THE SANDBOX

In this section, we highlight problems that anyone can play with, regardless of mathematical background. But just because these problems are easy to approach doesn't necessarily mean that they are easy to solve!

The first of two Sandbox problems involving inequalities is offered by Larry Lesser of The University of Texas at El Paso. He asks you to first consider a 9-student school consisting of two 2-student classrooms and one 5-student classroom. The mean class size on a *per-class basis* would of course be

$$\frac{2+2+5}{3}=3,$$

while the mean class size on a *per-student basis* would be

$$\frac{(2+2)+(2+2)+(5+5+5+5+5)}{9}=\frac{11}{3}.$$

This means that the average class size experienced by the teachers at this school, and likely advertised by the administration, is smaller than the average class size experienced by the students. **Problem 240, Meaningful Means,** asks if this is just one case of a more general phenomenon: if n students are distributed among k non-empty classes, must the per-student mean class size always be at least as large as the per-class mean class size? (A separate companion problem asks if the per-class standard deviation for class size must always be at least as large as the per-student standard deviation for class size. In the example given, you can check that the (sample) standard deviations are $\sqrt{3}$ and $\sqrt{2.5}$, respectively.)

Another Sandbox inequality is offered by Tuan Le, a student from Fairmont High School (Anaheim, CA). **Problem 241, As Easy As a, b, c,** asks you to prove that for non-negative real numbers a, b, and c,

$$a^4(a-b)(a-c)+b^4(b-a)(b-c)+c^4(c-a)(c-b)$$

is greater than or equal to

$$k((a-b)(b-c)(c-a))^2$$

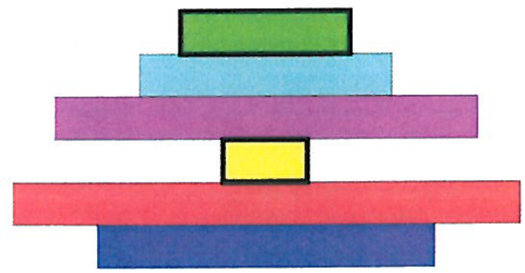
when $k=1$. (A challenge problem then asks you to find the largest possible value of k such that this relationship is guaranteed to hold.)

THE ZIP-LINE

This section offers problems with connections to articles that appear in this issue. Not all of the problems in this section require you to read the corresponding articles, but doing so can never hurt, of course.

"Bacterial Computing: Using *E. coli* to Solve the Burnt Pancake Problem" on pages 5-10 describes a method for solving a small 2-pancake instance of the Burnt Pancake Problem using living cells. The Zip-Line problem involves a larger 6-pancake stack, but you're allowed to use all of the living cells in your brain, plus a stand-alone *Mathematica* demo, to solve it!

A sloppy chef has prepared the colorful stack of six pancakes shown below:



Each pancake is burnt on one side, with the two burnt-side-up pancakes depicted with solid borders.

As in the article, you have two spatulas. You may insert the first spatula anywhere in the stack (even above the topmost pancake) and lift all of the pancakes above it. While they are suspended, you may insert a second spatula into the stack that remains, flip over as a group all of the pancakes resting on it, and then place the suspended pancakes in their original order back on top of the altered stack.

Problem 242, Flipjacks, asks you to turn the sloppy stack above into the standard smallest-to-largest all-burnt-sides-down stack in the shortest sequence of moves you can find. In case you're closer to a computer than batter and a griddle, a mouth-watering *Mathematica* demo is available at `www.maa.org/mathhorizons/supplemental.htm`.

THE JUNGLE GYM

Any type of problem may appear in the Jungle Gym—climb on!

Paolo Perfetti of Universita degli Studi di Roma "Tor Vergata" offers the Jungle Gym problem for this issue. Let $f:[0,1]\to \mathbf{R}$ be an increasing function on [0,1] that is differentiable on (0,1], such that $f':(0,1]\to\mathbf{R}$ is a decreasing function. **Problem 243, Not Quite $\pi^2/6$,** asks you to determine whether the series

$$\sum_{n=1}^{\infty}\frac{1}{n^2}f'(1/n)$$

converges.