)}} \right)^{1/n} = 0$$