

Let f be a function from $[-1, 1]$ to $\mathbb{R}$ with continuous derivatives of all orders up to $2n + 2$. Given $f(0) = f''(0) = \dots = f^{(2n)}(0) = 0$, prove

$$\frac{1}{2}((2n + 2)!)^2(4n + 5) \left(\int_{-1}^1 f(x) \, dx \right)^2 \leq \int_{-1}^1 (f^{(2n+2)}(x))^2 \, dx.$$