In conclusion:

$x(t) = 5\cos 3t \qquad y(t) = 2\sin 3t$

(b) $\underline{r}(t) = (5\cos 3t \ , 2\sin 3t)$

therefore the Cartesian equation of the path is:

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$ It is an ellipse.

(c) $T = \dfrac{2\pi}{3}$

$\|\!\!-\!\!-\!\!\|$

4| (a) We know that $\lambda_1 = 2$ and $\lambda_2 = -1$ must be eigenvalues of A. Therefore the third eigenvalue must be $\lambda_3 = \dfrac{\det(A)}{\lambda_1 \cdot \lambda_2} = \dfrac{6}{-2} = -3$

$\underline{v}$ is an eigenvector of λ_1
$\underline{w}$ is an eigenvector of λ_2
$\underline{u}$ is such that $\mathcal{B} = \{\underline{v}, \underline{w}, \underline{u}\}$ form a basis of V_3.
Hence $\underline{u}$ must be eigenvector of λ_3. In conclusion. A is diagonalizable. It follows that there is only one matrix A as required:

$$A = C^{-1}\,\mathrm{diag}(2, -1, -3)\, C$$

where $C = m^{\mathcal{B}}_{\mathcal{E}}(id) = \begin{pmatrix} 1 & 1 & -2 \\ 2 & -1 & 1 \\ 1 & -1 & 1 \end{pmatrix}$ $\square$

(b) The answer is that there is NO such matrix. Indeed $\{\underline{v}, \underline{w}, \underline{u}\}$ is a dependent set (in fact $\underline{u} = \underline{v} + \underline{w}$).