ESERCIZI 1 bis

1 . Let $A = (1, 1, -1)$, $B = (1, 2, 0)$, $C = (1, 1, 1)$. (a) Is $\{A, B, C\}$ a basis of $\mathcal{V}_3$? (b) If the answer to (a) is in the affirmative, compute the components of the vector $(0, 1, 1)$ with respect to the basis $\{A, B.C\}$.

2. Let $A = (1, 1, -1)$, $B = (1, 2, 0)$, $C = (-2, -5, -1)$. (a) Is $\{A, B, C\}$ a basis of $\mathcal{V}_3$? (b) If the answer to (a) is in the negative, express a A,B or C as a linear combination of the remaining two vectors.

3. Is there an orthogonal basis $\{A, B, C\}$ of $\mathcal{V}_3$ such that $\parallel A - 2B + C \parallel = 12$ and $\parallel A + B + C \parallel = 6$?

4. Let $\{A, B, C\}$ be a orthogonal basis such that $\parallel A \parallel = \parallel B \parallel = 2$ and $\parallel C \parallel = 1$. Moreover let $V = A+B+C$ and $W = -A + B - C$. Find the cosine of the angle between V and W.

5. For t varying in $\mathbf{R}$, let $A_t = (t + 1, 1, 1)$, $B_t = (1, t + 2, 1)$, $C_t = (1, 1, t + 2)$.
(a) Find all $t \in \mathbf{R}$ such that A_t, B_t, C_t form an independent set.
(b) Compute, for each $h \in \mathbf{R}$, the solutions of the system (depending on h): $\begin{cases} hx + y + z = 0 \\ x + hy + z = 0 \\ x + y + hz = 0 \end{cases}$.

6. For t varying in $\mathbf{R}$, let $A(t) = (t + 1, t + 2, 2t)$, $B(t) = (1, t^2 + 1, 1)$, $C(t) = (1, 2, 1)$. (a) Find all t such that $A(t)$, $B(t)$ and $C(t)$ do not form a basis of $\mathcal{V}_3$. (b) For all $t, s \in$ let us consider the system

$$xA(t) + yB(t) + zC(t) = (s, 2s, 1).$$

For which values of t and s there is no solution? For which values of t and s there is more than one solution?

7. For t, a varying in $\mathbf{R}$, let us consider the system $\begin{cases} x + y + z = 1 \\ 2x + ty + z = -1 \\ 6x + 7y + 3z = a \end{cases}$. Find for which values of t and a there is a unique solution, no solution, infinitely many solutions.

8. Let $A = (3, 4)$. Describe the set of all vectors $X \in \mathcal{V}_2$ such that the angle between A and X is $\pi/4$.