Linear Algebra and Geometry. Third intermediate test, april 4, 2014.

1. Let $P = (1, 2, -1)$ and $Q = (0, 1, -2)$. Which of the following points belong to the line containing P and Q?
(a) $P + Q$; (b) $Q + (-1, 2, 1)$; (c) $P + (3, 3, 3)$; (d) $P - 2Q$; (e) $Q + (5, 5, 5)$; (f) $P + (1, 3, 1)$

2. Let $L = \{(1, 2) + t(3, -4)\}$ and $O = (0, 0,)$. Compute $d(O, L)$ and the point of L closest to O.

3. Let $L = \{(-1, -1, -1) + t(1, 1, 0)\}$ and $S = \{(1, 4, -1) + t(0, 1, -1)\}$. Compute $L \cap S$.

SOLUTION

1. We know that a point R belongs to the line containing P and Q if and only if $R - P$ is a scalar multiple of $Q - P$. Equivalently, this can be expressed as: $R - Q$ is a scalar multiple of $Q - P$. Since $Q - P = (-1, -1, -1)$ this is even easier to check (the scalar multiples of that vector are the vectors whose coordinates are equal). Hence: (a) NO; (b) NO; (c) YES; (d) NO; (e) YES; (f) NO.

2. $d(O, L) = 2$. Closest point: $(1, 2) + (1/5)(3, -4) = (8/5, 6/5)$.
These solutions are obtained by applying directly the formulas. Closest point: writing the line as $\{P + tA\}$, we write $O - P = -P = \big((-P) \cdot A/(A \cdot A)\big)A + C = (1/5)(3, -4) + C$. We have that the closest point is $P + (1/5)A$ and $d(O, L) = \| C \|$.

3. $L \cap S = (1, 4, -1) - 2(0, 1, -1) = (1, 2, 1)$.
I recall, once again, how to di this calculation: a point of the intersection is both of the form $P + tA = Q + sB$. hence (t, s) are solutions of the system $tA + s(-B) = Q - P$. In our case: $t \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + s \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix}$. The solutions are easily found: $t = 3$, $s = -2$.