Short Distance Analysis of Localizable and Topological Charges

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References

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Contents

1. Motivations

- Renormalization Group

  $\rightsquigarrow$ Short distance properties of **QFT**

- RG formulated in terms of **unobservable fields**:

  conceptually unsatisfactory, fields are just a "coordinatization" of observables
  (Borchers classes, Schwinger model, Seiberg-Witten dualities in SUSY YM).

- Prominent example: **confinement**

  based on attaching a physical interpretation to unobservable degrees of freedom in the lagrangian
  (e.g. quark and gluon fields in QCD).

- **Algebraic approach to QFT**

  framework for an intrinsic description of the ultraviolet behaviour
  scaling algebras $\rightsquigarrow$ scaling (short distance) **limit** of a theory entirely in terms of **observables**.

- Analysis of *charges* and *particles* of the **scaling limit theory** through DHR theory of superselection sectors

  $\rightsquigarrow$ *ultracharges* and *ultraparticles* described by the given **theory at short distances**

- Unambiguous notion of **confinement** obtained through comparison of charge and ultracharge content of the theory

- Necessary to find a canonical way to **compare the two charge structures**

  $\rightsquigarrow$ identify ultracharges which are short distance remnants of (finite scales) charges.

- Idea $\rightsquigarrow$ characterize **ultraviolet stability** of charges through analysis of short distance behaviour of **associated fields**.

$\Longrightarrow$ **generalization** of scaling algebras to charge carrying (unobservable) fields.

2. Scaling algebras and scaling limits

$\mathscr{O} \to \mathfrak{A}(\mathscr{O})$ net of local observables

$\alpha_{(\Lambda,x)}$ Poincaré transformations

- Characteristic feature of conventional **RG transformations** R_λ:

  scale length by λ and 4-momentum by λ^{-1}

  $\Longrightarrow$ phase space occupation of orbits

  is fixed.

$$\Updownarrow$$

$$\lim_{(\Lambda,x)\to(\mathbb{1},0)} \sup_{\lambda>0} \|\alpha_{(\Lambda,\lambda x)}(R_\lambda(A)) - R_\lambda(A)\| = 0$$

- $\underline{A} : \mathbb{R}_+ \to \mathfrak{A}$ bounded:

$$\|\underline{A}\| := \sup_{\lambda>0} \|\underline{A}(\lambda)\|,$$

$$\underline{\alpha}_{(\Lambda,x)}(\underline{A})(\lambda) := \alpha_{(\Lambda,\lambda x)}(\underline{A}(\lambda))$$

- *Scaling algebra* $\underline{\mathfrak{A}}(\mathscr{O})$: bounded functions $\underline{A} : \mathbb{R}_+ \to \mathfrak{F}$ such that
  - $\underline{A}(\lambda) \in \mathfrak{A}(\lambda\mathscr{O})$
  - $\lim_{(\Lambda,x)\to(\mathbb{1},0)} \|\underline{\alpha}_{(\Lambda,x)}(\underline{A}) - \underline{A}\| = 0$

- Morally $\underline{A}(\lambda) = R_\lambda(A)$

  $\Longrightarrow$ $\underline{\mathfrak{A}}$ orbits of observables under *all possible RG transformations.*

- φ (locally normal) state on $\mathfrak{A}$:

$$\underline{\varphi}_\lambda(\underline{A}) := \varphi(\underline{A}(\lambda))$$

$$\mathrm{SL}_{\mathfrak{A}}(\varphi) := \left\{\text{weak* limit pts of } \left(\underline{\varphi}_\lambda\right)_{\lambda>0}\right\}.$$

<u>Theorem</u>[BV95] $\mathrm{SL}_{\mathfrak{A}}(\varphi)$ *is independent of* φ*. For* (π_0, Ω_0) *the GNS representation of* $\underline{\omega}_0 \in \mathrm{SL}_{\mathfrak{A}}$*,*

$$\mathfrak{A}_0(\mathscr{O}) := \pi_0(\underline{\mathfrak{A}}(\mathscr{O}))$$

is a covariant local net of observables with vacumm Ω_0 *(if* $d = 2$ *the vacuum may not be pure).*

$\mathfrak{A}_0$ *scaling limit net* of $\mathfrak{A}$. Possibilities:

- *degenerate scaling limit*: the various $\mathfrak{A}_0$ non-isomorphic;

- *unique (quantum) scaling limit*: the various $\mathfrak{A}_0$ isomorphic and non-trivial;

- *classical scaling limit*: each $\mathfrak{A}_0(\mathscr{O}) = \mathbb{C}\mathbb{1}$.

- Examples of ***unique* scaling limit**:

  <u>**Theorem**</u>[BV98] *Each scaling limit theory of the free scalar field in $d = 3, 4$ spacetime dimensions, is isomorphic to the massless free scalar field.*

- Examples of ***classical* scaling limit** [Lutz, Diploma (1997)]:

  ϕ generalized free field with constant mass measure, $\mathfrak{A}(\mathscr{O})$ generated by $\Box^{n(\mathscr{O})}\phi(x)$, $x \in \mathscr{O}$, $n(\mathscr{O}) \to +\infty$ as radius $\mathscr{O} \to 0$.

- **Schwinger model** (massless QED_2):

  $\mathfrak{A}(\mathscr{O})$ = massive free scalar field in $d = 2$

  $\Longrightarrow$ no charged states

  $\Longrightarrow$ conventional interpretation: confined electrons.

  Intrinsic?

  Scaling limit has nontrivial charged states [Buc96, BV98] $\Longrightarrow$ $\mathfrak{A}$ has *ultracharges*, intrinsically confined.

3. Superselection charges and reconstruction of fields

- DHR (resp. BF) charges described by classes of **localized morphisms**:

$$\rho : \mathfrak{A} \to B(\mathscr{H})$$
$$\rho(A) = A, \quad A \in \mathfrak{A}(\mathscr{O}')$$

(resp. $A \in \mathfrak{A}(\mathscr{C}')$).

$\Delta = \{\rho\}$ encodes charge carrying fields [DR90]:

- There exist unique
  - $\mathscr{O} \to \mathfrak{F}(\mathscr{O})$ generated by (unobservable) fields with normal commutation relations;
  - $V : G \to U(\mathscr{H}_{\mathfrak{F}})$ unitary representation of G (compact), $V(g)\mathfrak{F}(\mathscr{O})V(g)^* = \mathfrak{F}(\mathscr{O})$;

  such that

  - $\mathfrak{A}(\mathscr{O}) = \mathfrak{F}(\mathscr{O})^G := \{F \in \mathfrak{F}(\mathscr{O}) \, : \, V(g)FV(g)^* = F \; \forall \, g \in G\}$;

- $\forall \rho \in \Delta(\mathscr{O})$ irreducible $\exists\, \psi_1, \ldots, \psi_d \in \mathfrak{F}(\mathscr{O})$ and $v_{[\rho]}$ d-dimensional irreducible G-representation such that

$$\psi_i^* \psi_j = \delta_{ij} \mathbb{1}, \qquad \sum_{j=1}^{d} \psi_j \psi_j^* = \mathbb{1},$$

$$\beta_g(\psi_i) = \sum_{j=1}^{d} \psi_j v_{[\rho]}(g)_{ji},$$

$$\rho(A) = \sum_{j=1}^{d} \psi_j A \psi_j^*, \qquad A \in \mathfrak{A};$$

- $\mathfrak{F}(\mathscr{O})$ is generated by $\mathfrak{A}(\mathscr{O})$ and the multiplets ψ_j.

- Then:

$$\{\text{charges}\} \longleftrightarrow \left\{ \begin{array}{l} \text{irreducible representations} \\ \text{of global gauge group } G \end{array} \right\}$$

- Analogous result for BF charges, with net $\mathscr{C} \to \mathfrak{F}(\mathscr{C})$, i.e. fields localized in cones.

4. Scaling algebras for local fields and scaling limit of DHR charges

Reference [DMV04]

Construction parallel to the observable case.

- For $\underline{F} : \mathbb{R}_+ \to \mathfrak{F}$ bounded:

$$\underline{\beta}_g(\underline{F})(\lambda) := V(g)\underline{F}(\lambda)V(g)^*, \quad g \in G.$$

- *Scaling field algebra* $\underline{\mathfrak{F}}(\mathscr{O})$: bounded functions $\underline{F} : \mathbb{R}_+ \to \mathfrak{F}$ such that
  - $\underline{F}(\lambda) \in \mathfrak{F}(\lambda\mathscr{O})$;
  - $\lim_{(\Lambda,x)\to(\mathbb{1},0)} \|\underline{\alpha}_{(\Lambda,x)}(\underline{F}) - \underline{F}\| = 0$;
  - $\lim_{g\to e} \|\underline{\beta}_g(\underline{F}) - \underline{F}\| = 0$.

$\Longrightarrow$ We restrict to "dimensionless" charges.

- Clearly: $\underline{\mathfrak{A}}(\mathscr{O}) \subset \underline{\mathfrak{F}}(\mathscr{O})$.
- If φ state on $\mathfrak{F}$: $\left(\underline{\varphi}_\lambda\right)_{\lambda>0}$ and $\mathrm{SL}_{\mathfrak{F}}(\varphi)$ defined as for observables.

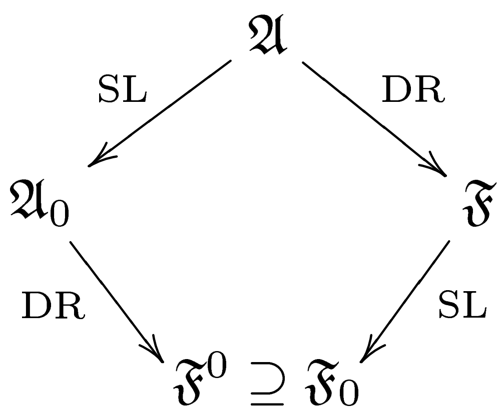
Theorem $\mathrm{SL}_{\mathfrak{F}}(\varphi)$ *is independent of* φ*. For* (π_0, Ω_0) *the GNS representation of* $\underline{\omega}_0 \in \mathrm{SL}_{\mathfrak{F}}$*,*

$$\mathfrak{F}_0(\mathscr{O}) := \pi_0(\underline{\mathfrak{F}}(\mathscr{O}))$$

is a covariant normal field net with an action of $G_0 := G/N_0$ *such that*

$$\mathfrak{A}_0(\mathscr{O}) := \pi_0(\underline{\mathfrak{A}}(\mathscr{O})) = \mathfrak{F}_0(\mathscr{O})^{G_0}.$$

$\rightsquigarrow$ General situation:



$\mathfrak{F}_0 \subsetneq \mathfrak{F}^0 \implies \mathfrak{A}$ has *confined ultracharges* [Buc96]

charges appearing at short distances but not at finite distances.

Intrinsic $\rightsquigarrow$ everything fixed by the observable net.

***Example*: Schwinger model**

$$\mathfrak{F} = \mathfrak{A} \implies \mathfrak{F}_0 = \mathfrak{A}_0 \subsetneq \mathfrak{F}^0.$$

"Converse" problem:

which charges *survive* to the scaling limit.

Physical picture $\rightsquigarrow$ **"pointlike" objects survive**

- $\psi_j(\lambda) \in \mathfrak{F}(\lambda\mathscr{O})$ of class $\xi \implies \psi_j(\lambda)\Omega$ charge ξ localized in $\lambda\mathscr{O}$

- ξ pointlike $\implies$ energy$(\psi_j(\lambda)\Omega) \sim \lambda^{-1}$

$$\underline{\alpha}_h \psi_j(\lambda) := \int_{\tilde{\mathscr{P}}_+^\uparrow} d\Lambda d^4x \, h(\Lambda, x) \alpha_{(\Lambda, \lambda x)}(\psi_j(\lambda))$$

<u>Theorem</u> *With ξ and $\psi_j(\lambda)$ as above, there exists*

$$\boldsymbol{\psi}_j = {}^*s\text{-}\lim_{h \to \delta} \pi_0(\underline{\alpha}_h \psi_j) \in \mathfrak{F}_0(\mathscr{O}_1)''$$

for each $\mathscr{O}_1 \supset \overline{\mathscr{O}}$, and $\boldsymbol{\psi}_j$ is a G_0-multiplet of class ξ, with $v_\xi^{(0)}(gN_0) := v_\xi(g)$ a well-defined irrep of G_0. Furthermore,

$$\boldsymbol{\rho}(\boldsymbol{a}) := \sum_{j=1}^d \boldsymbol{\psi}_j \boldsymbol{a} \boldsymbol{\psi}_j^*, \qquad \boldsymbol{a} \in \mathfrak{A}_0,$$

is a localized irreducible DHR endomorphism of $\mathfrak{A}_0$.

- *Example*: Majorana free field ϕ: $\psi(\lambda) = \phi(f_\lambda)$.

All sectors preserved $\implies$ much of the superselection structure can be determined locally:

- *Global intertwiners* between ρ, σ:

$$(\rho : \sigma) := \{T \in \mathfrak{A} \, : \, T\rho(A) = \sigma(A)T, \; A \in \mathfrak{A}\}$$

- *Local* intertwiners:

$$(\rho : \sigma)_{\mathscr{O}} := \{T \in \mathfrak{A} \, : \, T\rho(A) = \sigma(A)T, \; A \in \mathfrak{A}(\mathscr{O})\}$$

Basic building blocks of $\mathfrak{F}(\mathscr{O})$

Theorem *If all the $\mathfrak{F}(\mathscr{O})$ are factors, $\mathfrak{F}(\mathscr{O}) \cap \mathfrak{F}(\mathscr{O})' = \mathbb{C}\mathbb{1}$, and each sector is preserved in some scaling limit, then*

$$(\rho : \sigma) = (\rho : \sigma)_{\mathscr{O}}, \qquad \forall \mathscr{O}.$$

$\rightsquigarrow$ generalization of result of Roberts for **dilation invariant theories** [CMP **37** (1974)], useful on curved spacetimes.

5. Scaling algebras for cone-like fields and scaling limit of BF charges

- above analysis too narrow: if **quarks** are non-confined, they are localized in cones.
- **Problem**: spacelike cones not affected by rescaling $\Longrightarrow$ how to implement RG phase space?
- Asymptotically free theories $\rightsquigarrow$ charges in cones should become **localized** in the scaling limit (flux string vanishes)

  $\Longrightarrow$ phase space recovered asymptotically.
- Define $\underline{\mathfrak{F}}(\mathscr{C}, \mathscr{O})$, $\mathscr{O} \subset \mathscr{C}$: bounded functions $\underline{F} : \mathbb{R}_+ \to \mathfrak{F}$ such that:
  - $\underline{F}(\lambda) \in \mathfrak{F}(\lambda\mathscr{C})$;
  - $\lim_{(\Lambda,x)\to(\mathbb{1},0)} \|\underline{\alpha}_{(\Lambda,x)}(\underline{F}) - \underline{F}\| = 0$;
  - $\lim_{g\to e} \|\underline{\beta}_g(\underline{F}) - \underline{F}\| = 0$;
  - $\lim_{\lambda\to 0} \sup_{A\in\underline{\mathfrak{A}}(\mathscr{O}')_1} \|[\underline{F}(\lambda), \underline{A}(\lambda)]\| = 0$.

  $\underline{\mathfrak{F}}$ C*-algebra generated by all $\underline{\mathfrak{F}}(\mathscr{C}, \mathscr{O})$.

- φ normal state on $B(\mathscr{H}_{\mathfrak{F}})$
  $\rightsquigarrow$ states $\left(\underline{\varphi}_\lambda\right)_{\lambda>0}$ on $\underline{\mathfrak{F}}$ and $\mathrm{SL}_{\mathfrak{F}}(\varphi)$ defined as above

Theorem $\mathrm{SL}_{\mathfrak{F}}(\varphi)$ *is independent of* φ*. For* (π_0, Ω_0) *the GNS representation of* $\underline{\omega}_0 \in \mathrm{SL}_{\mathfrak{F}}$*,*

$$\mathscr{O} \to \mathfrak{F}_0(\mathscr{O}) := \bigcap_{\mathscr{C} \supset \mathscr{O}} \pi_0(\underline{\mathfrak{F}}(\mathscr{C}, \mathscr{O}))''$$

is a covariant normal field net with an action of $G_0 := G/N_0$*.*

- **net** indexed by ***double cones*** in the scaling limit, as expected in the asymptotically free case.
- Study of **preservation** similar to DHR case

To summarize:

$$\begin{array}{ccccc}
\mathfrak{A} & & \xrightarrow{\quad \mathrm{SL} \quad} & & \mathfrak{A}_0 \\
\downarrow & & & & \downarrow \\
\mathrm{BF}(\mathfrak{A}) & \hookleftarrow & \mathrm{BF}_0(\mathfrak{A}) & \dashrightarrow & \mathrm{DHR}(\mathfrak{A}_0)
\end{array}$$

$\Longrightarrow$ *charge confinement notion*:

$\xi \in \mathrm{DHR}(\mathfrak{A}_0) \setminus \mathrm{BF}_0(\mathfrak{A})$ is a confined ultracharge of $\mathfrak{A}$.

6. Conclusions and outlook

- **AQFT** $\rightsquigarrow$ tools for model-independent, intrinsic analysis of short-distance properties of QFT and classification of possible ultra-violet behaviour

- Formulation of unambiguous confinement criteria

  $\rightsquigarrow$ Schwinger model example

- Scaling algebra methods can be generalized to charge carrying fields

  $\rightsquigarrow$ short-distance properties of superselection charges

  $\rightsquigarrow$ characterize their preservation

- **Future developments**:
  - study of models

    $\rightsquigarrow$ non-preserved charges

    $\rightsquigarrow$ preserved BF charges

  - anomalous charge scaling
  - scaling of charges without DR theorem.