

$$\begin{aligned}
 * \quad \int_0^{1/2} \frac{1}{x |\log x|^\beta} dx &= \int_0^{1/2} \frac{1}{x (-\log x)^\beta} dx \\
 &= \lim_{c \rightarrow 0^+} \int_c^{1/2} \frac{1}{x (\log \frac{1}{x})^\beta} dx
 \end{aligned}$$

$$\begin{aligned}
 (-\log x &= \log x^{-1}) \\
 &= \log \frac{1}{x}
 \end{aligned}$$

$$\rightarrow \left(\frac{1}{x} = t, \quad dx = -\frac{1}{t^2} dt \right) = \int_{\frac{1}{c}}^2 t \frac{1}{(\log t)^\beta} \left(-\frac{1}{t^2} \right) dt$$

$$= \int_2^{\frac{1}{c}} \frac{1}{t (\log t)^\beta} dt$$

$$\left(-\int_{\frac{1}{c}}^2 f(t) dt = \int_2^{\frac{1}{c}} f(t) dt \right)$$

che dà un integrale convergente per $\beta > 1$.