Noncommutative geometry, Grand Symmetry and Higgs mass

in collaboration with F.Lizzi and P.Martinetti

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Overview

- NCG basic ideas
- The Standard Model
- Beyond the Standard Model
- Conclusion

Ideas

- Noncommutative geometry “a la Connes”: its main motivation is to extend the commutative duality between spaces and functions to the noncommutative setting.

GEL’FAND-NAIMARK THEOREM

$$\textit{Commutative } C^* - \textit{algebras } \mathscr{C}$$

$$\Longleftrightarrow$$

$$\textit{Hausdorff topological spaces } M$$

- We can reconstruct the space M from $\mathscr{C}(M)$ (points⇔IRR) such that $\mathscr{C}$ is $*-$*isomorphic* to the algebra of continuous functions $\mathscr{C}(M)$.

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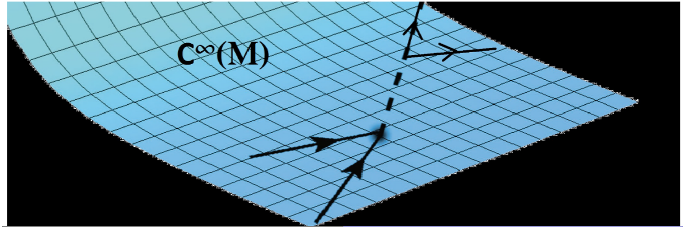
Motivations

- The next step is to give an algebric formulation of differential geometry:
- The introduction of a *canonical triple* $(\mathscr{A} = C^{\infty}(M), \mathscr{H} = L^2(M,S), \not{D})$ allow to fully describe a compact Riemannian spin manifold M.

(A. Connes, Noncommutative Geometry, 1994)



- The metric on the manifold will be determined by the Dirac operator $\not{D}$, $d_D(p,q) = sup_f \{ |f(p) - f(q)| : f \in \mathscr{A}, || [\not{D}, f] ||_{\mathscr{H}} \leq 1 \}$

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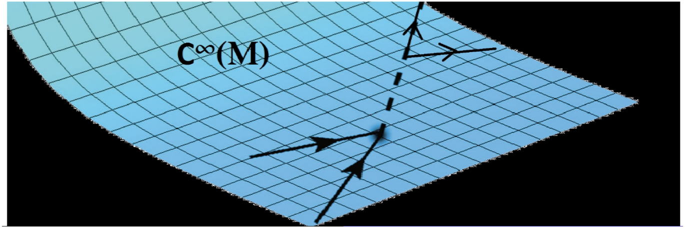

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NC extension

Spin Manifold M	$\leftrightarrow$	**Canonical triple** $\left(C^\infty(M), L^2(M,S), \not{D}\right)$
	$\Updownarrow$	
NC Manifold	$\leftrightarrow$	???

NC extension

Spin Manifold M	$\leftrightarrow$	**Canonical triple** $\left(C^\infty(M), L^2(M,S), \not{D}\right)$
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NC Manifold	$\leftarrow$	**Spectral triple** $(\mathscr{A}, \mathscr{H}, D)$

Spectral triple

- Gauge physical model $\leftarrow$ A set $(\mathscr{A}, \mathscr{H}, D)$
- $\mathscr{A}$ is $* -$ *algebra* represented on the Hilbert space $\mathscr{H}$.
- D, the generalized Dirac operator, is a self-adjoint operator
  1) Compact resolvent: $(D-\lambda)^{-1}$, $\lambda \notin \mathbb{R}$;
  2) $[D, a]$ is a bounded operator.
- Two others ingridients have to be introduced making the spectral triple a graded, real spectral triple: Γ, J
- Γ is a chirality operator that split the Hilbert space in the Left and Right part.
- J is called Real Structure is an anti-unitary operator on $\mathscr{H}$ linked to the charge conjugation operator

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The classical conditions

The graded real spectral triple has to satisfy the following conditions:

Conditions (*J.Noncommut.Geom. 7 (2013) 1-82 arXiv:0810.2088*)

1. Classical dimension n ($|D|^{-1}$infinitesimal of order n).
2. Regularity (a, $[D,a] \in \mathscr{B}(\mathscr{H})$)
3. Finiteness
4. Reality $J^2 = 1$; $JD = DJ$; $J\gamma = -\gamma J$; $[a,\gamma] = 0$(Chirality condition) $[a, Jb^*J] = 0$. *(Order zero condition)*
5. First order of the Dirac operator$[[D,a], Jb^*J] = 0$. *(Order one condition)*
6. Orientability
7. Poincarè duality

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Almost commutative spectral triple

Almost commutative spectral triple

$$(C^\infty(M)\otimes\mathscr{A}_F,\ Sp(L^2(M))\otimes\mathscr{H}_F,\ \mathscr{D},\ \gamma^5\otimes\gamma_F,\ \mathscr{J}\otimes J_F)$$

- Tensor product of the commutative algebra of function and a finite matrices algebra.
- The Dirac operator $\mathscr{D}$ is taken as the tensor product:

$$\begin{array}{ccccc}
\mathscr{D} & = & \partial\!\!\!/ & \otimes\mathbb{I}_F \oplus \gamma_5\otimes & D_F \\
 & & \downarrow & & \downarrow \\
\mathscr{A} & = & C^\infty(M) & \otimes & \mathscr{A}_F \\
 & & \downarrow & & \downarrow \\
\mathscr{H} & = & Sp(L^2(M)) & \otimes & \mathscr{H}_F
\end{array}$$

SM algebra

Standard Model

Gauge theory for electro-weak and strong interactions:

$$U(1) \times SU(2) \times SU(3)$$

Standard Model algebra

$$\mathscr{A}_{\mathsf{SM}} = C^\infty(M) \otimes [\mathbb{H} \oplus \mathbb{C} \oplus \mathbb{M}_3(\mathbb{C})]$$

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SM algebra

What is the most general form of the finite algebra?

Finite algebra

We look for algebras $\mathscr{A}_F$ in $\mathscr{H}_F$ such that

$$[a, Jb^* J^{-1}] = 0, \forall a, b \in \mathscr{A}_F$$

SM algebra

It is possible to show that the solution is,

Finite algebra (Why the Standard Model, A.Chamseddine, A.Connes)

$$\mathscr{A}_{\mathsf{F}} = \mathbb{M}_a(\mathbb{H}) \oplus \mathbb{M}_{2a}(\mathbb{C})$$

with $2(2a)^2 = n$, finite Hilbert space dimension.

SM algebra

Standard Model...and beyond

$$\mathscr{A}_{\mathsf{F}} = \mathbb{M}_a(\mathbb{H}) \oplus \mathbb{M}_{2a}(\mathbb{C}) \Rightarrow \begin{cases} a = 1, & n = 8, \ \textbf{Trivial grading} \\ a = 2, & n = 32, \ \textbf{Standard Model} \\ a = 3, & n = 72, \ \textbf{No physics mean} \\ a = 4, & n = 128 \ \textbf{Grand Symmetry} \end{cases}$$

Standard Model algebra (a=2)

$$\mathscr{A}_{\mathsf{sm}} \subset \mathbb{M}_2(\mathbb{H}) \oplus \mathbb{M}_4(\mathbb{C})$$

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Hilbert space

- The finite Hilbert space $\mathscr{H}_F$ contains all the 32 particle-degrees of freedom of the standard model (one particles family):

$$\mathscr{H}_F = \mathscr{H}_R \oplus \mathscr{H}_L \oplus \mathscr{H}_R^c \oplus \mathscr{H}_L^c = \mathbb{C}^{32}$$

Finite space $\mathscr{H}_F^{(32)}$

$$\begin{pmatrix} \nu_L^e \\ e_L \end{pmatrix}, \; SU(2)\,\text{doublets} \qquad \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}, \begin{cases} SU(2)\,\text{doublets} \\ SU(3)\,\text{triplets} \end{cases}$$

$$\begin{bmatrix} (e_R), \\ (\nu_R^e), \end{bmatrix}, \; U(1)\,\text{singlets} \qquad \begin{bmatrix} (u_R^i), \\ (d_R^i), \end{bmatrix}, \begin{cases} U(1)\,\text{singlets} \\ SU(3)\,\text{triplets} \end{cases} \quad \Rightarrow \Psi_\alpha^{CIm}(x) \in \mathscr{H}_F$$

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Hilbert space (2)

$\Psi_{\alpha}^{CIm}(x)$, the meaning of the indices is as follows:

- $\Psi_{\alpha}^{CIm}(x)$ particle-antiparticle index: C=0,1

  C = 0, 1 indicates whether we are considering “particles” (C = 0) or “antiparticles” (C = 1).

- $\Psi_{\alpha}^{CIm}(x)$ generation index: $m = 1, 2, 3$

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- $\Psi_{\alpha}^{CIm}(x)$ flavour index: α=$1, 2, \tilde{1}, \tilde{2}$

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Chirality

- Γ is a chirality operator that split the Hilbert space in the Left and Right part, for N families of fermions

$$\Gamma = \gamma^5 \otimes \gamma_F\,,\ \gamma_F = \begin{pmatrix} \mathbb{I}_{8N} & & & \\ & -\mathbb{I}_{8N} & & \\ & & -\mathbb{I}_{8N} & \\ & & & \mathbb{I}_{8N} \end{pmatrix}$$

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Real structure

- J is called Real Structure is an anti-unitary operator on $\mathscr{H}$ linked to the charge conjugation operator:

$$J = \mathscr{J} \otimes J_F$$

where $\mathscr{J}$ is the charge conjugation operator,

$$J_F = \begin{pmatrix} 0 & \mathbb{I}_{16N} \\ \mathbb{I}_{16N} & 0 \end{pmatrix} cc : \begin{pmatrix} \psi^P \\ \psi^A \end{pmatrix} \to \begin{pmatrix} \psi^{A*} \\ \psi^{P*} \end{pmatrix}$$

cc=complex conjugation

Finite Dirac operator

- The finite Dirac operator is a finite matrix,

$$D_F = \begin{pmatrix} 0 & \mathcal{M} & \mathcal{M}_R & 0 \\ \mathcal{M}^\dagger & 0 & 0 & 0 \\ \mathcal{M}_R^\dagger & 0 & 0 & \mathcal{M}^* \\ 0 & 0 & \mathcal{M}^T & 0 \end{pmatrix} \rightarrow \begin{pmatrix} \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}^P \\ \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}^A \end{pmatrix},$$

- containing Yukawa couplings so that the quantity $< \psi | D_F | \psi >$ will give the usual Standard Model fermionic action.
- $\mathcal{M}$ in D_F contains Yukawa coupling and its role is to mix ψ_L^P with ψ_R^{P*} giving an usual particle mass term.
- $\mathcal{M}_R$ in D_F contains Majorana coupling and mix ψ_R^A with ψ_R^{P*} giving a so called Majorana mass term $\rightarrow$ See-saw mechanism

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Spectral Action

- Given the Spectral Triple $(\mathscr{A}, \mathscr{H}, D, \Gamma, J)$ we define the Spectral Action as

$$S_B = \mathrm{Tr}[f\left(\frac{D_A^2}{\Lambda^2}\right)]$$

f a smooth approximation of the characteristic function [0,1].

- D_A is the fluctuated Dirac operator: $D_A = D + A + JAJ$ and A is the 1-form connection given by

$$A = \sum a_i[D, b_i]$$

- Heat kernel expansion $\Longrightarrow$SM bosonic action (gauge lagrangian and Higgs potential)
- The Dirac operator plays a multiple role:

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Metric of M Internal metric	Higgs and Gauge Bosons	Fermion mass (Dynamic)

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Reductions

- Mathematical reduction: starting from $[\mathbb{M}_2(\mathbb{H}) \oplus \mathbb{M}_4(\mathbb{C})]$ the manifold conditions reduce the algebra to $\mathbb{H} \oplus \mathbb{C} \oplus \mathbb{M}_3(\mathbb{C})$.

Chirality condition

$$[a, \Gamma] = 0$$
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Pati – Salam model

Order-1 condition

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Results

- $m_\nu \approx eV$ (See-saw mechanism)
- Gravity coupled to the Standard Model
- Top mass $m_{top} \sim 173$GeV
- Higgs field with $m_H \backsim 170 GeV$ ... ruled out by Tevratron in August 2008.

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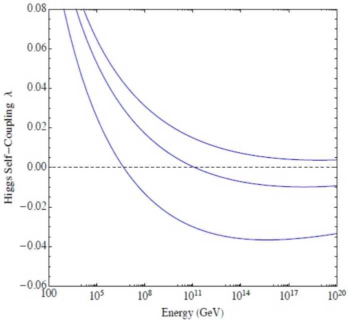
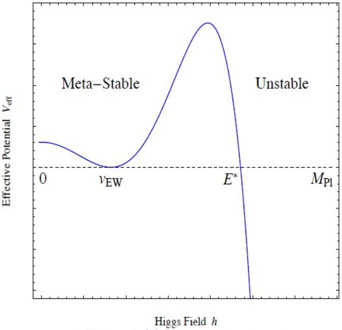
Problems

- Mass of the Higgs-Brout-Englert boson, official since July 2012, is 126 GeV.

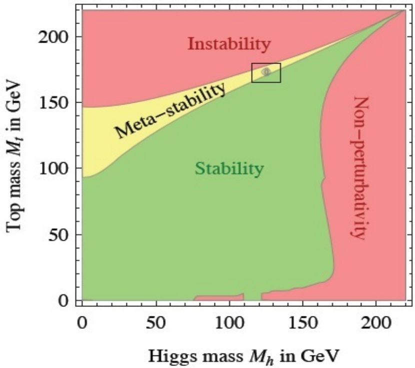
$$V_H = -\frac{\mu^2}{2} H^2 + \frac{\lambda}{4} H^4$$





Meta-stability

For $m_{top} = 173 GeV$ and $m_H = 126 GeV$ the model is meta-stable:



(Degrassi, Di Vita, Elias-Miro, Espinosa, Guidice, Isidori and A. Sturmia, Higgs mass and Vacuum Stability in the SM at NNLO, arXiv:1205.6497)

New scalar field σ

- Instability can be cured introducing a new scalar field σ:

$$V(H,\sigma) = \frac{1}{4}\left(\lambda H^4 + \lambda_\sigma \sigma^4 + 2\lambda_{H\sigma} H^2 \sigma^2\right)$$

- In the description of the SM in NCG, the field σ allows to pull m_H back to 126 GeV.

  (Resilience of the spectral SM, Chamseddine, Connes 2012)

- How is possible to derive this field in NCG?

- By turning the entry of the neutrino Majorana mass in D_F into a field,

$$k_R \rightarrow k_R \sigma(x)$$

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Standard model limits

- All the gauge fields of the SM (including the Higgs) are obtained by fluctuation of the metric

$$[D, a] = [\partial\!\!\!/ \otimes \mathbb{I}_F \oplus \gamma_5 \otimes D_F \, , f^i \otimes m_i]$$

contained in the 1-form connection.

- Unfortunately, the first order conditon prevents to do so for the field σ. Indeed, for D_R the Dirac with only the neutrino Majorana mass,

$$\left[[D_R, a], Jb^* J^{-1}\right] = 0 \Rightarrow [D_R, a] = 0$$

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Beyond the standard model

It's possible to find this scalar field going beyond the standard model:

- Enlarge the Hilbert space adding extra fermions (New Scalar Fields in Noncommutative Geometry, C.Stephan 2009)
- Renouncing to the 1-order condition $\mapsto$Generating the field σ, and retrieving the 1st-order condition dynamically, by minimizing the spectral action (Chamseddine, Connes, van Suijlekom, Inner fluctuation without first order condition 2013)
- Enlarge the starting algebra considering the case

$$\mathbb{M}_a(\mathbb{H}) \oplus \mathbb{M}_{2a}(\mathbb{C})$$

with $a = 4$, i.e. Grand Symmetry.

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Grand symmetry

Standard Model...and beyond

$$\mathscr{A}_{\mathbf{F}} = \mathbb{M}_a(\mathbb{H}) \oplus \mathbb{M}_{2a}(\mathbb{C}) \Rightarrow \begin{cases} a = 1, & n = 8, \textbf{Trivial grading} \\ a = 2, & n = 32, \textbf{Standard Model} \\ a = 3, & n = 72, \textbf{No physics mean} \\ a = 4, & \mathbf{n = 128\, GrandSymmetry} \end{cases}$$

Grand Symmetry algebra (a=4)

$$\mathscr{A}_{\mathbf{GS}} = \mathbb{M}_4(\mathbb{H}) \oplus \mathbb{M}_8(\mathbb{C}) \to \mathscr{H}^{(128)}(4_{(Lorentz)} \times 32_{SM})$$

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Hilbert space

- The Hilbert space will be the same as before

$$\mathscr{H} = Sp\left(L^2(M)\right) \otimes \mathscr{H}_F$$

- Let us write a vector in the Hilbert space in the following way:

$$\Psi^{Clm}_{s\dot{s}\alpha}(x) \in \mathscr{H}$$

- $\Psi^{Clm}_{s\dot{s}\alpha}(x)$ spinor indices: $s = r, l\,;\dot{s} = \dot{0}, \dot{1}$
  They take two values each, and together they make the four indices on an ordinary Dirac spinor. In the SM the algebra $\mathscr{A}_F$ acts diagonally on it.

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$$= L^2(M) \otimes \mathrm{H}_F \quad \left(1_{(Function)} \times 128_{GS}\right)$$

note the difference between $\mathscr{H}_F$, which is 32 dimensional (standard model degrees), and H_F which is 128 dimensional $4_{(Lorentz)} \times 32_{SM}$.

- $\mathrm{H}_F = \mathbb{C}^{128}$ takes into account both external (i.e. spin) and internal (i.e. particle) degrees of freedom.
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Grand algebra action

- Since the Hilbert space is not changed then neither Chirality Γ nor Reality J will change.
- The difference now is the action of the algebra on the Hilbert space. Explicitly, the algebra representations in term of these indices are:

SM algebra action

$$A^{t\dot{t}CI\beta}_{s\dot{s}DJ\alpha} = \delta^{t}_{s}\delta^{\dot{t}}_{\dot{s}}\left(\delta^{C}_{0}\,\delta^{I}_{J}\,Q^{\beta}_{\alpha} + \delta^{C}_{1}\,M^{I}_{J}\,\delta^{\beta}_{\alpha}\right)$$

$$\mathscr{A}_{SM} = \overbrace{C^{\infty}(M)}\otimes\overbrace{[\mathbb{M}_2(\mathbb{H})\oplus\mathbb{M}_4(\mathbb{C})]}$$

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Chirality reduction

- Chirality condition on quaternion part leads to:

Chirality condition

$$[A, \Gamma] = 0$$
$$\Downarrow$$
$$[\mathbb{M}_4(\mathbb{H}) \oplus \mathbb{M}_8(\mathbb{C})]$$
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Chirality condition

$$\left[Q^{\dot{t}\beta}_{\dot{s}\alpha}, \begin{pmatrix} \mathbb{I}_2 & 0 \\ 0 & -\mathbb{I}_2 \end{pmatrix}_{\alpha\beta} \right] = 0$$

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Chirality reduction (2)

- Chirality condition on complex matrix part:

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Complex matrix part

$$\left[M^{tI}_{sJ}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}_{st} \right] = 0$$

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Complex matrix part

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Order one reduction (1)

• Order one condition without Majorana coupling leads to two SM copies plus others two $\mathbb{C}$ algebras:

$$[[D, a], Jb^* J] = 0$$
$$\Downarrow$$
$$[\mathbb{M}_2(\mathbb{H}) \oplus \mathbb{M}_2(\mathbb{H})] \oplus [\mathbb{M}_4(\mathbb{C}) \oplus \mathbb{M}_4(\mathbb{C})]$$
$$\downarrow$$
$$\mathbb{C} \oplus \mathbb{H}_R \oplus \mathbb{C}' \oplus \mathbb{H}_L \oplus \mathbb{C}_R \oplus \mathbb{M}_3(\mathbb{C}) \oplus \mathbb{C}_L \oplus \mathbb{M}_3'(\mathbb{C})$$

Order one reduction (2)

•The 1°order condition with Majorana coupling leads to the identification of three $\mathbb{C}$ algebras:

$$[[D_R, a], Jb^* J] = 0$$

$$\Downarrow$$

$$\Big(\underbrace{\mathbb{C}}\oplus\mathbb{H}_R\Big)\oplus(\mathbb{C}'\oplus\mathbb{H}_L)\oplus\Big(\underbrace{\mathbb{C}_R}\oplus M_3(\mathbb{C})\Big)\oplus\Big(\underbrace{\mathbb{C}_L}\oplus M_3'(\mathbb{C})\Big)$$

$$\downarrow$$

$$\underbrace{\mathbb{C}}\oplus\mathbb{H}_R\oplus M_3(\mathbb{C})\oplus[\mathbb{C}'\oplus\mathbb{H}_L\oplus M_3(\mathbb{C})]$$

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$$\downarrow$$

$$\underbrace{\mathbb{C}_R}\oplus\mathbb{H}_R\oplus\mathbb{M}_3(\mathbb{C})\oplus[\mathbb{C}'_L\oplus\mathbb{H}_L\oplus\mathbb{M}_3(\mathbb{C})]$$

$$\begin{pmatrix} \begin{pmatrix} c_R & 0 \\ 0 & \bar{c_R} \end{pmatrix} & & & & & \\ & (\mathbb{H}_R) & & & & \\ & & \begin{pmatrix} c'_L & 0 \\ 0 & \bar{c}'_L \end{pmatrix} & & & \\ & & & (\mathbb{H}_L) & & \\ & & & & \begin{pmatrix} m_r & 0 \\ 0 & {M_r}^i_j \end{pmatrix} & \\ & & & & & \begin{pmatrix} m_l & 0 \\ 0 & {M_l}^i_j \end{pmatrix} \end{pmatrix}$$

$$c_R = m_r = m_l$$

$\sigma - field$ as 1-form

• Writing down the 1-forms generated by the Majorana part of the Dirac operator we have:

$$[D_R, a] =$$

$$= \begin{bmatrix} \begin{pmatrix} k_r(m_r - c_R) & 0 \\ 0 & k_r(m_r - c'_L) \end{pmatrix}_{\dot{s}}^{\dot{t}} \Xi^{I\beta}_{J\alpha} & \\ & \begin{pmatrix} k_l(m_l - c_R) & 0 \\ 0 & k_l(m_l - c'_L) \end{pmatrix}_{\dot{s}}^{\dot{t}} \Xi^{I\beta}_{J\alpha} \end{bmatrix}$$

with $\Xi^I_J = \Xi^\beta_\alpha = \begin{pmatrix} 1 & 0 \\ 0 & 0_3 \end{pmatrix}$.

σ – *field* as 1-form

- Reduction due to the order one condition requires

$$c_R = m_r = m_l$$

leading to

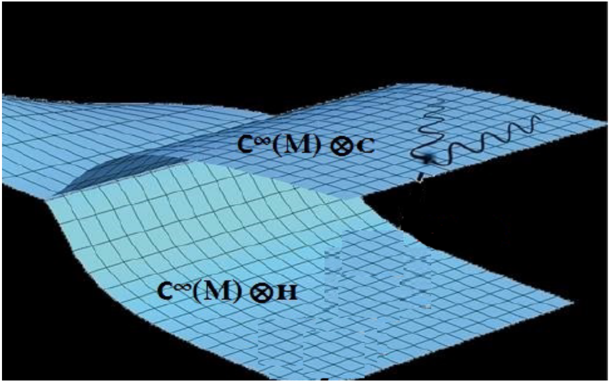
$$A \sim \begin{bmatrix} \begin{pmatrix} 0 & 0 \\ 0 & k_r(m_r - c'_L) \end{pmatrix}_{\dot{s}\dot{t}} \Xi^{I\beta}_{J\alpha} & \\ & \begin{pmatrix} 0 & 0 \\ 0 & k_l(m_r - c'_L) \end{pmatrix}_{\dot{s}\dot{t}} \Xi^{I\beta}_{J\alpha} \end{bmatrix}_{st}$$

The coefficient m_r and c'_L are functions on the manifold $\mathscr{M}$ and we can identify the σ field as

$$\sigma = k_r(m_r - c'_L)$$

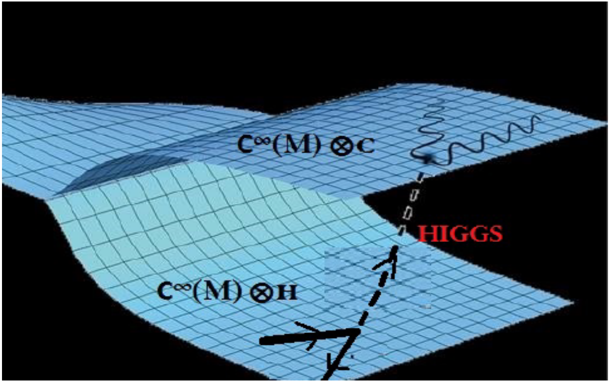
Higgs field role

- The Higgs field links the two manifold sheets mixing the elements of $C^\infty(M) \otimes \mathbb{H}$ and $C^\infty(M) \otimes \mathbb{C}$:



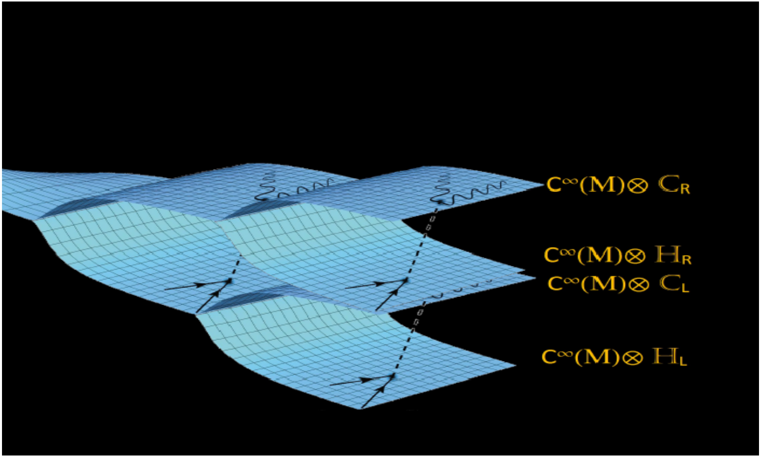
Higgs field role

- The Higgs field links the two space-time sheets mixing the elements of $C^\infty(M) \otimes \mathbb{H}$ and $C^\infty(M) \otimes \mathbb{C}$:



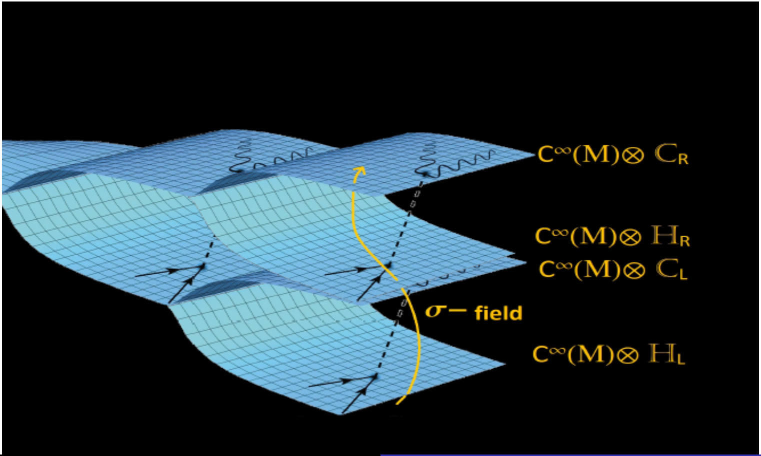
$\sigma - \textit{field}$ roles

- In this phase the σ field mixes the complex algebras of a bigger space $C^\infty(M) \otimes [\mathbb{C}_R \oplus \mathbb{H}_R \oplus \mathbb{C}_L \oplus \mathbb{H}_L]$



Grand Algebra space

- ...in fact $\sigma \sim (m_r - c'_L)$ in the 1-form connection links the different sheets $m_r \leftrightarrow c'_L$ $\{\mathbb{C}_\mathbb{R} \leftrightarrow \mathbb{C}_L\}$



Results

- The Grand Symmetry explains the presence of the σ field necessary for a correct fit of the Higgs mass and its stability.
- We pointed out there is a “next level” in noncommutative geometry, that intertwined the Riemannian and spin structure of spacetime.
- In this scenario the Majorana neutrino is the first field to appear and fluctuate, even before the geometric structure of spacetime emerges

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Open questions

- The final reduction to the usual standard model algebra $\mathscr{A}_{sm} = \mathbb{H} \oplus \mathbb{C} \oplus \mathbb{M}_3(\mathbb{C})$ is obtained using the order one condition on the free Dirac operator $\partial\!\!\!/$. However, the presence of a dynamical breaking, namely a minimization principle, is preferred...
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Thanks for your attention

Spectral Action

- In the SM case,

$$\mathrm{Tr}[f\left(\frac{D_A^2}{\Lambda^2}\right)] = \frac{24}{\pi^2} f_4 \Lambda^4 \int d^4x\sqrt{g} - \frac{2}{\pi^2} f_2 \Lambda^2 \int d^4x\sqrt{g}\left[R + \frac{1}{2} a\overline{H}H\right] +$$
$$+ \frac{1}{2\pi^2} f_0 \int d^4x\sqrt{g}[\frac{1}{30}\left(-18C^2_{\mu\nu\rho\sigma} + 11R^*R^*\right) +$$
$$+ \frac{5}{3} g_1^2 B^2_{\mu\nu} + g_2^2 \mathbf{W}^2_{\mu\nu} + g_3^2 \mathbf{V}^2_{\mu\nu}$$
$$+ \frac{1}{6} aR\overline{H}H + a\left(\overline{H}H\right)^2 + a(\nabla_\mu H)^2] + \ldots$$

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