L. A. G. test n. 3, april 21, 2016 Name:

1. Let $\mathcal{C}$ a parabola of focus F and directrix L. Let N be a unit normal vector to L and assume that $\mathcal{C}$ contains a point $X \in V_2$ such that $\| X - F \| = 4$ and the cosine of the angle between $X - F$ and N is equal to $-\frac{1}{5}$.

(a) Assuming that F is in the negative half-plane determined by N, compute $d(F, L)$.

(b) Assuming that F is in the positive half-plane determined by N, compute $d(F, L)$.

Solution. (a) In this case $d(F, L) + \frac{4}{5} = 4$. Hence $d(F, L) = \frac{16}{5}$ (you can also check this with the polar equation).
(b) In this case $4 + \frac{4}{5} = d(F, L)$. Hence $d(F, L) = \frac{24}{5}$.

2. (a) Let $P_1 = (-1, 1, -1)$, $P_2 = (1, 2, 2)$, $P_3 = (-3, 0, -4)$, $P_4 = (1, 1, 1)$. Is there a plane containing these four points? If yes, write down its cartesian equation, if no explain why.

(b) Let $P_1 = (-1, 1, -1)$, $P_2 = (1, 2, 2)$, $Q_3 = (1, 1, 2)$, $Q_4 = (1, 1, 1)$. Is there a plane containing these four points? If yes, write down its cartesian equation, if no explain why.

Solution. (a) Yes, $x + y - z = 1$ (note: the first three points are collinear). (b) No: the unique plane containing the first three points has equation $3x - 2z = -1$, and the fourth point is not contained in it.