L. A. G. final exam n.6, jan 23, 2017 Name:

1. Let M be the plane containing the points $(1,1,1)$, $(0,-1,-1)$ and $(1,0,-1)$.
Let L be the line $\{(1,1,2)+t(1,-2,1) \mid t \in \mathbf{R}\}$.
(a) Find the intersection of $M \cap L$.
(b) Find the points of L whose distance from M is 5

Solution. (a) The parametric equation of the plane M is $(1,1,1)+s((-1,-2,-2)+t(0,-1,-2)$. We compute the cartesian equation of M. We have that $(-1,-2,-2)\times(0,-1,-2)=(2,-2,1):=N$ is a normal vector and a cartesian equation is $N\cdot(x,y,z)=N\cdot(1,1,1)$, that is

$$2x-2y+z=1$$

Substituting the parametric equation of the line L we get

$$2(1+t)-2(1-2t)+2+t=1 \quad \leftrightarrow \quad t=-\frac{1}{7}$$

Therefore

$$L\cap M=(1,1,2)-\frac{1}{7}(1,-2,1)=\frac{1}{7}(6,9,13)$$

(b) Let $P_t=(1,1,2)+t(1,-2,1)=(1+t,1-2t,2+t)$ be a point of L. The distance between P_t and the plane M is

$$\frac{|2(1+t)-2(1-2t)+2+t-1|}{3}=\frac{|7t-1|}{3}$$

Solving the equation

$$\frac{|7t-1|}{3}=5$$

we get the two solutions $t=\frac{16}{7}$ and $t=-2$. Therefore the required points are

$$(1,1,2)+\frac{16}{7}(1,-2,1) \quad \text{and} \quad (1,1,2)-2(1,-2,1)$$

2. Let $\mathcal{C}$ a parabola of focus F and directrix L. Let N be a unit normal vector to L and assume that $\mathcal{C}$ contains a point $X\in V_2$ such that $\| X-F \|=7$ and the cosine of the angle between $X-F$ and N is equal to $-\frac{1}{2}$.

(a) Assuming that F is in the negative half-plane determined by N, compute $d(F,L)$.

(b) Assuming that F is in the positive half-plane determined by N, compute $d(F,L)$.

Solution. See Test 3 of the current year.

3. Let $T : V_3 \to V_4$ defined by $T((1,1,0)) = (1,0,-1,0)$, $T((0,1,-1)) = (0,1,0,1)$, $T((1,0,-1)) = (3,-2,-3,-2)$. Compute dimension and a basis of $T(V_3)$ and dimension and a basis of $N(T)$.

Solution. See Test 6 of the current year.

4. Let $T : V_3 \to V_3$, $T(x,y,z) = (2x+y+z, 2x+3y+2z, 3x+3y+4z)$. Find eigenvalues and eigenvectors of T. Find, if possible, a basis $\mathcal{B}$ of V_3 and a diagonal matrix D such that the matrix representing T with respect to $\mathcal{B}$ is D.

solution. This is Example 3 of §4.6 of the Textbook (Vol. II p. 105)

4. Version 2 Let $T : V_3 \to V_3$, $T(x,y,z) = (2x - y + z, 3y - z, 2x + y + 3z)$. Find eigenvalues and eigenvectors of T. Find, if possible, a basis $\mathcal{B}$ of V_3 and a diagonal matrix D such that the matrix representing T with respect to $\mathcal{B}$ is D.

solution. This is Example 2 of §4.6 of the Textbook (Vol. II p. 103)