Since we know that – as A is symmetric – ~~eigenvalues~~ different eigenvalues have orthogonal eigenvectors, it follows that $E_{10} = L((-2,1,0)\times(-2,0,1)) = L((1,2,2))$.

(otherwise you can compute it;).

To find an orthonormal basis of eigenvectors one should first orthogonalize the basis of E_1:

$\underline{u}_1 = (-2,1,0)$ $\qquad \underline{u}_2 = (-2,0,1) - \left(\dfrac{(-2,0,1)\cdot(-2,1,0)}{(-2,1,0)\cdot(-2,1,0)}\right)(-2,1,0) =$

$$= (-2,0,1) - \frac{4}{5}(-2,1,0) = \left(-\frac{2}{5}, -\frac{4}{5}, 1\right)$$

Dividing by the norms we find the orthonormal basis

$$\mathcal{B} = \Big\{ \underbrace{\frac{1}{\sqrt5}(-2,1,0)}_{\underline{w}_1}, \underbrace{\frac{1}{3\sqrt5}(-2,-4,1)}_{\underline{w}_2}, \underbrace{\frac{1}{3}(1,2,2)}_{\underline{w}_3} \Big\}.$$

The theory tells us that, if (x',y',z') are defined by $(x,y,z) = x'\underline{w}_1 + y'\underline{w}_2 + z'\underline{w}_3$ then

$Q(x,y,z) = 1\cdot(x')^2 + 1\cdot(y')^2 + 10(z')^2 = (x')^2 + (y')^2 + 10(z')^2$ $\square$

(b) The answer is NO.

Indeed if $Q(x,y,z) = 0$ then $(x')^2 + (y')^2 + 10(z')^2 = 0$

Hence $x' = 0$ $y' = 0$ $z' = 0$. But $(x,y,z) = x'\underline{w}_1 + y'\underline{w}_2 + z'\underline{w}_3$. Therefore $(x,y,z) = (0,0,0)$

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