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József Wildt International Mathematical Competition

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W1. Let ABC be a scalene triangle with semi-perimeter s and area S. Prove that it holds

$$\frac{a+s}{a(a-b)(a-c)}+\frac{b+s}{b(b-a)(b-c)}+\frac{c+s}{c(c-a)(c-b)}<\frac{3\sqrt{3}}{8S}.$$

José Luis Díaz-Barrero

W2. Let F_n be the n^{th} Fibonacci number defined by $F_0=0$, $F_1=1$ and $F_n=F_{n-1}+F_{n-2}$ for all $n\geq 2$. Prove that

$$\frac{F_{2n+1}+F_nF_{n+1}+1}{F_{n+2}+\sum\limits_{1\leq i<j\leq n}F_iF_j}$$

is an integer number and determine its value.

José Luis Díaz-Barrero

W3. If $f:[a,b]\to R$ three time differentiable, $f'''>0$, $t_1\in(a,(1-\lambda)\,a+\lambda b)\,,\ t_2\in((1-\lambda)\,a+\lambda b,b)$, $\lambda\in(0,1)$ then show:

$$f\,(b)-f\,(a)\geq(b-a)\,f'\,(\lambda t_1+(1-\lambda)\,t_2)$$

Seyran Ibrahimov

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W4. The Lucas numbers L_n satisfy $L_{n+2} = L_{n+1} + L_n$, $L_0 = 2$, $L_1 = 1$. Also, $\alpha = \frac{1+\sqrt{5}}{2}$, $\beta = \frac{1-\sqrt{5}}{2}$, and $L_n = \alpha^n + \beta^n$. For positive integers m, n, evaluate in closed form:

$$\sum_{j=1}^{n} \binom{2n}{n-j} \frac{L_{mn-4j} + L_{mn+4j}}{L_{mn}}.$$

Ángel Plaza

W5. Let $(B_n)_{n\geq 0}$ and $(C_n)_{n\geq 0}$ denote the balancing and Lucas-balancing numbers, respectively, i.e. $B_{n+1} = 6B_n - B_{n-1}$ and $C_{n+1} = 6C_n - C_{n-1}$ for all $n \geq 1$ and $B_0 = 0$, $B_1 = 1$, $C_0 = 1$, $C_1 = 3$. Show that

$$\sum_{n=1}^{\infty} \frac{B_n}{n6^n} = \frac{\ln(3+\sqrt{8})}{\sqrt{8}} \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{C_n}{n6^n} = \ln 6.$$

Ángel Plaza

W6. Evaluate

$$\sum_{n=0}^{\infty} \frac{2\,(5n+2)}{(n+1)\,(2n+1)\,(4n+1)}$$

Ángel Plaza

W7. Find the differential equation of the fourth order with constant coefficients, having particular solutions e^{-x}, e^{x} double e^{2x} and its non-homogeneous term being $2x^2$. Solve after its Cauchy problem

$$y\,(0) = 3,\ y'\,(0) = y''\,(0) = 2,\ y'''\,(0) = 1$$

Laurenţiu Modan

W8. i). For the natural numbers n, k , $n \geq k$ prove the inequality:

$$C_{n+1}^{k} \leq C_{n}^{k-1} + C_{n-1}^{k-1} + A_{n-1}^{k}$$

ii). Let the sequence:

$$a_k = C_{n+1}^2 - C_n^{k-1} - C_{n-1}^{k-1} - A_{n-1}^k,\ n \geq k \geq 1$$

Prove:

$$\sum_{k=0}^{n-1} a_k < 2^{n-1}$$

iii). Study the convergence of the series:

$$\sum_{k>1} \frac{C_{n-1}^k}{a_k}$$

Laurenţiu Modan

W9. In a village which is in a pandemic deseases, the number of the sick inhabitants is double at every four days. The maximal sick inhabitants appears after 32 days being a third from all the village inhabitants.
Let A, be the set of the young parents in this village, who represent a half of all the inhabitants.
Let B, be the set of the children and the adolescents in this village, they representing the sixth part of all the inhabitants.
Let now consider the function $f : B \to A$.
Find the probability to get down in a random way, an injection between the elements of the set

$$\{f/f : B \to A\}$$

Laurenţiu Modan

W10 . In the $[ABCD]$ tetrahedron is denoted by h_x, r_x, S_x the length of the height corresponding to the vertex X, the radius of the exinscribed sphere opposite the vertex X, and respectively the area of the face opposite to the vertex X, $(\forall)\, X \in \{A, B, C, D\}$. If S is the total area of the $[ABCD]$ tetrahedron, to show that inequality is taking place:

$$\left(\frac{r_A}{h_A}\right)^{S_A} \cdot \left(\frac{r_B}{h_B}\right)^{S_B} \cdot \left(\frac{r_C}{h_C}\right)^{S_C} \cdot \left(\frac{r_D}{h_D}\right)^{S_D} \geq \left(\frac{1}{2}\right)^{S}$$

Marius Olteanu

W11. In the regular triangual pyramid $[ABCD]$ it is denoted by m_X, h_X the lengths of the median and the height corresponding to the vertex $X \in \{A, B, C, D\}$. If s, R is the radius of the inscribed respectivelly circumscribed sphere of the $[ABCD]$ tetrahedron, show that

$$\frac{R}{3r} \geq \frac{m_X}{h_X}, \ \ (\forall)\, X \in \{A, B, C, D\}$$

Marius Olteanu

W12. Prove that:

$$\int_0^{\frac{\pi}{2}} \frac{\sin^2 nx}{\sin^2 x} dx = \frac{n\pi}{2}, \ n \in N$$

Šefket Arslanagić

W13. Let r_a, r_b and r_c be the length of the radii of excircles of a triangle $\triangle ABC$ with circumradius R and inradius r. Let a, b and c be the length of the sides of $\triangle ABC$. Prove that:

$$\frac{a^2}{r_b r_c} + \frac{b^2}{r_c r_a} + \frac{c^2}{r_a r_b}$$

Šefket Arslanagić

W14. Prove that:

$$\sqrt{1 + \sqrt[3]{u} + \sqrt[3]{\overline{u}}} + \sqrt{\sqrt[3]{u} + \sqrt[3]{\overline{u}}} - \sqrt[3]{u} - \sqrt[3]{\overline{u}} \in N$$

where

$$u = 2 + \frac{2}{3}\sqrt{\frac{11}{3}} \text{ and } \overline{u} = 2 - \frac{2}{3}\sqrt{\frac{11}{3}}$$

Ionel Tudor

W15. Let the sequences $(a_n)_{n\geq 1}, (b_n)_{n\geq 1}$:

$$a_n = \sum_{k=1}^{n} \arctan \frac{1}{k^2 - k + 1} \text{ and } \lim_{n\to\infty} \frac{b_{n+1}}{nb_n} = b \in R_+^*$$

Compute:

$$\lim_{n\to\infty}\left(\frac{\pi}{2}-a_n\right)\sqrt[n]{b_n}$$

D.M. Bătineţu-Giurgiu and Neculai Stanciu

W16. If $\gamma_n = -\ln n + \sum\limits_{k=1}^{n}\frac{1}{k}$ with $\lim\limits_{n\to\infty}\gamma_n = \gamma =$ Euler-Mascheroni constant, then compute the limits
(i)

$$\lim_{n\to\infty}(\gamma_n-\gamma)\,n$$

(ii)

$$\lim_{n\to\infty}\left(\gamma_n\gamma_{n+1}-\gamma^2\right)n$$

(iii)

$$\lim_{n\to\infty}\left(\gamma_n\gamma_{n+1}\gamma_{n+2}-\gamma^3\right)n$$

(iv)

$$\lim_{n\to\infty}\left(\gamma_n\gamma_{n+1}\gamma_{n+2}\cdots\gamma_{n+m-1}\gamma_{n+m}-\gamma^{m+1}\right)n$$

D.M. Bătineţu-Giurgiu and Neculai Stanciu

W17. If $x, y, z > 0$ and $A_1B_1C_1, A_2B_2C_2$ are two triangles with the circumradius R_1, respectively R_2 then holds the following inequality:

$$\frac{x+y}{z\sqrt{a_1a_2}}+\frac{y+z}{x\sqrt{b_1b_2}}+\frac{z+x}{y\sqrt{c_1c_2}}\geq\frac{2\sqrt{3}}{\sqrt{R_1R_2}}$$

D.M. Bătineţu-Giurgiu and Neculai Stanciu

W18. Calculate in terms of known constants

$$\lim_{a\to 0^+}\left(\int\limits_a^\infty\frac{1}{e^{2\pi x}-1}dx+\int\limits_a^{1-a}\frac{x^3\cos(\pi x)}{2\sin(\pi x)}dx\right)$$

Paolo Perfetti

W19. Evaluate:

$$\int_0^1 \sqrt[4]{x(1-x)(x-2)(x-4)}dx + \int_2^4 \sqrt[4]{x(1-x)(x-2)(x-4)}dx +$$

$$+\sqrt{2}\int_1^2 \sqrt[4]{x(x-1)(x-2)(x-4)}dx$$

Results: $= \pi\sqrt{2}\frac{35}{32}$.

Paolo Perfetti

W20. Let p, q be integers, $|a| < 1$, $|b| < 1$, $b^p \neq a^q$, $p = m\alpha$, $q = n\alpha$, n and m coprime. Evaluate in terms of a, b, m, n the sum

$$\frac{1}{p}\sum_{k=0}^{p-1} \frac{1-b^2}{((1+b^2) - ba^{\frac{q}{p}}e^{\frac{i\pi k2q}{p}} - ba^{\frac{-q}{p}}e^{\frac{-i\pi k2q}{p}})} +$$

$$+\frac{1}{q}\sum_{k=0}^{q-1} \frac{1-a^2}{((1+a^2) - ab^{\frac{p}{q}}e^{\frac{i\pi k2p}{q}} - ab^{\frac{-p}{q}}e^{\frac{-i\pi k2p}{q}})}$$

Answer: $\frac{1+b^m a^n}{1-b^m a^n}$

Paolo Perfetti

W21. Calculate

$$\int_0^\infty \left(\frac{\cosh x}{\sinh^2 x} - \frac{1}{x^2}\right)(\ln x)^3 dx$$

Answer:
$6\gamma_1 \ln 2 - \ln^3 2 + 3\gamma \ln^2 2 - 3\ln\pi \ln^2 2 + 6\gamma \ln\pi \ln 2 - 3\ln^2\pi \ln 2 - \frac{3}{4}\pi^2 \ln 2$
γ is the Euler constant, $\gamma_1 = \lim_{m\to\infty}\left(\sum_{k=1}^{m} \frac{\ln k}{k} - \frac{\ln^2 m}{2}\right)$

Paolo Perfetti

W22. In triangle ABC we have $A\measuredangle = 90°$, $AB = AC$. The inscribed circle in triangle ABC have tangent ponts D, E with sides AB and AC. Let be $K \in (AB)$, $L \in (AC)$ such that $AD = BK$ and $AE = CL$ and let be $\{P\} = KE \cap DL$.

Prove that the incircle of triangles DKP and ELP are congruent with incircle of triangle ABC.

Ion Pătraşcu

W23. If $E\left(x,y\right)=\sqrt{xy+\left(1-x\right)\left(1-y\right)}$, $x,y\in\left(0,1\right)$, show that

$$\sqrt{abc}+\sqrt{\left(1-a\right)\left(1-b\right)\left(1-c\right)}\leq\min\left\{E\left(a,b\right),E\left(b,c\right),E\left(c,a\right)\right\}$$

for any $a,b,c\in\left(0,1\right)$

Ovidiu Pop

W24. Let $n\in N,n\geq2$, $i,j\in\{1,2,...,n\}$, $i\neq j$ and the transposition (ij). To determine all permutations $\sigma\in S_n$ such that

$$\sigma\left(ij\right)=\left(ij\right)\sigma$$

Ovidiu Pop

W25. For $n\in N^*$, with notation,

$$r_n=\sqrt{n\cdot\sqrt{\left(n-1\right)\cdot\sqrt{\left(n-2\right)\cdot\ldots\cdot\sqrt{3\cdot\sqrt{2\cdot\sqrt{1}}}}}},$$

prove that:

a). $r_n\leq\left(\frac{(n-1)\cdot2^n+1}{2^n-1}\right)^{1-\frac{1}{2^n}}$ b). $\frac{kn}{n}\in(0,1]$

Dorin Mărghidanu

W26. If a,b,c are strictly positive real numbers, then:

$$\left(a^a\cdot b^b\cdot c^c\right)^{\frac{1}{a+b+c}}+\left(a^b\cdot b^c\cdot c^a\right)^{\frac{1}{a+b+c}}+\left(a^c\cdot b^a\cdot c^b\right)^{\frac{1}{a+b+c}}\leq$$
$$\leq a+b+c$$

Dorin Mărghidanu

W27. Let u, v, w complex numbers such that:

$$u+v+w=1,\ u^2+v^2+w^2=3,\ uvw=1$$

Prove that:
a). u, v, w are distinct numbers two by two;
b). if $S^{(k)} := u^k + w^k$, then $S^{(k)}$ is an odd natural number;
c). the expression

$$\frac{u^{2n+1}-v^{2n+1}}{u-v}+\frac{v^{2n+1}-w^{2n+1}}{v-w}+\frac{w^{2n+1}-u^{2n+1}}{w-u}$$

is an integer number.

Dorin Mărghidanu

W28. For any fixed natural n let
$D_n := \{(x,y)\,|x,y\in Z \text{ and } F_{n+1}x-F_ny=1\}$, where F_n is n^{th} Fibonacci number. Find:

a). $\min\limits_{(x,y)\in D_n}|x+y|$; b). $\min\limits_{(x,y)\in D_n}(|x|+|y|)$; c). $\min\limits_{(x,y)\in D_n}\left(x^2+y^2\right)$

Arkady Alt

W29. Let a, b, c be side lengths of a triangle ABC and x, y, z be non-negative real numbers such that $x+y+z=1$ and let R be circumradius of this triangle. Prove that

$$a^2yz+b^2zx+c^2xy\le R^2$$

Arkady Alt

W30. Let $P_m(x) := \sum\limits_{k=0}^{m}\frac{(-1)^k x^{m-k}}{k+1}, m\in N$. For any $m\in N$ calculate:

$$\lim_{n\to\infty}\frac{\left(1+\frac{1}{n}\right)^{n^m}}{e^{P_{m-1}(n)}}$$

Arkady Alt

W31. Find $\inf\limits_{(x,y)\in D}(c-1)(y-1)$ where

$$D := \{(x,y)\,|x,y\in R_+, x\neq y \text{ and } x^y=y^x\}$$

Arkady Alt

W32. Calculate

$$\sum_{n=1}^{\infty} H_n\left[\zeta^2(2)-\left(1+\frac{1}{2^2}+\cdots+\frac{1}{n^2}\right)^2-\frac{2\zeta(2)}{n}\right],$$

where $H_n = 1+\frac{1}{2}+\cdots+\frac{1}{n}$ denotes the n^{th} harmonic number.

Ovidiu Furdui and Alina Sîntămărian

W33. Let $k\geq 1$ be an integer, $f:[0,1]\to\mathbb{R}$ be a Riemann integrable function and $g:[0,1]\to\mathbb{R}$ be a continuous function.
Calculate

$$\lim_{n\to\infty}\int_0^1 f(\left\{nx^k\right\})g(x^n)dx,$$

where $\{x\}$ denotes the fractional part of the real number x.

Ovidiu Furdui and Alina Sîntămărian

W34. Let be $m,n\in\mathbb{R}_+=[0,\infty); m+n>0$ and
$f:\mathbb{R}_+^*\times\mathbb{R}_+^*\to\mathbb{R}_+^*=(0,\infty),\ f(x,y)=\frac{x^{12}+y^4}{\sqrt{x^{12}-x^6y^2+y^4}}$. If ABC is a triangle with the area F, then:

$$\frac{f(a,z)}{(mx+ny)h_a}+\frac{f(b,x)}{(my+nz)h_b}+\frac{f(c,y)}{(mz+nx)h_c}\geq\frac{16}{m+n}\cdot F$$

D.M. Bătineţu-Giurgiu and Daniel Sitaru

W35. Prove without any software:

$$(\ln(e-1)+\ln(e+1))\ln\pi<\ln(\pi-1)+\ln(\pi+1)$$

Daniel Sitaru

W36. Let be $f(x) = ax^3 + bx^2 + cx + d; a, b, c, d \in \mathbb{R};\ a \neq 0;\ a + c \neq 0$. If f has three real roots which all lies in $(-1, 1)$ then:

$$\left|\frac{b+d}{a+c}\right| < 1$$

Daniel Sitaru

W37. If $f : [0,1] \to [0,1], f$ continuous, $0 \leq a, b \leq 1$ then:

$$3(b-a)^2 \int_a^b f(x)dx - \left(\int_a^b f(x)dx\right)^3 \leq 2(b-a)^3$$

Daniel Sitaru

W38. In $\Delta ABC; a, b, c \in (0,1)$. Prove that:

$$\frac{(s-2)^2 + r^2 + 4rR - 1}{(s-1)^2 + r^2 + 4R(r-s)} \geq \frac{3}{\sqrt[3]{(1-a)(1-b)(1-c)}}$$

Daniel Sitaru

W39. Let $a, b, c \in \mathbb{R}$. Solve the equations:

$$x^3 - 3ax^2 + 3(a^2 - b^2)x - a^3 + 3ab^2 - 2b^3 = 0 \quad (1)$$

$$x^3 - 3ax^2 + (3a^2 - b^2 - c^2 - bc)x - a^3 + ab^2 + ac^2 - b^2c - bc^2 + abc = 0 \quad (2)$$

Daniel Sitaru

W40. In a tetrahedron $ABCD$ let be r the radius of inscribed sphere and $r_A, r_B; r_C, r_D$ radii of exinscribed spheres. Prove that:

$$\frac{2r_A - r}{2r_A + r} + \frac{2r_B - r}{2r_B + r} + \frac{2r_C - r}{2r_C + r} + \frac{2r_D - r}{2r_D + r} \geq \frac{12}{5}$$

D.M. Bătineţu-Giurgiu and Daniel Sitaru

W41. Let $(x_n)_{n\geq 1}$, $(y_n)_{n\geq 1}$ be two sequences of real numbers such that $\lim\limits_{n\to\infty} \frac{y_n}{n} = 0$, $\lim\limits_{n\to\infty} \frac{y_n^2}{n} = \beta \in \mathbb{R}$, $\lim\limits_{n\to\infty} x_n = 0$,

$$\lim_{n\to\infty}\left(\sum_{k=1}^{n} x_k - y_k\right) = \alpha \in \mathbb{R}.$$

Then

$$\lim_{n\to\infty}\left(n\left(\left(1+\frac{x_1}{n}\right)\left(1+\frac{x_2}{n}\right)\dots\left(1+\frac{x_n}{n}\right)-1\right)-y_n\right) = \alpha + \frac{1}{2}\beta.$$

Marius Drăgan

W42. Let $a, b, c > 0$ and $a+b+c=1$.Then

$$(a+2ab+2ac+bc)^a(b+2bc+2ba+ca)^b(c+2ca+2cb+ab)^c \leq 1. \qquad (1)$$

Marius Drăgan

W43. Prove that:

i). $$\left[\tfrac{2}{3t-1}\left(\sqrt{(t+k)^3}+\sqrt{(t+k+1)^3}+\sqrt{(t+k+2)^3}-\sqrt{\left(k+\tfrac{1}{3}\right)^3}- -\sqrt{\left(k+\frac{4}{3}\right)^3}-\sqrt{\left(k+\frac{7}{3}\right)^3}\right)\right] = ???$$

$\forall t \in \left(\frac{1}{3}, \frac{4}{9}\right]$

ii). $$\left[\tfrac{2}{3t-2}\left(\sqrt{(t+k)^3}+\sqrt{(t+k+1)^3}+\sqrt{(t+k+2)^3}-\sqrt{\left(k+\tfrac{2}{3}\right)^3}- -\sqrt{\left(k+\frac{5}{3}\right)^3}-\sqrt{\left(k+\frac{5}{3}\right)^3}\right)\right] = ???$$

$\forall t \in \left(\frac{2}{3}, \frac{7}{9}\right]$

Mihály Bencze and Marius Drăgan

W44. Compute

$$\lim_{n\to\infty}\left\{n\left[\left(1+\frac{1}{n}\right)\left(1+\frac{1}{n\sqrt{2}}\right)\left(1+\frac{1}{n\sqrt{3}}\right)...\left(1+\frac{1}{n\sqrt{n}}\right)\right]\right\}-2\sqrt{n}$$

Mihály Bencze and Marius Drăgan

W45. Let $z_1, z_2, z_3, z_4, z_5, z_6$ the affixes of vertices $A_1, A_2, A_3, A_4, A_5, A_6$ of an regular hexagon. Then

$$\left(\frac{z_3-z_1}{z_6-z_1}\right)^n+\frac{1}{\sqrt{3}}\left(\frac{z_4-z_2}{z_1-z_2}\right)^{n+1}+\frac{1}{3}\left(\frac{z_5-z_3}{z_2-z_3}\right)^{n+2}+$$

$$+\frac{1}{3\sqrt{3}}\left(\frac{z_6-z_1}{z_3-z_4}\right)^{n+3}+\frac{1}{9}\left(\frac{z_1-z_5}{z_4-z_5}\right)^{n+4}=\left(\sqrt{3}i\right)^n$$

Mihály Bencze and Marius Drăgan

W46. Let n be a natural nonzero number. Study the convexity of the function $g:[0,+\infty)\to R$

$$g(x)=\sqrt[n]{x^n+C_n^1x^{n-1}+...+C_n^kx^{n-k}}$$

Marius Drăgan and Sorin Rădulescu

W47. In every triangle ABC is true the inequality:

i). $4\sum tg^2\frac{A}{2}tg^2\frac{B}{2}-3\sum tg^3\frac{A}{2}tg^3\frac{B}{2}\leq 1$
ii). $2\sum tg^2\frac{A}{2}+9\sum tg^2\frac{A}{2}tg^2\frac{B}{2}\leq 1$

Marius Drăgan

W48. Let p and q be complex numbers, for which we define the sequence $(w_n)_{n\geq 0}$ by the recurrence relation

$$w_{n+2}=pw_{n+1}+qw_n,\quad w_0=1, w_1=\sqrt{2},\quad n=0,1,\ldots.$$

1). Find the general formula for w_n, for $n=1,2,\ldots$.

2). Find p and q such that $(w_n)_{n\geq 0}$ is periodic of period 2021.

3). How many sequences $(w_n)_{n\geq 0}$ of period 2021 do exist, if $p^2+4q\neq 0$?

Ovidiu Bagdasar

W49. The sequence $(x_n)_{n\geq 0}$ is defined by $x_0=1$ and

$$x_{n+1}=\frac{\sqrt{3}x_n-1}{x_n+\sqrt{3}},\ n=0,1,\dots.$$

Find x_{2021}.

Ovidiu Bagdasar

W50. If $x>0$, then prove:

$$2arctgx\cdot arctgx-\frac{\pi}{2arctgx\cdot arctgx}<\frac{\pi^2}{4}-3\sqrt[3]{2}$$

Rovsen Pirguliyev

W51. In all triangle ABC holds:

1). $\sum\left(\frac{2a^3}{b^3}+\frac{b^3}{c^3}\right)r_a^2\geq\frac{12s^2(2R-r)^2}{s^2+r^2+4Rr}$
2). $\sum\left(\frac{2r_a^3}{r_b^3}+\frac{r_b^3}{r_c^3}\right)a^2\geq 12\left(2R-r\right)^2$

Mihály Bencze

W52. If $a_k\in\left(0,\frac{1}{4}\right]$ $(k=1,2,...,n)$ then

$$\sum_{1\leq i<j\leq n}\frac{1}{1-\sqrt{a_ia_j}}\leq\frac{n-1}{9}\left(5n+16\sum_{k=1}^{n}a_k^2\right)$$

If $a_k>\frac{1}{4}$ $(k=1,2,...,n)$ then holds the reverse inequality.

Mihály Bencze

W53. If $a, b, c > 0$ then

$$\prod_{k=1}^{n} \frac{a^{2k} + b^{2k} + c^{2k}}{a^k + b^k + c^k} \geq \left(\frac{a^2 + b^2 + c^2}{a + b + c} \right)^{\frac{n(n+1)}{2}}$$

Mihály Bencze

W54. Let $(f_n)_{n \geq 0}$ be the Fibonacci sequence and let the matrix

$$A = \begin{pmatrix} f_n & f_{n+1} & f_{n+2} & f_{n+3} \\ f_{n+1} & -f_n & f_{n+3} & -f_{n+2} \\ f_{n+2} & -f_{n+3} & -f_n & f_{n+1} \\ f_{n+3} & f_{n+2} & -f_{n+1} & -f_n \end{pmatrix}$$

Show that $\det A < 0$ and $\max \left\{ k \in N \middle| \det A \equiv 0 \left(\bmod 3^k \right) \right\} = 2.$

Nicuşor Minculete, Diana Savin and Andreea Dobre

W55. If $f : [0, 1] \to [0, \infty)$, $f(0) = 0$, it is convex and continuous function, show that the inequality

$$(t + 2) \int_0^1 f^t(x) dx \geq 2^t \left(\int_0^1 f(x) dx \right)^t$$

for all $t \geq 1$.

Stănescu Florin

W56. If n is a nonzero natural number, show that:

$$\frac{\cot\left(\frac{\pi}{2n}\right)}{\sin\left(\frac{\pi}{2n}\right)} + \frac{\cot\left(\frac{3\pi}{2n}\right)}{\sin\left(\frac{3\pi}{2n}\right)} + ... + \frac{\cot\left(\frac{(2n-1)\pi}{2n}\right)}{\sin\left(\frac{(2n-1)\pi}{2n}\right)} = 0$$

Stănescu Florin

W57. Let $f : [a, b] \to [0, \infty)$ two continuous functions, such that f it is convex, and g concave and increasing. If

$$f(a) \int_a^b g(x) dx = g(a) \int_a^b f(x) dx$$

show inequality:

$$\int_a^b f(x)g(x)h(x)dx \geq \frac{1}{b-a}\int_a^b h(x)dx \int_a^b f(x)g(x)dx$$

where $h : [a,b] \to R$ is an increasing function.

Stănescu Florin