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József Wildt International Mathematical Competition

The Edition XXXVth, 2025 [1]

The solution of problems W1. - W70 must be mailed before 30 October 2025, to Mihály Bencze, str. Hărmanului 6, 505600 Săcele - Négyfalu, Jud. Braşov, Romania,
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W1. Find without any software:

$$\Omega = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\cos x + 1 - x^2}{(1 + x\sin x)\sqrt{1 - x^2}} dx$$

Daniel Sitaru

W2. If $x, y, z \geq 0$ then:

$$\frac{[2^x] + [2^y]}{[2^x + 2^y]} + \frac{[2^y] + [2^z]}{[2^y + 2^z]} + \frac{[2^z] + [2^x]}{[2^z + 2^x]} \leq 3$$

$[*]$ - great integer function.

Daniel Sitaru

W3. If in $\Delta ABC; \mu(A), \mu(B), \mu(C) \in (\frac{\pi}{6}, \frac{\pi}{2})$ then:

$$24\ln(\sin A \sin B \sin C) + 12\sum_{cyc} \mu^2(A) > 7\pi^2$$

$(\mu(A), \mu(B), \mu(C)$ are the measures of A, B, C in radians)

Daniel Sitaru

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W4. Calculate the integral

$$\int_0^1 \left\{\frac{1}{x}\right\}\left(\left\{\frac{1}{x}\right\} - \frac{x}{1+x}\right) dx.$$

Li Yin

W5. Assuming $x, y, z > 0$, $x^k + y^k + z^k = 1$, and satisfying $\lambda + \mu \geq k$, we have

$$\frac{x^\lambda}{1-x^\mu} + \frac{y^\lambda}{1-y^\mu} + \frac{z^\lambda}{1-z^\mu} \geq \frac{3^{1+\frac{\mu}{k}-\frac{\lambda}{k}}}{3^{\frac{\mu}{k}}-1},$$

if and only if the equal sign of $x = y = z$ holds.

Li Yin

W6. For any $a, b, c, k, l, m \in R^+$, we have

$$\arctan\frac{ka}{b+c} + \arctan\frac{lb}{c+a} + \arctan\frac{mc}{a+b}$$
$$\geq \arctan\left(\frac{1}{2}\left(\frac{1}{lm} + \frac{1}{mk} + \frac{1}{kl}\right) + \frac{9}{2\sqrt[3]{k^2l^2m^2}} - \frac{1}{2}\right).$$

Li Yin

W7. Evaluate the series

$$\sum_{k=0}^{\infty}(2k+1)\int_{2k\pi}^{(2k+2)\pi} \frac{\sin(px)}{x^2+a^2}dx, \quad a \neq 0,\ |p| < 1$$

Paolo Perfetti

W8. Evaluate the series

$$\sum_{n=1}^{\infty}\left(n\left(\frac{1}{n} - \frac{1}{n+1} + \frac{1}{n+2} - \frac{1}{n+3} + ...\right) - \frac{1}{2}\right)^3$$

Paolo Perfetti

W9. Evaluate

$$PV\int_0^{\infty} \frac{\sqrt[3]{x}\ln^2 x}{(x^2-1)^2(x-2)}dx$$

where “PV” means “ Cauchy Principal Value”

Paolo Perfetti

W10. Solve in R the equation

$$2\ln(1+x) - \frac{x^2+2x}{2(x+1)} = \sqrt{x}arctg\sqrt{x}$$

Pirkuliyev Rovsen

W11. Compute

$$\lim_{n\to\infty} e^{-H_n} \sqrt[n]{\sqrt{3!!}\cdot\sqrt[3]{5!!}\cdot\ldots\cdot\sqrt[n]{(2n-1)!!}}$$

where $(2k-1)!! = 1\cdot 3\cdot\ldots\cdot(2k-1)$, $\forall k \in N^*$ and $H_n = \sum\limits_{k=1}^{n} \frac{1}{k}$

D.M. Bătineţu-Giurgiu and Neculai Stanciu

W12. If $(a_n)_{n\geq 1}$ is a positive real sequence such that $\lim\limits_{n\to\infty}(a_{n+1}-a_n) = a$, $a > 0$, then compute

$$\lim_{n\to\infty} \sqrt{n}\left(\sqrt{a_{n+1}} - \sqrt{a_n}\right)$$

D.M. Bătineţu-Giurgiu and Neculai Stanciu

W13. Solve the equation

$$x - \frac{2x}{x+2} - \frac{6x^2}{x^2+6x+12} - \frac{24x^4}{x^4+24x^3+192x^2+576x+576} = \frac{243}{1694}$$

Béla Kovács

W14. Solve the equation

$$z^4 - 7z^3 + 50z - 50 = iz(7z^2 - 39z + 50)$$

Béla Kovács

W15. In any $\triangle ABC$ the following relationship holds:

$$\frac{\sin^2\frac{A}{2}}{\sin\frac{C}{2}} + \frac{\sin^2\frac{B}{2}}{\sin\frac{A}{2}} + \frac{\sin^2\frac{C}{2}}{\sin\frac{B}{2}} \leq \frac{1}{3}\left(\frac{4R}{r} + \frac{r}{R} - 4\right)$$

Marian Ursărescu and Florică Anastase

W16. Let be $z_1, z_2, z_3 \in C^*$ different in pairs such that $|z_1| = |z_2| = |z_3| = 1$. If

$$\sum_{cyc} \sqrt{(2z_1 - z_2 - z_3)(2z_2 - z_3 - z_1)} = 9$$

then z_1, z_2, z_3 are the affixes of an equilateral triangle.

Marian Ursărescu and Florică Anastase

W17. Let $F : (0, +\infty) \to R$ be a primitive of the functions $f : (0, \infty) \to R$, $f(x) = \frac{2(xarctgx)}{(1+x^2)^2 arctg^3 x}$. Find:

$$L = \lim_{x\to\infty} \frac{x}{F(x)} \cdot \int\limits_{\frac{1}{x+1}}^{\frac{1}{x}} \frac{\sin t}{t^2} dt$$

Marian Ursărescu and Florică Anastase

W18. Let be triangle ABC, AA_1, BB_1, CC_1 internal bisectors and A_2, B_2, C_2 contact points to the bisectors with the circumcircle of the triangle. Prove that:

$$A_1A_2 \cdot B_2C_2 + B_1B_2 \cdot A_2C_2 + C_1C_2 \cdot A_2B_2 \geq Rs$$

where p represent the semiperimeter and R the circumradii of triangle ABC.

Marian Ursărescu and Florică Anastase

W19. Let be the function $f : [0, 1] \to R$ integrable such that $f(1) = 1$ and $\int\limits_x^y f(t)dt = \frac{1}{2}(yf(y) - xf(x)), \forall x, y \in [0, 1]$. Find:

$$I = \int\limits_0^{\frac{\pi}{4}} f(x) \cdot tg^2 x dx$$

Marian Ursărescu and Florică Anastase

W20. Calculate

$$\lim_{n\to\infty} n(-1)^n \sum_{k=1}^{n-1} \frac{(-1)^{k-1}}{k(n-k)^2}.$$

Ovidiu Furdui and Alina Sîntămărian

W21. Let $a \in \mathbb{R}$. Find all continuous functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(x) = a - \int_0^x \cos\left(\frac{t}{2}\right) f(x-t)\mathrm{d}t, \quad \forall x \in \mathbb{R}.$$

Ovidiu Furdui and Alina Sîntămărian

W22. Consider a system of two particles of masses m_1 and m_2, whose Lagrangian is given by

$$L = \frac{1}{2}m_1\dot{\mathrm{x}}_1^2 + \frac{1}{2}m_2\dot{\mathrm{x}}_1^2 + m_1\dot{\mathrm{x}}_1\dot{\mathrm{x}}_2 - m_1 g x_1 + m_2 g x_2$$

Let the total mass and reduced mass be M and μ then show that $\mu x_2 + [M - \mu]x_1$ is constant of motion.

Toyesh Prakash Sharma and Etisha Sharma

W23. If n belongs to the set of odd numbers then show that:

$$\frac{2}{3} + \frac{(\log_{F_{2n}} F_n^{F_n} L_n^{L_n})^2 + (\log_{F_{2n}} F_n^{L_n} L_n^{F_n})^2}{6} < F_n^2$$

Toyesh Prakash Sharma and Etisha Sharma

W24. Solve in $\mathcal{M}_2(\mathbb{R})$ the equation $\sin A = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$.

Ovidiu Furdui and Alina Sîntămărian

W25. If $a_k > 0$, $k = 1, 2, ..., n$ are arbitrary real numbers, prove that

$$\sum_{k=1}^{n} \sqrt{(a_k^2 - a_k a_{k+1} + a_{k+1}^2)(a_{k+1}^2 - a_{k+1}a_{k+2} + a_{k+2}^2)} \geq \sum_{k=1}^{n} a_k^2$$

where $a_{n+1} = a_1, a_{n+2} = a_2$

Dorin Mărghidanu

W26. If a, b, c are real numbers for which $ab + bc + ca = 1$, prove that

$$2a^2 + 3b^2 + 4c^2 > 2\sqrt{2}$$

Dorin Mărghidanu

W27. For $a, b > 0$, let

$$E(a,b) = \frac{2a+3b}{4a+5b} + \frac{2b+3a}{4b+5a}$$

Prove that $E(a,b) \in \left(\frac{11}{10}, \frac{10}{9}\right]$ and that $\frac{11}{10}$ and $\frac{10}{9}$ are the best constants with this property (i.e. the interval $\left(\frac{11}{10}, \frac{10}{9}\right]$ is the largest possible)

Dorin Mărghidanu

W28. If $a, b, c > 0$ and $abc \geq 8$ then prove that

$$2(a^2+b^2+c^2) + 3\left(\frac{ab}{a^2-ab+b^2} + \frac{bc}{b^2-bc+c^2} + \frac{ca}{c^2-ca+a^2}\right) \geq 33$$

Dorin Mărghidanu

W29. For given positive real a, b, c find maximal value of constant t such that inequality

$$ax^2 + by^2 + cz^2 \geq 2t\,(xy + yz + zx)$$

holds for any real $x, y, z \geq 0$ such that $xy + yz + zx \neq 0$.

Arkady Alt

W30. Let a_i is number of rings on the rod at the vertices $i = 1, 2, ..., 6$ of a regular hexagon. Find the greatest value of $3-$rings chains constructed from rings that taken by one from any 3 neighboring rods.

Arkady Alt

W31. Find maximal value of remainder from division n by k, where $n \in \mathbb{N} \setminus \{1\}$ and $k \in \{1, 2, ..., n\}$.

Arkady Alt

W32. Let ABC be an acute triangle with circumradius R.Prove that :

$$\sum a^2 \cdot \left(\frac{1}{2R^2} + \sum \frac{1}{b^2+c^2}\right) \leq 9.$$

Arkady Alt

W33. Let $D := \{(x_1, x_2, ..., x_n) \mid x_i \in \mathbb{N}, i = 1, 2, ..., n,$
$x_1 < x_2 < ... < x_n$ and $x_1 + x_2 + ... + x_n = a\}$ and let $1 \le k < n/2$.

a). Find

$$\min_{x \in D} (x_n + x_{n-1} + ... + x_{n-k+1})$$

(or $\max\limits_{x \in D} (x_1 + x_2 + ... + x_m)$);

b). For which k, a and n holds inequality

$$x_n + x_{n-1} + ... + x_{n-k+1} \ge x_{n-k} + x_{n-k-1} + ... + x_1 \iff$$

$$\iff x_n + x_{n-1} + ... + x_{n-k+1} \ge a/2.$$

Arkady Alt

W34. If $f : [0, 1] \to R$ it is differentiable and concave function, with $f(0) = -f(1)$, prove that:

$$\int_0^1 \frac{f(x)dx}{x^2 - x + 1} \le \frac{4\pi}{3\sqrt{3}} \int_0^1 f(x)dx$$

Stănescu Florin

W35. Let $I \subseteq R$ an interval and $f : I \to R$ one function differentiable on I, which has the following property: for any $a, b \in I$, $a < b$ and for all $x \in (a, b)$, exist $h > 0$, which depends on x, such that:

$$h + x \in (a, b) \qquad \text{and} \qquad \frac{f(x + h) - f(x)}{h} \ge f'(x)$$

Show that f' is an increasing function on I

Stănescu Florin

W36. We consider $A, B \in M_n(R)$, $n \ge 2$, with AB=BA and $\omega_k = \cos\frac{2k\pi}{n+1} + i \sin\frac{2k\pi}{n+1}$, $k = \overline{0, n}$ such that:

a).

$$\det(A + \omega_n B) = 0$$

b).

$$Im\left[\sum_{k=0}^{n-1}\omega_k^{2p+1}\sin\left(\frac{k\pi}{n+1}\right)\cdot\det(A+\omega_k B)\right]\leq 0$$

$(\forall)p\in\{0,1,...,n-1\}$
c).

$$\det(a+\omega_n B)=0$$

Show that:
a). $A^{n+1}+(-B)^{n+1}=O_n$
b). $\det(A+b)=(n+1)\det B$
c). $Tr\left[B\cdot(A+B)^{-1}\right]=\frac{n}{2}$; $Tr=$ trace of matrix A
d). $Tr\left[B^2\cdot(A+B)^{-2}\right]=-\frac{n(n-4)}{12}$
Note. We write the complex number $z=Rez+Imz\cdot i$
Remark. *If I_n, then: We consider $A\in M(R)$, $n\geq 2$ and* $\omega_k=\cos\frac{2k\pi}{n+1}+i\sin\frac{2k\pi}{n+1}$, $k=\overline{0,n}$, *such that:*

a). $det(A+\omega_n I_n)=0$
b).

$$Im\left[\sum_{k=0}^{n-1}\omega_k^{2p+1}\sin\left(\frac{k\pi}{n+1}\right)\cdot\det(A+\omega_k B)\right]\leq 0$$

$(\forall)p\in\{0,1,...,n-1\}$
Show that:
a). $A^{n+1}+(-I_n)^{n+1}=O_n$
b). $det(A+I_n)=n+1$
c). $Tr(A+I_n)^{-1}=\frac{n}{2}$
d).$Tr(A+I_n)^{-2}=\frac{n(n-4)}{12}$

Stănescu Florin

W37. Let $f:[0,1]\to\mathbb{R}$ be continuous, with $\int_0^1 x(x-1)^2f(x)dx=0$. Let us consider the set

$$\mathcal{R}=\mathbb{R}\setminus\left[\frac{2}{3},1\right).$$

(a). For $\omega\in\mathcal{R}$, show that there exists a real number $c\in(0,1)$ such that

$$\frac{1}{c}\int_0^c t^2f(t)dt=\omega\int_0^c tf(t)dt. \qquad (1)$$

(b). If $\omega \notin \mathcal{R}$, show that (1) has no solution in c for some function $f := f_\omega$ satisfying the hypothesis of the problem and find such a function.

Remarks: The problem in TCMJ was proposed by Maubinool Omarjie and the one in the Monthly was proposed by Florin Stanescu about a year earlier. Problem 1283 is substantially easier. Our generalization is perhaps quite difficult.

Eugen J. Ionaşcu

W38. Let $\omega_1(O_1, r_1)$ and $\omega_2(O_2, r_2)$ two circles with radii $r_1 < r_2$ which intersect in two distinct points P and Q, let $A(\neq P; Q)$ a fix point on ω_1. Contruct with ruler and compass the cords PB and PD in circle ω_2 such that P, A, B are cellinear and $AB = CD$ when $\{C\} = \omega_1 \cap PD$.

Ion Pătraşcu and Alin Creţu

W39. For $p > 1$, $\frac{1}{p} + \frac{1}{q} = 1$ and $r > 1$. If $x_{00}, y_{00}, a_{00}, b_{00} > 0$ and reals $x_{ij}, y_{ij}, a_{ij}, b_{ij}, i = 1, 2, \ldots, n, j = 1, 2, \ldots, m$, then

$$\frac{\left(\sum_{j=1}^m \sum_{i=1}^n [(x_{ij} + y_{ij})^r + (a_{ij} + b_{ij})^r]\right)^p}{[(x_{00} + y_{00})^r + (a_{00} + b_{00})^r]^{p/q}} \leq$$

$$\leq \left(\frac{\left(\sum_{j=1}^m \sum_{i=1}^n x_{ij}^r\right)^{p/r}}{x_{00}^{p/q}} + \frac{\left(\sum_{j=1}^m \sum_{i=1}^n y_{ij}^r\right)^{p/r}}{y_{00}^{p/q}}\right)^r$$

$$+ \left(\frac{\left(\sum_{j=1}^m \sum_{i=1}^n a_{ij}^r\right)^{p/r}}{a_{00}^{p/q}} + \frac{\left(\sum_{j=1}^m \sum_{i=1}^n b_{ij}^r\right)^{p/r}}{b_{00}^{p/q}}\right)^r. \tag{1.1}$$

with equality if and only if either $x_{ij} = y_{ij} = 0$ and $a_{ij} = b_{ij} = 0$ for $i = 1, \ldots, n$ and $j = 1, \ldots, m$ or $x_{ij} = \alpha y_{ij}$ and $a_{ij} = \beta b_{ij}$ for $i = 0, 1, \ldots, n$ and $j = 0, 1, \ldots, m$ and some $\alpha, \beta > 0$, and

$$\left(\frac{\left(\sum_{j=1}^m \sum_{i=1}^n x_{ij}^r\right)^{p/r}}{x_{00}^{p/q}} + \frac{\left(\sum_{j=1}^m \sum_{i=1}^n y_{ij}^r\right)^{p/r}}{y_{00}^{p/q}}\right) :$$

$$: \left(\frac{\left(\sum_{j=1}^m \sum_{i=1}^n a_{ij}^r\right)^{p/r}}{a_{00}^{p/q}} + \frac{\left(\sum_{j=1}^m \sum_{i=1}^n b_{ij}^r\right)^{p/r}}{b_{00}^{p/q}}\right) =$$

$$= (x_{00} + y_{00}) : (a_{00} + b_{00}).$$

Chang-Jian Zhao and Mihály Bencze

W40. For $p>1$, $\frac{1}{p}+\frac{1}{q}=1$ and $r>1$. If $u(x,y),v(x,y),u'(x,y),v'(x,y)>0$, and $f(x,y),g(x,y),f'(x,y),g'(x,y)$ are continuous functions on $[a,b]\times[c,d]$, then

$$\frac{\left(\int_a^b\int_c^d[(f(x,y)+g(x,y))^r+(f'(x,y)+g'(x,y))^r]dxdy\right)^p}{[(u(x,y)+v(x,y))^r+(u'(x,y)+v'(x,y))^r]^{p/q}}$$
$$\le\left(\frac{\left(\int_a^b\int_c^d f(x,y)^r dxdy\right)^{p/r}}{u(x,y)^{p/q}}+\frac{\left(\int_a^b\int_c^d g(x,y)^r dxdy\right)^{p/r}}{v(x,y)^{p/q}}\right)^r+$$
$$+\left(\frac{\left(\int_a^b\int_c^d f'(x,y)^r dxdy\right)^{p/r}}{u'(x,y)^{p/q}}+\frac{\left(\int_a^b\int_c^d g'(x,y)^r dxdy\right)^{p/r}}{v'(x,y)^{p/q}}\right)^r. \tag{1.2}$$

with equality if and only if either $f(x,y)=g(x,y)=0$ and $f'(x,y)=g'(x,y)=0$ or $(f(x,y),g(x,y))=\alpha(u(x,y),v(x,y))$ and $(f'(x,y),g'(x,y))=\beta(u'(x,y),v'(x,y))$ and some $\alpha,\beta>0$, and

$$\left(\frac{\left(\int_a^b\int_c^d f(x,y)^r dxdy\right)^{p/r}}{u(x,y)^{p/q}}+\frac{\left(\int_a^b\int_c^d g(x,y)^r dxdy\right)^{p/r}}{v(x,y)^{p/q}}\right):$$
$$:\left(\frac{\left(\int_a^b\int_c^d f'(x,y)^r dxdy\right)^{p/r}}{u'(x,y)^{p/q}}+\frac{\left(\int_a^b\int_c^d g'(x,y)^r\right)^{p/r}}{v'(x,y)^{p/q}}\right)$$
$$=(u(x,y)+v(x,y)):(u'(x,y)+v'(x,y)).$$

Chang-Jian Zhao and Mihály Bencze

W41. Let $f:[a,b]\to R$ be a Rolle function on $[a,b]$ with the property that f' is continuous on (a,b). Show that $(a_n)_{n\geq 0}$ is a sequence such that $a_n\in[a,b]$, $a_n\neq a_{n+1}$ for any $n\geq 0$, there exists $\lim\limits_{n\to\infty}a_n$ and $\lim\limits_{n\to\infty}a_n=l\in(a,b)$, then $\lim\limits_{n\to\infty}\frac{f(a_{n+1}-f(a_n))}{a_{n+1}-a_n}=f'(e)$.
Application: $\lim\limits_{n\to\infty}n^2(e_{n+1}-e_n)$, where $e_n=\left(1+\frac{1}{n}\right)^n$, $n\in N^*$

Ovidiu Pop

W42. Show that the equation $\frac{1}{x} = \frac{1}{[x]} + \frac{1}{\{x\}}$ has no solutions in the set R, where $[x], \{x\}$ are the integer part and respectively the fractional part of x.

Ovidiu Pop

W43. The number of 1's in the binary expansion of the positive integer n is customarily denoted by $s_2(n)$ and sometimes called the Hamming weight of n. For every natural number n, we let

$$M(n) := \min_{k \geq 1}\{s_2(kn)\}.$$

Find $M(2025 + k)$, for all $k \in \{0, 1, 2, \cdots, 10\}$, with proof (avoid huge computations).

Eugen J. Ionaşcu

W44. If $a, b, c > 0$, and

$$2(a + b + c) = ab + bc + ca$$

and $\lambda \geq 0$, then

$$\sum \frac{a}{a(b + c) + \lambda} \leq \lambda \frac{a + b + c}{abc + \lambda}$$

Marin Chirciu and Daniel Văcaru

W45. If $n \in N$, $a_k > 0$ for any $k \in \{1, 2, 3, ..., n\}$ prove that

$$\frac{2a_1 + n}{2\sqrt{a_2} + 3\sqrt[3]{a_3} + ... + n\sqrt[n]{a_n}} + \frac{2a_2 + n}{2\sqrt{a_3} + 3\sqrt[3]{a_4} + ... + n\sqrt[n]{a_1}} + ...$$
$$+ \frac{2a_n + n}{2\sqrt{a_1} + 3\sqrt[3]{a_2} + ... + n\sqrt[n]{a_{n-1}}} \geq \frac{2n}{n-1}$$

Nicolae Papacu

W46. Let $p \in N$, $p \geq 2$ be and sequence $(x_n)_{n \geq 1}$,

$$x_n = \frac{1}{n} \cdot \sqrt[p]{\sum_{k=1}^{n} (k^p - (k-1)^p) \left(\frac{k-1}{k}\right)^p}$$

for any $n \in N^*$
a). Prove that $x_n \in [0, 1]$ and $x_n \leq x_{n+1}$ for any $n \in N^*$
b). Prove that $\lim_{n \to \infty} x_n = 1$ and calculate $\lim_{n \to \infty} \left(n x_n^p - (n-1) x_{n-1}^p\right)$

Nicolae Papacu

W47. Let $x > 0$. Evaluate

$$\frac{1\cdot\binom{2n}{0}}{x} - \frac{2\cdot\binom{2n}{1}}{x+1} + \frac{3\cdot\binom{2n}{2}}{x+2} + \ldots + \frac{(2n+1)\cdot\binom{2n}{2n}}{x+2n}$$

Paolo Perfetti

W48. Let us define the sequence

$$n_0 = n_1 = n_2 = 1,\ n_{r+1} = n_r + n_{r-2},\ (r \geq 3).$$

The first few terms are $\{n_{r\geq 0}\} = \{1, 1, 1, 2, 3, 4, 6, 9, 13, \ldots\}$.
Prove the following identity: For $r \geq 6$, $n_r = n_{r-2} + 2n_{r-4} + n_{r-6}$.

Ángel Plaza

W49. Let $\alpha = (1+\sqrt{5})/2$, $\beta = (1-\sqrt{5})/2$, $f(x) = \dfrac{e^{\alpha x} - e^{\beta x}}{\sqrt{5}}$, and let $T(f)_n(x)$ be the n-th Taylor polynomial of $f(x)$ at $x = 0$, for $n \in N$. Prove

$$\sum_{k=0}^{n} (-1)^{n-k} \binom{n}{k} T(f)_n(k) = F_n$$

where F_n denotes the n-th Fibonacci number.

Ángel Plaza

W50. Let V is the volume of a sphere of radius R, centred at the origin. Then evaluate the volume integral

$$\int\int\int \frac{e^{-(r/R)^2}}{1+(r/R)^2} \bigtriangledown \cdot \left(\frac{\hat{\mathrm{r}}}{4r^2}\right) d^3r$$

Ankush Kumar Parcha

W51. Let $n \geq 1$ and $P_n(x)$ denotes Legendre's polynomial then show that

$$\sum_{n=1}^{\infty} \int_{-1}^{1} x^n P_{n-1}(x)dx \int_{-1}^{1} x^{n-1} P_n(x)dx < 2$$

Etisha Sharma and Toyesh Prakash Sharma

W52. Let us define the sequence

$$n_0 = n_1 = n_2 = 1,\ n_{r+1} = n_r + n_{r-2},\ (r \geq 3).$$

The first few terms are $\{n_r\}_{r\geq 0} = \{1,1,1,2,3,4,6,9,13,\ldots\}$.
Prove the following identity: For $m, p \geq 2$,

$$n_{m+p} = n_m n_p + n_{m-1} n_{p-2} + n_{m-2} n_{p-1}.$$

Ángel Plaza

W53. Let n be a positive integer, r is the number of distinct prime factors of n, $n \geq 2$. If φ is the Euler's totient function, then

$$\prod_{\substack{p|n \\ p\, prime}} p \geq \frac{n}{(n^{1/r} - \varphi^{1/r}(n))^r}.$$

Nicuşor Minculete

W54. In any triangle ABC

$$\sum \cos^3 \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \leq \frac{s}{8}\left(\frac{1}{r} - \frac{1}{2R}\right).$$

Marin Chirciu and Daniel Văcaru

W55. Let $f, g, h : [a,b] \to R$ integrable function such that

$$\int_a^b f(x)g(x)dx = \int_a^b g(x)h(x)dx = \int_a^b h(x)f(x)dx =$$

$$= \left(\int_a^b f^2(x)dx\right)\left(\int_a^b g^2(x)dx\right)\left(\int_a^b h^2(x)dx\right) = 1$$

Prove that:

$$\int_a^b f^2(x)dx = \int_a^b g^2(x)dx = \int_a^b h^2(x)dx = 1$$

Marius Drăgan

W56. Find the best constant k such that the inequality

$$\sum \frac{s+a}{b+c} \leq \frac{135R+30kr}{30R+(8k+12)r}$$

in true in every ABC triangle.

Marius Drăgan

W57. If $x, y, z > 0$ and $n \geq 2$ then

$$\frac{x^n}{y^{n-1}} + \frac{y^n}{z^{n-1}} + \frac{z^n}{x^{n-1}} + x + y + z \geq \frac{(x+y)^n}{(y+z)^{n-1}} + \frac{(y+z)^n}{(z+x)^{n-1}} + \frac{(z+x)^n}{(x+y)^{n-1}}$$

Marius Drăgan and Mihály Bencze

W58. In all triangle ABC holds:

$$\sum \frac{tg\frac{A}{2}}{\sqrt{tg\frac{B}{2} + \lambda tg\frac{C}{2}}} \geq \frac{1}{\sqrt{\lambda+1}} \left(\frac{4R+r}{s}\right)^{\frac{3}{2}}$$

Mihály Bencze and Marius Drăgan

W59. Compute $\int \frac{x(4x^6+9x^5+x^3+22x-6)dx}{x^{10}+2x^8+3x^7+9x^5+2x^3+3x^2+2}$

Mihály Bencze and Ovidiu Bagdasar

W60. Compute $I = \int \frac{\left(\lambda(1+x^2)+2(\sin x+\cos x)(1+x^2)+\cos x-\sin x\right)dx}{(1+x^2)((arctgx)^2+\lambda(\sin x+\cos x)arctgx+\lambda^2 \sin x\cos x)}$ when $\lambda \in R^*$

Mihály Bencze

W61. In all triangle ABC holds:
1). $\sum \frac{ab^2}{a^2+ab+3b^2} \leq \frac{2s}{5}$ 2). $\sum \frac{r_a r_a^b}{r_a^2+r_a r_b+3r_b^2} \leq \frac{4R+r}{5}$

Mihály Bencze and Ovidiu Bagdasar

W62. Prove that $\int\limits_0^1 \frac{xdx}{(x^2+x+3)(x^2+3x+5)} \leq \frac{1}{25} \ln \frac{9}{5}$

Mihály Bencze and Chang-Jian Zhao

W63. If $a, b, c > 0$ and $a + b + c = \lambda$ then

$$\frac{3\lambda}{3\sum a^2 + \sum ab} \leq \sum \frac{a}{\lambda a + bc} \leq \frac{9}{4\lambda}$$

Mihály Bencze and Chang-Jian Zhao

W64. In all triangle ABC holds:

$$\sum \frac{r_b}{r_a^3(r^2 + r_a r_b)} \geq \frac{s^2}{3r^4(s^2 + (4R + r)r)}$$

Mihály Bencze and Rovsen Pirguliyev

W65. Prove that: $\int\limits_1^e \frac{x \ln x dx}{x^2+x+3} \leq \frac{e^2+7}{50}$

Mihály Bencze and Gabriel Prăjitură

W66. In all triangle ABC holds:

$$12r \leq \sum \frac{m_a}{\cos^2 \frac{A}{2}} \leq \frac{4\sqrt{2R^2 + r^2}(5R^2 + 3Rr + r^2)}{s^2}$$

Mihály Bencze

W67. Calculate the following sum

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left(\frac{1}{2n-1} - \frac{1}{2n} + \frac{1}{2n+1} - \dots \right)$$

Paolo Perfetti

W68. Evaluate

$$\int\limits_{-\infty}^{\infty} e^{-x^2} \frac{(x^3 - cotx)}{x^2 secx} dx$$

Etisha Sharma

W69. If $a_k > 0$ $(k = 1, 2, ..., n)$ then

$$\frac{\sum\limits_{k=1}^{n} a_k}{\sqrt[n]{\prod\limits_{k=1}^{n} a_k}} \leq \frac{n-1}{n} \sum_{cyclic} \frac{a_1}{a_2} + \frac{1}{n} \sum_{cyclic} \frac{a_2^{n-2}}{a_3 a_4 ... a_n}$$

Mihály Bencze and Florentin Smarandache

W70. Minimize the function

$$F(x_1, x_2, ..., x_n) =$$

$$= \left(\log_P x_1^{a_1} x_2^{a_2} ... x_n^{a_n}\right)^m + \left(\log_P x_1^{a_2} x_2^{a_3} ... x_n^{a_1}\right)^m + ... \left(\log_P x_1^{a_n} x_2^{a_1} ... x_n^{a_{n-1}}\right)^m$$

when $x_k, a_k > 0 (k = 1, 2, ..., n)$, $P = \prod\limits_{k=1}^{n} x_k$ and $m \in N$

Mihály Bencze, Toyesh Prakash Sharma and Etisha Sharma