L. A. G. 2015. Second intermediate test, april 1, 2015

1. Let $A = (3, -4)$ and $Q = (1, 1)$. Find a point $p \in \mathcal{V}_2$ such that the distance between Q and the line $\{P + tA \mid t \in \mathbf{R}\}$ is equal to 1.

Solution. First of all: in there is such a point $\bar{P}$, then there are infinitely many (ant point P belonging to the line $\{\bar{P} + tA \mid t \in \mathbf{R}\}$ will do the job).
A cartesian equation of a line parallel to the vector $A = (3, -4)$ is $4x + 3y = c$. We find c such that the distance between the line and Q is equal to 1: the distance is $\frac{|4+3-c|}{5}$ hence we want $c \in \mathbf{R}$ such that $\frac{|4+3-c|}{5} = 1$ that is $c = 2$ and $c = 12$ For example, for $c = 2$ we get the line of cartesian equation $4x + 3y = 2$. For example $P = (1/2, 0)$ is a point of that line.

2. Let $A = (1, 2, -3)$ and $B = (3. - 2, 4)$. (a) is there a vector $C \in \mathcal{V}_3$, non-parallel to A or B, such that $\| C \| = 1$ and such that $\det \begin{pmatrix} A \\ B \\ C \end{pmatrix} = 0$?. (b) If the answer is yes, show an example of such a vector C.

Solution. (a) We have that

$$\det \begin{pmatrix} A \\ B \\ C \end{pmatrix} = 0 \tag{1}$$

if and only if $\{A, B, C\}$ is a dependent set. Therefore, any linear combination of A and B, will do the job. hence the answer to the first question is yes.
(b) for example $C = A + B = (4, 0, 1)$, is such that (1) holds. This C is not a unit vector, but dividing by its norm we get the unit vector

$$\frac{1}{\| C \|} C = \frac{1}{\| C \|}(A + B) = \frac{1}{\| C \|} A + \frac{1}{\| C \|} B,$$

which is still a linear combination of A and B. This vector will do the job (as well as any other unit vector of $L(A, B)$). Therefore a vector as required is:

$$\frac{1}{\sqrt{17}}(4, 0, 1)$$