* $\displaystyle\int_0^{1/2} \frac{1}{x\,|\log x|^\beta}\,dx = \int_0^{\frac{1}{2}} \frac{1}{x(-\log x)^\beta}\,dx$

$\left(-\log x = \log x^{-1} = \log\frac{1}{x}\right)$

$\displaystyle = \lim_{c\to 0^+} \underbrace{\int_c^{\frac{1}{2}} \frac{1}{x}\,\frac{1}{\left(\log\frac{1}{x}\right)^\beta}\,dx}$

$\displaystyle \left(\frac{1}{x}=t,\quad dx = -\frac{1}{t^2}\,dt\right) = \int_{\frac{1}{c}}^{2} t\,\frac{1}{(\log t)^\beta}\left(-\frac{1}{t^2}\right)dt$

$\displaystyle = \int_2^{\frac{1}{c}} \frac{1}{t\,(\log t)^\beta}\,dt$

$\displaystyle\left(-\int_{\frac{1}{c}}^{2} \dots\,dt = \int_2^{\frac{1}{c}}\right)$

che dà un integrale convergente per $\beta > 1$.