Linear Algebra and Geometry. Fourth intermediate test, april 11, 2014.

1. For each $t \in \mathbf{R}$ and $a \in \mathbf{R}$ consider the system $\begin{cases} 3x + ty + 2z & =2 \\ 3tx - ty + z & =\text{a} \\ 6x + ty + z & =\text{t} \end{cases}.$
Determine all pairs (t, a) such that the system has a unique solution, no solutions, more than one solution.

2. Let $P = (1, 1, -1)$, $Q = (1, 0, 1)$, $R = (2, 2, -1)$. (a) Find the cartesian equation of the plane M containing P, Q and R. (b) Find the cartesian equations of the planes M' parallel to M such that $d(Q, M') = 4$.

SOLUTION

1. Let $\mathcal{A}(t)$ be the matrix of coefficients of the system. We find that $\det \mathcal{A}(t) = 3t^2 + 12t$. Therefore the system has a unique solution if and only if $t \neq 0, -4$. If $t = 0$: one finds (with gaussian elimination, for example) that there are solutions (necessarily infinitely many) if and only if $a = 4/3$. If $t = -4$ one finds (in the same way) that there are solutions if and only if $a = 16$. In conclusion: MORE THAN ONE (infinitely many) solutions: $(t, a) = (0, 4/3)$ and $(t, a) = (-4, 16)$. NO solutions for $(0, a)$ with $a \neq 4/3$ and for $(-4, a)$ with $a \neq 16$. ONE solution: for every (t, a) with $t \neq 0, -4$.

2. The equation of the plane M is $(X-P)\cdot(Q-P)\times(R-P) = 0$, that is: $-2x+2y+z = -1$. A parallel plane M' has equation as follows: $-2x + 2y + z = d$, and one has to find the d's such that $d(Q, M') = 4$. We know that $d(Q, M') = |(-2) \cdot 1 + 1 \cdot 1 - d|/3 = | -1 - d|/3$. hence the planes are two, one for $d = -13$ and the other for $d = 11$.