EXERCISES, MAY 22, 2012

Ex. 0.1. *In* $V_4(\mathbb{R})$, *let* $W = L((1,2,1,-1),(1,1,-1,-1),(-1,1,5,1)$. *Find dimension and a basis of* W. *Find dimension and a basis of* $W^{\perp}$.

Ex. 0.2. *Find a basis of* $V_4(\mathbb{R})$ *containing the vectors* $(1,1,-1,1)$ *and* $(1,2,0,1)$.

Ex. 0.3. *In* $V_4(\mathbb{R})$, *with usual dot product. (a) Find a orthonormal basis* $\{e_1,e_2,e_3,e_4\}$ *of* $V_4(\mathbb{R})$ *such that* e_1 *is parallel to* $(0,3,4,0)$. *(b) Compute the distance between* $(1,0,0,0)$ *and* $L((0,3,4,0))^{\perp}$.

Ex. 0.4. *In* $V_4(\mathbb{R})$, *with the usual dot product. Let* $u_1 = (1,2,-2,3)$ *and* $u_2 = (2,3,-1,-1)$. *(a) Find a orthogonal basis* $\{v_1,v_2,v_3,v_4)$ *of* $V_4(\mathbb{R})$ *such that* $L(v_1,v_2) = L(u_1,u_2)$. *(b) Find an element* $u \in L(u_1,u_2)$ *and an element* $w \in L(u_1,u_2)^{\perp}$ *such that* $(1,0,0,0) = u + w$. *(c) Compute* $d((1,0,0,0),L(u_1,u_2))$ *and the point of* $L(u_1,u_2)$ *nearest to* $(1,0,0,0)$.

Ex. 0.5. *(a) In* V_3, *find a system of linear equations whose space of solutions is the line* $L((1,2,-1)) = \{t(1,2,-1) \mid t \in \mathbb{R}\}$. *(b) In* $V_4(\mathbb{R})$ *find a system of linear equations whise space of solutions is the two-dimensional linear subspace* $W = L((1,1,-1,0),(1,2,1,1)) = \{t(1,1,-1,0)+s(1,2,1,1) \mid t,s \in \mathbb{R}\}$.

Ex. 0.6. *In* $V_2(\mathbb{R})$ *define an inner product by*

$$< (x_1,x_2),(y_1,y_2) >= 3x_1y_1 - 2x_1y_2 - 2x_2y_1 + 3x_2y_2.$$

(a) Verify that this is in fact an inner product. (b) Find the orthogonal complement of $L((1,1))$ *with respect to this inner product. (c) Find the distance between* $(1,0)$ *and* $L((1,1))$ *with respect to this inner product. (d) Find an orthonormal basis of* $V_2(\mathbb{R})$ *with respect to this inner product.*

Ex. 0.7. *Are the vectors of* $V_2(\mathbb{C})$ $(i+1,3-2i)$ *and* $(5,5-5i)$ *parallel?*

Ex. 0.8. *Are the vectors of* $V_3(\mathbb{C})$ $A = (1+i,2,2-3i)$, $B = (2-i,1-i,-1-i)$ *and* $C = (1,1+i,3)$ *dependent or independent?*

Ex. 0.9. *In* $V_n(\mathbb{C})$, *with the usual dot product* $(x_1,\ldots,x_n)\cdot(y_1,\ldots,y_n) = \sum_{i=1}^{n} x_i\bar{y}_i$.
(a) In $V_2(\mathbb{C})$, *find* $L((i+1,2-5i))^{\perp}$.
In $V_3(\mathbb{C})$ *find* $L((1+i,i,1+2i),(2+i,2,2-i))^{\perp}$.