L. A. G. final exam n.5, nov 7, 2016 Name:

1. Let $P=(1,2,-1)$ and $Q=(-1,3,5)$. (a) Find the plane M of V_3 such that the point of M which is closest to P is Q. $[2x-y-6z=-35]$
(b) Find the parametric equation of the line contained in M passing through Q and parallel to the plane $\pi: 3x-2y+z=0$. $[\{(-1,3,5)+t(-13,-20,-1) \mid t\in\mathbf{R}\}]$

2. Find the cartesian equation of the hyperbola passing trough the point $P=(0,2)$ and such that its vertices are $(1,-5)$ and $(1,1)$. Draw a rough picture. $[-\frac{7}{9}(x-1)^2+\frac{(y+2)^2}{9}=1]$

3. Let U be the space of polynomials of degree ≤ 2.
Let $S: U\to U$ defined by $S(P(x))=P'(x)(x+1)P(1)$. Is S a linear trasformation? Prove your answer. If the answer is yes, find the inverse (that is, given $P(x)$ find explicitly $S^{-1}(P(x))$). [*No*]
Let $T: U\to U$ defined by $T(P(x))=P'(x)(x+1)+P(1)x^2$. Is T a linear transformation? Prove your answer. If the answer is yes, find the inverse (that is, given $P(x)$ find explicitly $T^{-1}(P(x))$). [*Yes.* $T^{-1}(a+bx+ct^2)=(\frac{a}{2}+b-\frac{c}{2})+(-\frac{3a}{2}+\frac{c}{2})x+ax^2$]

4. Let us consider V_4 equipped with the usual dot product.
Let $U=\{(x,y,z,t)\in V_4 \mid 3x-2y+z-2t=0\}$ and let $\mathbf{v}=(1,-1,1,1)$.
Compute the distance between $\mathbf{v}$ and U and find the element of U which is closest to $\mathbf{v}$.
[$d(\mathbf{v},U)=\frac{2\sqrt{2}}{3}$. *Closest element:* $\frac{1}{9}(3,-5,7,13)$]

5. Let us consider the quadratic form $Q((x,y,z))=x^2+8xy+8xz+y^2+8yz+z^2$.
(a) Find an orthonormal basis $\mathcal{B}$ of V_3 and scalars λ_1, λ_2, λ_3 such that

$$Q((x,y,z))=\lambda_1 {x'}^2+\lambda_2 {y'}^2+\lambda_3 {z'}^2$$

where (x',y',z') is the vector of components of (x,y,z) with respect to the basis $\mathcal{B}$. [*Orthonormal basis such that...: there are infinitely many. One is*
$\mathcal{B}=(\frac{1}{\sqrt{2}}(1,-1,0), \frac{1}{\sqrt{6}}(1.1.-2), \frac{1}{\sqrt{3}}(1,1,1))$.
Normal form with respect to this basis: $Q(x,y,z)=-3(x')^2-3(y')^2=9(z')^2$]
(b) Consider the function

$$\{(x,y,z)\in V_3 \mid \ \| (x,y,z) \|=1\} \longrightarrow \mathbf{R}, \quad (x,y,z)\mapsto Q(x,y,z).$$

Compute maximum and minimum of this function. Find all points of maximum and all points of minimum. [*Maximum:* 9. *Points of maximum:* $\pm\frac{1}{\sqrt{3}}(1,1,1)$. *Minimum:* -3. *Points of minimum* $\{\lambda\frac{1}{\sqrt{2}}(1,-1,0)+\mu\frac{1}{\sqrt{6}}(1.1.-2) \mid \lambda,\mu$ *such that* $\lambda^2+\mu^2=1\}$]