LINEAR ALGEBRA AND GEOMETRY

TEXTBOOK: T. Apostol: Calculus, Vol. I and II, Wiley (1969). Vol. I: Chapetrs 12, 13, 14, 15, 16. Vol. II: Chapters 3, 4, 5.

SYLLABUS:

STANDARD n-DIMENSIONAL EUCLIDEAN SPACES: vector algebra, dot product, norm and distance, orthogonal projection, orthogonality, angles, spanning sets, linear independence, bases. Standard complex euclidean spaces (Apostol, Vol. I Chapter 12) .

APPLICATIONS TO ANALYTIC GEOMETRY: lines, planes (parametric and cartesian equations), normal vectors, distance point-line and point-plane, cross product and area, mixed product and volume. Conic sections: polar equation, conic sections with center of symmetry, cartesian equations (Apostol, Vol., Chapter 13).

VECTOR-VALUED FUNCTIONS: curves, velocity vector and speed, normal vector, acceleration vector and curvature, osculating plane, arc-length, curves in polar coordinates, motion with central acceleration, Kepler's laws (Apostol, Vol. I, Chapter 14).

LINEAR SPACES: vector algebra, linear subspaces, spanning sets, linear independence, bases, dimension. Euclidean spaces: dot product, norms, orthogonality, orthogonal projection, Gram-Schmidt orthogonalization, orthogonal decomposition, explicit calculations in infinite-dimensional function spaces (Apostol, Vol. I, Chapter 15).

LINEAR TRANSFORMATIONS AND MATRICES: linear transformations (examples in finite and infinite dimension), null-space and rank, injectivity and surjectivity of linear transformations, linear transformations with prescribed values on a basis, matrices, matrices representing a linear transformation, systems of linear equations, inverses (Apostol, Vol. I, Chapter 16).

DETERMINANTS: definition via axioms, Laplace expansions, determinant of a product, determinant and linear independence, determinant and inverses, Cramer's rule (Apostol, Vol. II, Chapter 3).

EIGENVALUES AND EIGENVECTORS: basic properties, characteristic polynomial, trace, diagonalization of a linear operator (Apostol, Vol. II, Chapter 4).

EIGENVALUES OF OPERATORS ACTING ON EUCLIDEAN SPACES: hermitian and self-hermitian operators, symmetric and skew-symmetric matrices, hermitian and skew-hermitian matrices, orthogonal and unitary matrices, diagonalization of hermitian and skew-hermitian operators (spectral theorem). Quadratic forms and their normal forms. Applications to conic sections. (Apostol, Vol. II, Chapter 5).

IN THE LAG COURSE A CENTRAL ROLE IS PLAYED BY EXERCISES: FIRST OF ALL,THOSE IN THE TEXTBOOK, AND ALSO THOSE FOUND IN THE EXERCISES FILES.