Scaling Limit for Subsystems and Doplicher-Roberts Reconstruction

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Operator Algebras and Conformal Field Theory

ESI, September 8th, 2008

Motivation

- Study of subsystems $\mathscr{A} \subset \mathscr{B}$ aims at an intrinsic description of the observables of a QFT and their superselection charges.
- Buchholz-Verch Scaling Algebras provide tools for analysing the short distance (high energy) properties of QFT in a model independent setting.
- Natural question: given subsystem $\mathscr{A} \subset \mathscr{B}$, what can be said about scaling limit theories $\mathscr{A}_0$, $\mathscr{B}_0$? In particular:
  - It is possible to find $\mathscr{A} \subsetneq \mathscr{B}$ such that $\mathscr{A}_0 = \mathscr{B}_0$?
  - Relation between type of scaling limits of $\mathscr{A}$ and $\mathscr{B}$ (in Buchholz-Verch classification).
  - Relation between superselection structures of $\mathscr{A}_0$ and $\mathscr{B}_0$.

Outline

Superselection Theory 1/2

Data:

- $\mathscr{H}$ separable;
- $O \subset \mathbb{R}^4 \to \mathscr{A}(O) \subset B(\mathscr{H})$ net of observable algebras satisfying Haag duality
$$\mathscr{A}(O) = \mathscr{A}(O')';$$
- $\gamma \to U(\gamma)$ unitary representation of $\mathscr{P}_+^\uparrow$ with positive energy, such that
$$U(\gamma)\mathscr{A}(O)U(\gamma)^* = \mathscr{A}(\gamma.O);$$
- $\Omega \in \mathscr{H}$ unique such that $U(x)\Omega = \Omega$ (vacuum).

Superselection sectors are described by classes of localized endomorphisms:

$$\Delta(O) := \{\rho \in \mathrm{End}(\mathscr{A}) \,:\, \rho(A) = A \;\forall A \in \mathscr{A}(O')\}$$

Superselection Theory 2/2

Theorem ([Doplicher-Roberts '90])

$\exists O \to \mathscr{F}(O)$ field net, $g \in G \to V(g)$, G compact, such that:

- *$\mathscr{F}(O)^G = \mathscr{A}(O)$;*
- *$\forall \rho \in \Delta(O)\ \exists \psi_1, \dots, \psi_d \in \mathscr{F}(O)$ orthogonal isometries, $v_{[\rho]}$ d-dimensional irrep of G, with*

$$\mathrm{Ad}\, V(g)(\psi_i) = \sum_{j=1}^{d} v_{[\rho]}(g)_{ij}\psi_j, \quad \rho(A) = \sum_{j=1}^{d} \psi_j A \psi_j^*;$$

- *$\mathscr{F}$ has normal commutation relations, defined by $k \in Z(G)$ with $k^2 = e$.*

Superselection Theory for Subsystems 1/2

$\mathscr{B}, \mathscr{F}$ field nets.

Definition

$\mathscr{B} \subset \mathscr{F}$ is a subsystem if

$$\mathscr{B}(O) \subset \mathscr{F}(O), \qquad U(\gamma)\mathscr{B}(O)U(\gamma)^* = \mathscr{B}(\gamma.O).$$

Main examples:

- $\mathscr{B} \subset \mathscr{F} = \mathscr{B}\hat{\otimes}\tilde{\mathscr{B}}$ for some net $\tilde{\mathscr{B}}$;
- $\mathscr{A} \subset \mathscr{F}$ with $\mathscr{A}$ observable net and $\mathscr{F}$ its DR-net;
- $\mathscr{C} \subset \mathscr{F}$ with $\mathscr{C}$ the net generated by the canonical local implementation of translations of $\mathscr{F}$.

Superselection Theory for Subsystems 2/2

Theorem ([Conti-Doplicher-Roberts '01])

$\mathscr{A} \subset \mathscr{B}$ observable nets.

- *There holds*

$$\begin{array}{ccc} \mathscr{B} & \subset & \mathscr{F}(\mathscr{B}) \\ \cup & & \cup \\ \mathscr{A} & \subset & \mathscr{F}(\mathscr{A}) \end{array}$$

  and $\exists$ homomorphism $\phi : G(\mathscr{B}) \to G(\mathscr{A})$ induced by restriction.
- *If $\mathscr{A} \subset \mathscr{B} \subset \mathscr{F}(\mathscr{A})$ and $\mathscr{A}$ has no infinite statistics sectors, then $\mathscr{B}$ has no infinite statistics sectors and $\mathscr{F}(\mathscr{A}) = \mathscr{F}(\mathscr{B})$ ($\implies G(\mathscr{B})$ is a closed subgroup of $G(\mathscr{A})$).*

In particular for $\mathscr{B} = \mathscr{F}(\mathscr{A})^b$ (bosonic part), $\mathscr{F}(\mathscr{A})^b = \mathscr{F}(\mathscr{A})^{\mathbb{Z}_2}$ has precisely two sectors.

Classification of Subsystems 1/2

$\mathscr{F}$ field net satisfying standard assumptions plus

- geometric modular action: modular groups of wedges act like Lorentz boosts;
- split property: for every $O_1 \subset\subset O_2$ exists a type I factor

$$\mathscr{F}(O_1) \subset \mathscr{N} \subset \mathscr{F}(O_2);$$

- triviality of superselection structure: $\mathscr{F}^b$ has precisely two sectors (assuming $\mathscr{F}^b \subsetneq \mathscr{F}$).

A subsystem $\mathscr{B} \subset \mathscr{F}$ is full if the coset subsystem

$$\mathscr{B}^c(O) := \mathscr{B}' \cap \mathscr{F}(O)$$

is trivial.

Classification of Subsystems 2/2

Theorem ([Carpi-Conti '05])

A local subsystem $\mathscr{B} \subset \mathscr{F}$ is canonically isomorphic to

$$\mathscr{F}(\mathscr{B})^{G(\mathscr{B})} \otimes \mathbb{C}\mathbb{1} \subset \mathscr{F}(\mathscr{B})\hat{\otimes}\mathscr{B}^c.$$

In particular: $\mathscr{B} \subset \mathscr{F}$ full $\implies \mathscr{F} = \mathscr{F}(\mathscr{B})$.

Applications:

- $\mathscr{F}$ generated by a free scalar field of mass $m \geq 0 \implies \mathscr{F}^{\mathbb{Z}_2}$ is the only proper subsystem;
- $\mathscr{C}^d = \mathscr{F}^{G_{\max}}$, with $G_{\max}$ the (compact) group of $V \in \mathscr{U}(\mathscr{H}_\mathscr{F})$ such that $V\mathscr{F}(O)V^* = \mathscr{F}(O)$, $V\Omega = \Omega$;
- hence $\mathscr{C}^d = \mathscr{A} \iff G(\mathscr{A}) = G_{\max}$.

Scaling Algebras 1/2

On C*-algebra of bounded functions $\lambda \in \mathbb{R}_+^\times \to \underline{F}_\lambda \in \mathscr{F}$ define:

$$\|\underline{F}_\lambda\| := \sup_\lambda \|\underline{F}_\lambda\|,$$

$$\underline{\alpha}_{(\Lambda,x)}(\underline{F})_\lambda := \mathrm{Ad}\, U(\Lambda, \lambda x)(\underline{F}_\lambda), \qquad (\Lambda, x) \in \mathscr{P}_+^\uparrow,$$

$$\underline{\beta}_g(\underline{F})_\lambda := \mathrm{Ad}\, V(g)(\underline{F}_\lambda), \qquad g \in G.$$

Definition

Local scaling algebra of O:

$$\underline{\mathfrak{F}}(O) := \Big\{ \underline{F} : \underline{F}_\lambda \in \mathscr{F}(\lambda O), \lim_{\gamma \to e} \|\underline{\alpha}_\gamma(\underline{F}) - \underline{F}\| = 0, \lim_{g \to e} \|\underline{\beta}_g(\underline{F}) - \underline{F}\| = 0 \Big\}$$

Scaling Algebras 2/2

- Continuity condition w.r.t. translations $\iff$ $\underline{F}_\lambda$ has a "phase space occupation" independent of λ $\iff$ $\hbar$ not rescaled.
- Continuity condition w.r.t. G $\iff$ $\underline{F}_\lambda$ has a "charge transfer" independent of λ
- Typical elements

$$\underline{F}_\lambda = \int dx\, dg\, h(x,g) V(g) U(\lambda x) e^{i\phi_\lambda(f)} U(\lambda x)^* V(g)^*,$$

  where $\phi_\lambda(x) = Z_\lambda \phi(\lambda x)$ is the usual renormalized field.
- We consider "all possible renormalization schemes" compatible with above requirements.
- Keep in mind: construction of $\underline{\mathfrak{F}}$ depends on G (acting faithfully).

Scaling Limits 1/2

φ locally normal state on $\mathscr{F} \rightsquigarrow \underline{\varphi}_\lambda(\underline{F}) := \varphi(\underline{F}_\lambda)$ states on $\underline{\mathfrak{A}}$,

$$\mathrm{SL}^{\mathscr{F}}(\varphi) := \{\text{weak* limit points of } (\underline{\varphi}_\lambda)_{\lambda>0} \text{ for } \lambda \to 0\}.$$

Theorem ([D'Antoni-M.-Verch '04])

- $\mathrm{SL}^{\mathscr{F}}(\varphi) = (\underline{\omega}_{0,\iota})_{\iota \in I}$ *is independent of* φ.
- $\underline{\omega}_{0,\iota} \in \mathrm{SL}^{\mathscr{F}}$ *with GNS representation* $\pi_{0,\iota}$. *Then* $\mathscr{F}_{0,\iota}(O) := \pi_{0,\iota}(\underline{\mathfrak{F}}(O))''$ *is a field net in vacuum representation.*
- $\exists G_{0,\iota} = G/N_{0,\iota}$ *such that* $\mathscr{A}_{0,\iota} = \mathscr{F}_{0,\iota}^{G_{0,\iota}}$.

$O \to \mathscr{F}_{0,\iota}(O)$ is the scaling limit net of $\mathscr{F}$.

Physical interpretation: $\mathscr{F}_{0,\iota}$ describes the short-distance (i.e. high-energy) behaviour of $\mathscr{A}$.

Scaling Limits 2/2

Classification:

- If $\mathscr{F}_{0,\iota} = \mathbb{C}\mathbb{1}$ for all $\underline{\omega}_{0,\iota}$ then $\mathscr{F}$ has trivial scaling limit.
- If all nets $\mathscr{F}_{0,\iota}$ corresponding to different $\underline{\omega}_{0,\iota}$ are isomorphic and non-trivial $\mathscr{F}$ has unique scaling limit.
- Otherwise $\mathscr{F}$ has degenerate scaling limit.

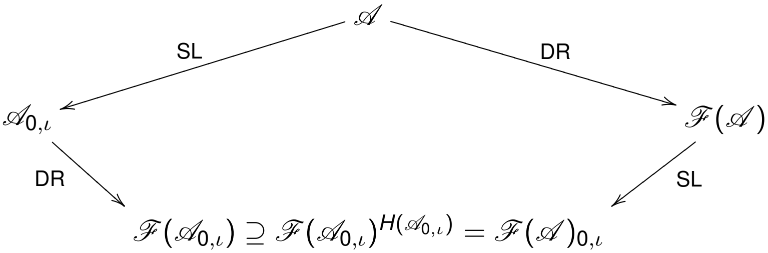
Examples:

- $\mathscr{F}^{(m)}$ net generated by free scalar field of mass $m \geq 0$ $\implies \mathscr{F}^{(m)}_{0,\iota} \simeq \mathscr{F}^{(0)}$: $\mathscr{F}^{(m)}$ has unique limit [Buchholz-Verch '97];
- $\mathscr{F}$ Lutz model (suitable subnet of a generalized free field) has trivial limit [Lutz '98].

Scaling Limits and Superselection Sectors 1/3

$\mathscr{F}(\mathscr{A})_{0,\iota}$ is not in general the canonical DR field net for $\mathscr{A}_{0,\iota}$

General situation:



$H(\mathscr{A}_{0,\iota}) \subset G(\mathscr{A}_{0,\iota})$ normal subgroup such that

$$G(\mathscr{A})_{0,\iota} = G(\mathscr{A})/N(\mathscr{A})_{0,\iota} = G(\mathscr{A}_{0,\iota})/H(\mathscr{A}_{0,\iota}).$$

$\mathscr{F}(\mathscr{A}_{0,\iota}) \supsetneq \mathscr{F}(\mathscr{A})_{0,\iota} \implies \mathscr{A}$ has confined charges.

E.g. in the Schwinger model:

$\mathscr{F}(\mathscr{A}) = \mathscr{A} \implies \mathscr{F}(\mathscr{A})_{0,\iota} = \mathscr{A}_{0,\iota} \subsetneq \mathscr{F}(\mathscr{A}_{0,\iota})$ [Buchholz-Verch '97]

Scaling Limits and Superselection Sectors 2/3

Which sectors *survive* the scaling limit?
Physical picture $\rightsquigarrow$ pointlike charges survive.

- $\psi_j(\lambda) \in \mathscr{F}(\lambda O)$ of class $[\rho] \implies \psi_j(\lambda)\Omega$ charge $[\rho]$ in λO.
- $[\rho]$ pointlike $\implies$ energy of $\psi_j(\lambda)\Omega \sim \lambda^{-1}$.

Theorem ([D'Antoni-M.-Verch '04])

With $\psi_j(\lambda)$ as above and

$$(\underline{\alpha}_h \psi_j)_\lambda := \int_{\mathbb{R}^4} dx\, h(x) \mathrm{Ad}\, U(\lambda x)(\psi_j(\lambda)),$$

there exists

$$\psi_j^0 = \operatorname*{s^*\text{-}lim}_{h \to \delta} \pi_{0,\iota}(\underline{\alpha}_h \psi_j) \in \mathscr{F}_{0,\iota}(O)$$

and ψ_j^0 is a $G_{0,\iota}$-multilplet which implements a DHR sector of $\mathscr{A}_{0,\iota}$.

Scaling Limits and Superselection Sectors 3/3

Non-preserved sectors can actually appear:

Theorem ([D'Antoni-M. '06])

For each Lie group G and closed normal subgroup $N \subset G$, exists $\mathscr{F}$ with action of G such that only sectors corresponding to representations of G/N are preserved.

Proof uses Lutz model and free fields.

Scaling Limit of Subsystems 1/3

Data:

- $\mathscr{B} \subset \mathscr{F}$ subsystem;
- $E : \mathscr{F} \to \mathscr{B}$ normal conditional expectation of nets, such that

$$\mathrm{Ad}\, U(\gamma) E = E\, \mathrm{Ad}\, U(\gamma), \quad \mathrm{Ad}\, V(k) E = E\, \mathrm{Ad}\, V(k), \quad \omega E = \omega.$$

$\rightsquigarrow \underline{E} : \underline{\mathfrak{F}} \to \underline{\mathfrak{B}}$ conditional expectation defined by $\underline{E}(\underline{F})_\lambda := E(\underline{F}_\lambda)$.

Theorem

- $\underline{\omega}_{0,\iota} \in \mathrm{SL}^{\mathscr{B}} \to \underline{\omega}_{0,\iota} \circ \underline{E} \in \mathrm{SL}^{\mathscr{F}}$ *is bijection;*
- $\mathscr{B}_{0,\iota}, \mathscr{F}_{0,\iota}$ *defined by corresponding s.l. states* $\implies \mathscr{B}_{0,\iota} \subset \mathscr{F}_{0,\iota}$ *and* $\exists E_{0,\iota}; \mathscr{F}_{0,\iota} \to \mathscr{B}_{0,\iota}$ *normal conditional expectation determined by*

$$E_{0,\iota}(\pi^{\mathscr{F}}_{0,\iota}(\underline{F})) = \pi^{\mathscr{B}}_{0,\iota}(\underline{E}(\underline{F})).$$

Scaling Limit of Subsystems 2/3

Application:

- $\mathscr{F}^{(m)}$ generated by free scalar field ϕ of mass $m \geq 0$;
- $\mathscr{B}^{(m)} = (\mathscr{F}^{(m)})^{\mathbb{Z}_2}$ with respect to $\phi \to -\phi$;
- then $\mathscr{B}^{(m)}_{0,\iota} \simeq \mathscr{B}^{(0)}$.

Can be generalized to free field multiplets.

Question: $\mathscr{F}$ has unique limit $\implies$ $\mathscr{B}$ has unique limit?

In free field case, this depends on

$$\theta_{\iota',\iota} \circ E_{0,\iota} = E_{0,\iota'} \circ \theta_{\iota',\iota},$$

where $\theta_{\iota',\iota} : \mathscr{F}_{0,\iota} \xrightarrow{\simeq} \mathscr{F}_{0,\iota'}$.

Scaling Limit of Subsystems 3/3

Definition ([Bostelmann-D'Antoni-M., next talk])

$\mathscr{F}$ has convergent scaling limit if $\exists$ subalgebra $\hat{\underline{\mathfrak{F}}} \subset \underline{\mathfrak{F}}$ such that

- $\underline{F} \in \hat{\underline{\mathfrak{F}}} \implies \exists \lim_{\lambda \to 0} \omega(\underline{F}_\lambda)$;
- $\pi_{0,\iota}(\hat{\underline{\mathfrak{F}}}(O))'' = \mathscr{F}_{0,\iota}(O)$.

Facts:

- convergent limit $\implies$ unique limit;
- free field has convergent limit.

Theorem

$\mathscr{B} \subset \mathscr{F}$ with $E : \mathscr{F} \to \mathscr{B}$. Then $\mathscr{F}$ has convergent limit $\implies$ $\mathscr{B}$ has convergent limit.

Scaling for Subsystems and DR Theory 1/5

$\mathscr{A} \subset \mathscr{B}$ subsystem of observable nets with $\mathscr{F}(\mathscr{A}) = \mathscr{F}(\mathscr{B}) +$ technical hypotheses ($\mathscr{B}_{0,\iota}$ Haag dual with separable Hilbert space) $\rightsquigarrow$

- nets $\mathscr{A}_{0,\iota}, \mathscr{B}_{0,\iota}, \mathscr{F}(\mathscr{A})_{0,\iota}, \mathscr{F}(\mathscr{B})_{0,\iota}, \mathscr{F}(\mathscr{A}_{0,\iota}), \mathscr{F}(\mathscr{B}_{0,\iota})$;
- groups $G(\mathscr{A})_{0,\iota}, N(\mathscr{A})_{0,\iota}, G(\mathscr{A}_{0,\iota}), H(\mathscr{A}_{0,\iota})$ and the same for $\mathscr{B}$.

Remember: $\mathscr{F}(\mathscr{A}) = \mathscr{F}(\mathscr{B}) \not\Rightarrow \mathscr{F}(\mathscr{A})_{0,\iota} = \mathscr{F}(\mathscr{B})_{0,\iota}$ because $G(\mathscr{B}) \subsetneq G(\mathscr{A})$.

Scaling for Subsystems and DR Theory 2/5

Theorem

- *There holds:*

$$\begin{array}{ccccc} \mathscr{B}_{0,\iota} & \subset & \mathscr{F}(\mathscr{B})_{0,\iota} & \subset & \mathscr{F}(\mathscr{B}_{0,\iota}) \\ \cup & & \cup & & \cup \\ \mathscr{A}_{0,\iota} & \subset & \mathscr{F}(\mathscr{A})_{0,\iota} & \subset & \mathscr{F}(\mathscr{A}_{0,\iota}) \end{array}$$

- $N(\mathscr{B})_{0,\iota} \subset N(\mathscr{A})_{0,\iota}$ *and* $\phi : G(\mathscr{B}_{0,\iota}) \to G(\mathscr{A}_{0,\iota})$ *satisfies* $\phi(H(\mathscr{B}_{0,\iota})) \subset H(\mathscr{A}_{0,\iota})$ *and* $\tilde{\phi}(gN(\mathscr{B})_{0,\iota}) = gN(\mathscr{A})_{0,\iota}$*;*
- $\mathscr{F}(\mathscr{A}_{0,\iota}) = \mathscr{F}(\mathscr{B}_{0,\iota}) \implies \phi$ *injective;*
- $\mathscr{F}(\mathscr{A})_{0,\iota} = \mathscr{F}(\mathscr{B})_{0,\iota} \implies N(\mathscr{B})_{0,\iota} = N(\mathscr{A})_{0,\iota} \cap G(\mathscr{B}), H(\mathscr{B}_{0,\iota}) = H(\mathscr{A}_{0,\iota})$ *and* $\tilde{\phi}$ *injective.*

Scaling for Subsystems and DR Theory 3/5

Question: when $\mathscr{F}(\mathscr{A}_{0,\iota}) = \mathscr{F}(\mathscr{B}_{0,\iota})$?
Examples:

- $\mathscr{A} \subset \mathscr{B} \subset \mathscr{F} = \mathscr{F}(\mathscr{A}) = \mathscr{F}(\mathscr{B})$ dilation covariant $\implies$

$$\begin{array}{ccccc} \mathscr{B}_{0,\iota} & \subset & \mathscr{F}(\mathscr{B})_{0,\iota} & = & \mathscr{F}(\mathscr{B}_{0,\iota}) \\ \cup & & \shortparallel & & \shortparallel \\ \mathscr{A}_{0,\iota} & \subset & \mathscr{F}(\mathscr{A})_{0,\iota} & = & \mathscr{F}(\mathscr{A}_{0,\iota}) \end{array}$$

- $\mathscr{B} = \mathscr{F}(\mathscr{B})$ generated by G-multiplet of free scalar fields, $\mathscr{A} = \mathscr{B}^G$ ($\implies \mathscr{F}(\mathscr{A}) = \mathscr{B}, G(\mathscr{A}) = G$). From [D'Antoni-M. '06]:

$$\begin{array}{ccccc} \mathscr{B}_{0,\iota} & = & \mathscr{F}(\mathscr{B})_{0,\iota} & = & \mathscr{F}(\mathscr{B}_{0,\iota}) \\ \cup & & \shortparallel & & \shortparallel \\ \mathscr{A}_{0,\iota} & \subset & \mathscr{F}(\mathscr{A})_{0,\iota} & = & \mathscr{F}(\mathscr{A}_{0,\iota}) \end{array}$$

Scaling for Subsystems and DR Theory 4/5

Theorem

$\mathscr{A} \subset \mathscr{B}$ with $\mathscr{F}(\mathscr{B}_{0,\iota})$ with trivial superselection structure. Assume $\exists Q$ such that $\mathscr{A}_{0,\iota} = \mathscr{F}(\mathscr{B})^Q_{0,\iota}$. Then $\mathscr{F}(\mathscr{A}_{0,\iota}) = \mathscr{F}(\mathscr{B}_{0,\iota})$.

In particular $\mathscr{A}_{0,\iota} = \mathscr{F}(\mathscr{B})^Q_{0,\iota}$ if $\mathscr{F}(\mathscr{A})_{0,\iota} = \mathscr{F}(\mathscr{B})_{0,\iota}$.

Theorem

$\mathscr{A} \subset \mathscr{B}$ as above. Assume:

- *$G(\mathscr{B})$ normal in $G(\mathscr{A})$;*
- *$\mathscr{B}$ has convergent scaling limit;*
- *$\underline{\hat{\mathfrak{B}}} \subset \underline{\mathfrak{F}}(\mathscr{B})$ globally $G(\mathscr{A})$-invariant.*

Then $\mathscr{F}(\mathscr{A}_{0,\iota}) = \mathscr{F}(\mathscr{B}_{0,\iota})$.

In these hypotheses, $\mathscr{A}_{0,\iota} = \mathscr{F}(\mathscr{B})^{G(\mathscr{A})}_{0,\iota} \implies$ apply above theorem to $Q = G(\mathscr{A})^-$.

Scaling for Subsystems and DR Theory 5/5

Application:

- $\mathscr{C} \subset \mathscr{A}$ net generated by canonical local implementation of translations of $\mathscr{F}(\mathscr{A})$;
- $\tilde{\mathscr{C}}_{0,\iota} \subset \mathscr{A}_{0,\iota}$ net generated by canonical local implementation of translations of $\mathscr{F}(\mathscr{A}_{0,\iota})$;
- above assumptions;
- then $\tilde{\mathscr{C}}_{0,\iota} \subset \mathscr{C}_{0,\iota}$.

In short: scaling limit of net generated by energy-momentum tensor contains net generated by the energy-momentum tensor in the scaling limit.

Easy to build examples of theories with the split property but with scaling limit without the split property using [Mohrdieck '02].

Summary & Outlook

- Subsystems and Scaling Algebras are useful in understanding structural properties of superselection charges in QFT.
- First steps in the analysis of the interplay between them.
- In particular, sufficient conditions for

$$\mathscr{A} \subset \mathscr{B} \subset \mathscr{F}(\mathscr{A}) = \mathscr{F}(\mathscr{B}) \implies \mathscr{A}_{0,\iota} \subset \mathscr{B}_{0,\iota} \subset \mathscr{F}(\mathscr{A}_{0,\iota}) = \mathscr{F}(\mathscr{B}_{0,\iota}).$$

- Outlook
  - Discuss conditions on $\mathscr{A}$, $\mathscr{B}$ which entail hypotheses made on scaling limit nets (Haag duality for $\mathscr{B}_{0,\iota}$, triviality of superselection structure of $\mathscr{F}(\mathscr{B}_{0,\iota})$).
  - Generalize to $\mathscr{F}(\mathscr{A}) \subsetneq \mathscr{F}(\mathscr{B})$ ($\implies \mathscr{F}(\mathscr{B}) = \mathscr{F}(\mathscr{A}) \hat{\otimes} \tilde{\mathscr{F}}$).
  - Applications to full quantum Noether theorem? Non-existence of infinite statistics sectors?

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