First Name: Last Name:

Linear Algebra and Geometry, Midterm exam, 02.16.2011

NOTE: you must give reasons for every solution/assertions. Final solutions, without adequate explanations, will not be evaluated. In practice, for each exercise you must (briefly) explain the steps of the reasoning.

1. Let $\{A, B, C\}$ be an orthogonal basis of $\mathcal{V}_3$ such that $\| A \| = 2$, $\| B \| = \sqrt{2}$, $\| C \| = \sqrt{3}$. Let $V = A + B + C$ and let respectively θ_A, θ_B, θ_C be the angles between V and A, B, C. (a) Compute $\cos\theta_A$, $\cos\theta_B$ and $\cos\theta_C$. (b) Compute $\| A \times V \|$ and $\| A \times B \|$. (c) Compute $|V \cdot (A \times B)|$.

2. Let A, B, C be three vectors in $\mathcal{V}_3$. Prove or disprove the following assertions:
(a) If A, B and C are linearly independent then the vectors $A+2B$, $A+B-C$, $A+B$ are linearly independent.
(b) The vectors $A + 2B$, $A + B - C$, $A + B$ can be linearly independent even if A, B and C are linearly dependent
(c) The vectors $A+2B$, $A+B-C$, $-A+2C$ are always linearly dependent, regardless of the linear dependence or independence of A, B, C.

3. In $\mathcal{V}_3$, let us consider the plane $M = \{(0, 0, 1) + t(1, 0, 1) + s(1, -1, 0)\}$, and the line $L = \{t(1, 1, 1)\}$. (a) Find all points in L such that their distance from M is equal to $\sqrt{3}$. (b) For each such point P find the cartesian equation of the plane parallel to M containing P.

4. Let $\mathcal{C}$ be a parabola with vertical directrix and the focus at the origin. Suppose that the point P of polar coordinates $(\rho, \theta) = (4, \pi/3)$ belongs to $\mathcal{C}$. (a) Find: the directrix, the vertex, the polar equation and the cartesian equation of $\mathcal{C}$ if the directrix lies in the half-plane $x > 0$. (b) Find: the directrix, the vertex, the cartesian equation of $\mathcal{C}$ if the directrix lies in the half-plane $x < 0$.

5. A curve in $\mathcal{V}_3$ is such that the velocity vector makes a constant angle with a fixed unit vector $\mathbf{c} \in \mathcal{V}_3$. (a) Suppose that the curve lies in a plane containing $\mathbf{c}$. Describe the curve. (b) Give an example of such a curve which is not contained in a plane.