Therefore $\underline{u}$ can't be eigenvector of A:

$$A\underline{u} = A(\underline{v}+\underline{w}) = A\underline{v} + A\underline{w} = 2\underline{v} + \underline{w}.$$ Hence

$A\underline{u}$ cannot be of the form $\lambda\underline{u} = \lambda(\underline{v}+\underline{w}) = \lambda\underline{v} + \lambda\underline{w}$ for $\lambda \in \mathbb{R}$.

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5| [(a)] The matrix of the quadratic form Q is

$$A = \begin{pmatrix} 2 & 2 & 2 \\ 2 & 5 & 4 \\ 2 & 4 & 5 \end{pmatrix}$$

To find the canonical form of A amounts to find the eigenvalues of A and an orthonormal basis of V_3 of eigenvectors of A. (see the theory).

Calculation of the eigenvalues: $P_A(\lambda) = \det \begin{pmatrix} \lambda-2 & -2 & -2 \\ -2 & \lambda-5 & -4 \\ -2 & -4 & \lambda-5 \end{pmatrix} =$

$$\underset{A_3 \to A_3 - 2}{=} \det \begin{pmatrix} \lambda-2 & -2 & -2 \\ -2 & \lambda-5 & -4 \\ 0 & -\lambda+1 & \lambda-1 \end{pmatrix} = (\lambda-1)\det\begin{pmatrix} \lambda-2 & -2 \\ -2 & \lambda-5 \end{pmatrix} + (\lambda-1)\det\begin{pmatrix} \lambda-2 & -2 \\ -2 & -4 \end{pmatrix} =$$

$$= (\lambda-1)\left((\lambda-2)(\lambda-5) - 4 - 4(\lambda-2) - 4\right) = (\lambda-1)\left(\lambda^2 - 11\lambda + 10\right)$$

$$= (\lambda-1)^2(\lambda-10)$$

Therefore the eigenvalues are $\lambda_1 = 1$ and $\lambda_2 = 10$.

Calculation of the eigenspaces.

$$E_1: \begin{pmatrix} -1 & -2 & -2 \\ -2 & -4 & -4 \\ -2 & -4 & -4 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\{\, x + 2y + 2z = 0 \qquad x = -2y - 2z.$$ Therefore

$$E_1 = \{(-2y-2z, y, z) \mid y, z \in \mathbb{R}\} = L((-2,1,0), (-2,0,1))$$