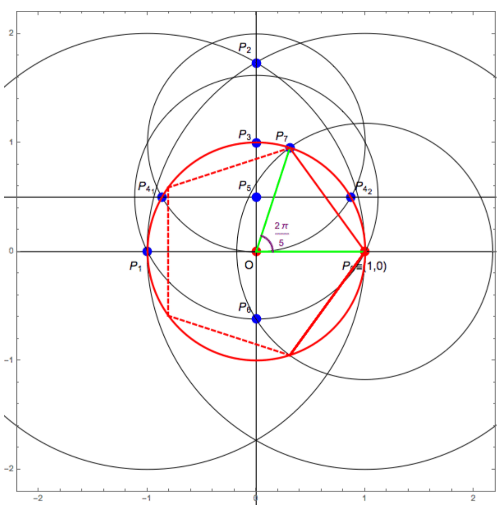
Mathematica at Work

Introduzione ad un Software estremamente potente e ad alcune sue applicazioni nella ricerca e nel mondo del lavoro

INCA - ABACO Infrastutture di Calcolo a Basso Costo
Università degli Studi di Roma Tor Vergata



Corrado Falcolini
Dipartimento di Architettura
Università degli Studi di Roma Tre

Poligoni regolari ed approssimazione di π .

Rilievo, nuvole di punti e modelli in Architettura.

Equazioni differenziali.

Coesistenza di attrattori.

Poliedri semi-regolari.

Poligoni regolari ed approssimazione di π .

Sia P(*n*) un poligono regolare di *n* lati.

Domande:

1) Qual è il minimo numero *n* tale che il rapporto tra il perimetro di P(*n*) e il diametro della circonferenza circoscritta sia maggiore di 3.14?

2) Qual è il minimo valore di *n* tale che P(*n*) non sia distinguibile da un cerchio?

```
Manipulate[Graphics[{{Red, Circle[{0, 0}, 1]},
   Line[Table[{Cos[k * 2 Pi / n], Sin[k * 2 Pi / n]}, {k, 0, n, 1}]]},
  ImageSize → 500, PlotRange → {{-1.1, 1.1}, {-1.1, 1.1}}], {n, 3, 100, 1}]
```

```
Manipulate[Graphics[{Line[Table[{Cos[k * 2 Pi / n], Sin[k * 2 Pi / n]}, {k, 0, n, 1}]]},
  ImageSize → 500, PlotRange → {{-1.1, 1.1}, {-1.1, 1.1}}], {n, 3, 100, 1}]
```

(* $\ell_n = 2r * \sin\left(\frac{\pi}{n}\right)$, $\pi_n = n * \frac{\ell_n}{2r} = n * \sin\left(\frac{\pi}{n}\right)$ *)

```
TableForm[Table[{n, N[n * Sin[Pi / n], 7]}, {n, 3, 60, 1}],
 TableHeadings → {None, {"n", "πn"}}]
```

n	π_n
3	2.598076
4	2.828427
5	2.938926
6	3.000000
7	3.037186
8	3.061467
9	3.078181
10	3.090170
11	3.099058
12	3.105829
13	3.111104
14	3.115293
15	3.118675
16	3.121445
17	3.123742
18	3.125667
19	3.127297
20	3.128689
21	3.129888
22	3.130926
23	3.131833
24	3.132629
25	3.133331
26	3.133954
27	3.134509
28	3.135005
29	3.135452
30	3.135854
31	3.136218
32	3.136548
33	3.136849
34	3.137124
35	3.137376
36	3.137607
37	3.137819
38	3.138015
39	3.138196
40	3.138364
41	3.138519
42	3.138664
43	3.138799
44	3.138924
45	3.139041
46	3.139151
47	3.139254
48	3.139350
49	3.139441
50	3.139526
51	3.139606
52	3.139682

53	3.139753
54	3.139821
55	3.139885
56	3.139945
57	3.140002
58	3.140057
59	3.140108
60	3.140157

```
appk[k_] := Block[{n = 3}, While[n * Sin[Pi / n] < ToExpression[
      StringJoin[Drop[Characters[ToString[N[Pi, k + 1]]], -1]]], n++]; Return[n];];

TableForm[Table[{p, appk[p]}, {p, 1, 10, 1}]]
```

1	6
2	12
3	57
4	94
5	237
6	1396
7	2812
8	9820
9	37 942
10	93 602

Poligoni regolari ed approssimazione di π .

Sia P(n) un poligono regolare di n lati.

Domande:

3) Per quali interi n il numero $\cos\left(\frac{2\pi}{n}\right)$ è rappresentabile per radicali?

4) Per quali interi n *il poligono* P(n) è costruibile con riga e compasso?

Cosa si può fare con Mathematica:

The function Cos[ ]

The function ToRadicals[ ]

note

```
c[n_] := HoldForm[Cos[2 Pi / n]]
```

```
Table[{c[k], Cos[2 Pi / k]}, {k, 1, 8}]
```

$$\left\{\{\text{Cos}[2\,\pi],1\},\left\{\text{Cos}\left[\frac{2\,\pi}{2}\right],-1\right\},\left\{\text{Cos}\left[\frac{2\,\pi}{3}\right],-\frac{1}{2}\right\},\left\{\text{Cos}\left[\frac{2\,\pi}{4}\right],0\right\},\left\{\text{Cos}\left[\frac{2\,\pi}{5}\right],\frac{1}{4}\left(-1+\sqrt{5}\right)\right\},\left\{\text{Cos}\left[\frac{2\,\pi}{6}\right],\frac{1}{2}\right\},\left\{\text{Cos}\left[\frac{2\,\pi}{7}\right],\text{Sin}\left[\frac{3\,\pi}{14}\right]\right\},\left\{\text{Cos}\left[\frac{2\,\pi}{8}\right],\frac{1}{\sqrt{2}}\right\}\right\}$$

```
TableForm[Table[{k, c[k], Cos[2 Pi / k]}, {k, 1, 8}],
 TableHeadings → {None, {"k", "HoldForm", "Cos[2 π/k]"}}]
```

k	HoldForm	$\text{Cos}\left[\frac{2\pi}{k}\right]$
1	$\text{Cos}[2\,\pi]$	1
2	$\text{Cos}\left[\frac{2\pi}{2}\right]$	-1
3	$\text{Cos}\left[\frac{2\pi}{3}\right]$	$-\frac{1}{2}$
4	$\text{Cos}\left[\frac{2\pi}{4}\right]$	0
5	$\text{Cos}\left[\frac{2\pi}{5}\right]$	$\frac{1}{4}\left(-1+\sqrt{5}\right)$
6	$\text{Cos}\left[\frac{2\pi}{6}\right]$	$\frac{1}{2}$
7	$\text{Cos}\left[\frac{2\pi}{7}\right]$	$\text{Sin}\left[\frac{3\pi}{14}\right]$
8	$\text{Cos}\left[\frac{2\pi}{8}\right]$	$\frac{1}{\sqrt{2}}$

```
TableForm[Table[{n, c[n], "=", Cos[2 Pi / n], ToRadicals[Cos[2 Pi / n]]}, {n, 1, 50}],
 TableHeadings → {None, {"k", "Hold", "", "Cos[2π/k]", "ToRadicals"}}]
```

k	Hold		$\mathrm{Cos}[\frac{2\pi}{k}]$	ToRadicals
1	$\mathrm{Cos}[2\pi]$	=	1	1
2	$\mathrm{Cos}[\frac{2\pi}{2}]$	=	-1	-1
3	$\mathrm{Cos}[\frac{2\pi}{3}]$	=	$-\frac{1}{2}$	$-\frac{1}{2}$
4	$\mathrm{Cos}[\frac{2\pi}{4}]$	=	0	0
5	$\mathrm{Cos}[\frac{2\pi}{5}]$	=	$\frac{1}{4}\left(-1+\sqrt{5}\right)$	$\frac{1}{4}\left(-1+\sqrt{5}\right)$
6	$\mathrm{Cos}[\frac{2\pi}{6}]$	=	$\frac{1}{2}$	$\frac{1}{2}$
7	$\mathrm{Cos}[\frac{2\pi}{7}]$	=	$\mathrm{Sin}[\frac{3\pi}{14}]$	$-\frac{1}{2}(-1)^{2/7}(-1+(-1)^{3/7})$
8	$\mathrm{Cos}[\frac{2\pi}{8}]$	=	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
9	$\mathrm{Cos}[\frac{2\pi}{9}]$	=	$\mathrm{Cos}[\frac{2\pi}{9}]$	$-\frac{1}{2}(-1)^{7/9}(1+(-1)^{4/9})$
10	$\mathrm{Cos}[\frac{2\pi}{10}]$	=	$\frac{1}{4}\left(1+\sqrt{5}\right)$	$\frac{1}{4}\left(1+\sqrt{5}\right)$
11	$\mathrm{Cos}[\frac{2\pi}{11}]$	=	$\mathrm{Cos}[\frac{2\pi}{11}]$	$-\frac{1}{2}(-1)^{9/11}(1+(-1)^{4/11})$
12	$\mathrm{Cos}[\frac{2\pi}{12}]$	=	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$
13	$\mathrm{Cos}[\frac{2\pi}{13}]$	=	$\mathrm{Cos}[\frac{2\pi}{13}]$	$-\frac{1}{2}(-1)^{11/13}(1+(-1)^{4/13})$
14	$\mathrm{Cos}[\frac{2\pi}{14}]$	=	$\mathrm{Cos}[\frac{\pi}{7}]$	$-\frac{1}{2}(-1)^{6/7}(1+(-1)^{2/7})$
15	$\mathrm{Cos}[\frac{2\pi}{15}]$	=	$\mathrm{Cos}[\frac{2\pi}{15}]$	$\frac{1}{4}\sqrt{\frac{3}{2}\left(5-\sqrt{5}\right)}+\frac{1}{8}\left(1+\sqrt{5}\right)$
16	$\mathrm{Cos}[\frac{2\pi}{16}]$	=	$\mathrm{Cos}[\frac{\pi}{8}]$	$\frac{\sqrt{2+\sqrt{2}}}{2}$
17	$\mathrm{Cos}[\frac{2\pi}{17}]$	=	$\mathrm{Cos}[\frac{2\pi}{17}]$	$\frac{1}{4\sqrt{\frac{2}{15+\sqrt{17}-\sqrt{2\left(17-\sqrt{17}\right)}+\sqrt{2\left(34+6\sqrt{17}+\sqrt{2\left(17-\sqrt{17}\right)}-\sqrt{34\left(17-\sqrt{17}\right)}+8\sqrt{2\left(17+\sqrt{17}\right)}\right)}}}}$
18	$\mathrm{Cos}[\frac{2\pi}{18}]$	=	$\mathrm{Cos}[\frac{\pi}{9}]$	$-\frac{1}{2}(-1)^{8/9}(1+(-1)^{2/9})$
19	$\mathrm{Cos}[\frac{2\pi}{19}]$	=	$\mathrm{Cos}[\frac{2\pi}{19}]$	$-\frac{1}{2}(-1)^{17/19}(1+(-1)^{4/19})$
20	$\mathrm{Cos}[\frac{2\pi}{20}]$	=	$\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}$	$\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}$
21	$\mathrm{Cos}[\frac{2\pi}{21}]$	=	$\mathrm{Cos}[\frac{2\pi}{21}]$	$-\frac{1}{2}(-1)^{19/21}(1+(-1)^{4/21})$
22	$\mathrm{Cos}[\frac{2\pi}{22}]$	=	$\mathrm{Cos}[\frac{\pi}{11}]$	$-\frac{1}{2}(-1)^{10/11}(1+(-1)^{2/11})$
23	$\mathrm{Cos}[\frac{2\pi}{23}]$	=	$\mathrm{Cos}[\frac{2\pi}{23}]$	$-\frac{1}{2}(-1)^{21/23}(1+(-1)^{4/23})$
24	$\mathrm{Cos}[\frac{2\pi}{24}]$	=	$\frac{1+\sqrt{3}}{2\sqrt{2}}$	$\frac{1+\sqrt{3}}{2\sqrt{2}}$
25	$\mathrm{Cos}[\frac{2\pi}{25}]$	=	$\mathrm{Cos}[\frac{2\pi}{25}]$	$-\frac{1}{2}(-1)^{23/25}(1+(-1)^{4/25})$
26	$\mathrm{Cos}[\frac{2\pi}{26}]$	=	$\mathrm{Cos}[\frac{\pi}{13}]$	$-\frac{1}{2}(-1)^{12/13}(1+(-1)^{2/13})$
27	$\mathrm{Cos}[\frac{2\pi}{27}]$	=	$\mathrm{Cos}[\frac{2\pi}{27}]$	$-\frac{1}{2}(-1)^{25/27}(1+(-1)^{4/27})$

28	$\text{Cos}\left[\frac{2\pi}{28}\right]$	=	$\text{Cos}\left[\frac{\pi}{14}\right]$	$-\frac{1}{2}(-1)^{13/14}\left(1+(-1)^{1/7}\right)$
29	$\text{Cos}\left[\frac{2\pi}{29}\right]$	=	$\text{Cos}\left[\frac{2\pi}{29}\right]$	$-\frac{1}{2}(-1)^{27/29}\left(1+(-1)^{4/29}\right)$
30	$\text{Cos}\left[\frac{2\pi}{30}\right]$	=	$\text{Cos}\left[\frac{\pi}{15}\right]$	$\frac{1}{8}\left(-1+\sqrt{5}\right)+\frac{1}{4}\sqrt{\frac{3}{2}\left(5+\sqrt{5}\right)}$
31	$\text{Cos}\left[\frac{2\pi}{31}\right]$	=	$\text{Cos}\left[\frac{2\pi}{31}\right]$	$-\frac{1}{2}(-1)^{29/31}\left(1+(-1)^{4/31}\right)$
32	$\text{Cos}\left[\frac{2\pi}{32}\right]$	=	$\text{Cos}\left[\frac{\pi}{16}\right]$	$\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2}}}$
33	$\text{Cos}\left[\frac{2\pi}{33}\right]$	=	$\text{Cos}\left[\frac{2\pi}{33}\right]$	$-\frac{1}{2}(-1)^{31/33}\left(1+(-1)^{4/33}\right)$
34	$\text{Cos}\left[\frac{2\pi}{34}\right]$	=	$\text{Cos}\left[\frac{\pi}{17}\right]$	$\frac{1}{4\sqrt{\frac{2}{15+\sqrt{17}+\sqrt{2\left(17-\sqrt{17}\right)}+\sqrt{2\left(34+6\sqrt{17}-\sqrt{2\left(17-\sqrt{17}\right)}+\sqrt{34\left(17-\sqrt{17}\right)}-8\sqrt{2\left(17+\sqrt{17}\right)}\right)}}}}$
35	$\text{Cos}\left[\frac{2\pi}{35}\right]$	=	$\text{Cos}\left[\frac{2\pi}{35}\right]$	$-\frac{1}{2}(-1)^{33/35}\left(1+(-1)^{4/35}\right)$
36	$\text{Cos}\left[\frac{2\pi}{36}\right]$	=	$\text{Cos}\left[\frac{\pi}{18}\right]$	$-\frac{1}{2}(-1)^{17/18}\left(1+(-1)^{1/9}\right)$
37	$\text{Cos}\left[\frac{2\pi}{37}\right]$	=	$\text{Cos}\left[\frac{2\pi}{37}\right]$	$-\frac{1}{2}(-1)^{35/37}\left(1+(-1)^{4/37}\right)$
38	$\text{Cos}\left[\frac{2\pi}{38}\right]$	=	$\text{Cos}\left[\frac{\pi}{19}\right]$	$-\frac{1}{2}(-1)^{18/19}\left(1+(-1)^{2/19}\right)$
39	$\text{Cos}\left[\frac{2\pi}{39}\right]$	=	$\text{Cos}\left[\frac{2\pi}{39}\right]$	$-\frac{1}{2}(-1)^{37/39}\left(1+(-1)^{4/39}\right)$
40	$\text{Cos}\left[\frac{2\pi}{40}\right]$	=	$\text{Cos}\left[\frac{\pi}{20}\right]$	$\frac{\sqrt{5-\sqrt{5}}}{4}+\frac{1+\sqrt{5}}{4\sqrt{2}}$
41	$\text{Cos}\left[\frac{2\pi}{41}\right]$	=	$\text{Cos}\left[\frac{2\pi}{41}\right]$	$-\frac{1}{2}(-1)^{39/41}\left(1+(-1)^{4/41}\right)$
42	$\text{Cos}\left[\frac{2\pi}{42}\right]$	=	$\text{Cos}\left[\frac{\pi}{21}\right]$	$-\frac{1}{2}(-1)^{20/21}\left(1+(-1)^{2/21}\right)$
43	$\text{Cos}\left[\frac{2\pi}{43}\right]$	=	$\text{Cos}\left[\frac{2\pi}{43}\right]$	$-\frac{1}{2}(-1)^{41/43}\left(1+(-1)^{4/43}\right)$
44	$\text{Cos}\left[\frac{2\pi}{44}\right]$	=	$\text{Cos}\left[\frac{\pi}{22}\right]$	$-\frac{1}{2}(-1)^{21/22}\left(1+(-1)^{1/11}\right)$
45	$\text{Cos}\left[\frac{2\pi}{45}\right]$	=	$\text{Cos}\left[\frac{2\pi}{45}\right]$	$-\frac{1}{2}(-1)^{43/45}\left(1+(-1)^{4/45}\right)$
46	$\text{Cos}\left[\frac{2\pi}{46}\right]$	=	$\text{Cos}\left[\frac{\pi}{23}\right]$	$-\frac{1}{2}(-1)^{22/23}\left(1+(-1)^{2/23}\right)$
47	$\text{Cos}\left[\frac{2\pi}{47}\right]$	=	$\text{Cos}\left[\frac{2\pi}{47}\right]$	$-\frac{1}{2}(-1)^{45/47}\left(1+(-1)^{4/47}\right)$
48	$\text{Cos}\left[\frac{2\pi}{48}\right]$	=	$\text{Cos}\left[\frac{\pi}{24}\right]$	$\frac{1}{4}\sqrt{3\left(2-\sqrt{2}\right)}\left(1+\sqrt{2}\right)+\frac{1}{4}\left(-1+\sqrt{2}\right)\sqrt{2+\sqrt{2}}$
49	$\text{Cos}\left[\frac{2\pi}{49}\right]$	=	$\text{Cos}\left[\frac{2\pi}{49}\right]$	$-\frac{1}{2}(-1)^{47/49}\left(1+(-1)^{4/49}\right)$
50	$\text{Cos}\left[\frac{2\pi}{50}\right]$	=	$\text{Cos}\left[\frac{\pi}{25}\right]$	$-\frac{1}{2}(-1)^{24/25}\left(1+(-1)^{2/25}\right)$

◄ \| ►

"constructible" numbers

```
supscr[a_, b_] := If[b == 1, a, Superscript[a, b]]

prod[n_] := Block[{com = Apply[supscr, FactorInteger[n], 1], res = n},
  If[Length[com] > 1, com = Apply[CenterDot, com], com = com[[1]]];
  Return[com]]

costr[n_] := Block[{com = Complement[FactorInteger[n],
      {{3, 1}, {5, 1}, {17, 1}, {257, 1}, {65 537, 1}}], res},
   res = If[Length[com] == 0 || Length[com] == 1 && com[[1, 1]] == 2, True, False, False];
   Return[res];
  ];
```

a table with divisors

constructible numbers: an error?

```
TableForm[
 Select[Table[{Style[n, If[costr[n], FontColor → Red, FontColor → Black]], prod[n],
    c[n], If[costr[n], "=", ""], If[costr[n], ToRadicals[Cos[2 Pi / n]], ""]},
   {n, 2, 300}], ! FreeQ[#, "="] &]]
```

2	2	$\mathrm{Cos}\left[\frac{2\pi}{2}\right]$	=	-1
3	3	$\mathrm{Cos}\left[\frac{2\pi}{3}\right]$	=	$-\frac{1}{2}$
4	2^2	$\mathrm{Cos}\left[\frac{2\pi}{4}\right]$	=	0
5	5	$\mathrm{Cos}\left[\frac{2\pi}{5}\right]$	=	$\frac{1}{4}\left(-1+\sqrt{5}\right)$
6	$2 \cdot 3$	$\mathrm{Cos}\left[\frac{2\pi}{6}\right]$	=	$\frac{1}{2}$
8	2^3	$\mathrm{Cos}\left[\frac{2\pi}{8}\right]$	=	$\frac{1}{\sqrt{2}}$
10	$2 \cdot 5$	$\mathrm{Cos}\left[\frac{2\pi}{10}\right]$	=	$\frac{1}{4}\left(1+\sqrt{5}\right)$
12	$2^2 \cdot 3$	$\mathrm{Cos}\left[\frac{2\pi}{12}\right]$	=	$\frac{\sqrt{3}}{2}$
15	$3 \cdot 5$	$\mathrm{Cos}\left[\frac{2\pi}{15}\right]$	=	$\frac{1}{4}\sqrt{\frac{3}{2}\left(5-\sqrt{5}\right)}+\frac{1}{8}\left(1+\sqrt{5}\right)$
16	2^4	$\mathrm{Cos}\left[\frac{2\pi}{16}\right]$	=	$\frac{\sqrt{2+\sqrt{2}}}{2}$
17	17	$\mathrm{Cos}\left[\frac{2\pi}{17}\right]$	=	$\frac{1}{4\sqrt{\frac{2}{15+\sqrt{17}-\sqrt{2\left(17-\sqrt{17}\right)}+\sqrt{2\left(34+6\sqrt{17}+\sqrt{2\left(17-\sqrt{17}\right)}-\sqrt{34\left(17-\sqrt{17}\right)}+8\sqrt{2\left(17+\sqrt{17}\right)}\right)}}}}$
20	$2^2 \cdot 5$	$\mathrm{Cos}\left[\frac{2\pi}{20}\right]$	=	$\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}$
24	$2^3 \cdot 3$	$\mathrm{Cos}\left[\frac{2\pi}{24}\right]$	=	$\frac{1+\sqrt{3}}{2\sqrt{2}}$
30	$2 \cdot 3 \cdot 5$	$\mathrm{Cos}\left[\frac{2\pi}{30}\right]$	=	$\frac{1}{8}\left(-1+\sqrt{5}\right)+\frac{1}{4}\sqrt{\frac{3}{2}\left(5+\sqrt{5}\right)}$
32	2^5	$\mathrm{Cos}\left[\frac{2\pi}{32}\right]$	=	$\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2}}}$
34	$2 \cdot 17$	$\mathrm{Cos}\left[\frac{2\pi}{34}\right]$	=	$\frac{1}{4\sqrt{\frac{2}{15+\sqrt{17}+\sqrt{2\left(17-\sqrt{17}\right)}+\sqrt{2\left(34+6\sqrt{17}-\sqrt{2\left(17-\sqrt{17}\right)}+\sqrt{34\left(17-\sqrt{17}\right)}-8\sqrt{2\left(17+\sqrt{17}\right)}\right)}}}}$
40	$2^3 \cdot 5$	$\mathrm{Cos}\left[\frac{2\pi}{40}\right]$	=	$\frac{\sqrt{5-\sqrt{5}}}{4}+\frac{1+\sqrt{5}}{4\sqrt{2}}$
48	$2^4 \cdot 3$	$\mathrm{Cos}\left[\frac{2\pi}{48}\right]$	=	$\frac{1}{4}\sqrt{3\left(2-\sqrt{2}\right)}\left(1+\sqrt{2}\right)+\frac{1}{4}\left(-1+\sqrt{2}\right)\sqrt{2+\sqrt{2}}$

51	$3 \cdot 17$	$\text{Cos}\left[\frac{2\pi}{51}\right]$	$= \dfrac{1}{8\sqrt{\dfrac{2}{15-\sqrt{17}+\sqrt{2\left(17+\sqrt{17}\right)}-\sqrt{2\left(34-6\sqrt{17}+8\sqrt{2\left(17-\sqrt{17}\right)}-\sqrt{2\left(17+\sqrt{17}\right)}-\sqrt{34\left(17+\sqrt{17}\right)}\right)}}}}$
60	$2^2 \cdot 3 \cdot 5$	$\text{Cos}\left[\frac{2\pi}{60}\right]$	$= \frac{1}{4}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}+\frac{1}{8}\sqrt{3}\left(1+\sqrt{5}\right)$
64	2^6	$\text{Cos}\left[\frac{2\pi}{64}\right]$	$= \frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}$
68	$2^2 \cdot 17$	$\text{Cos}\left[\frac{2\pi}{68}\right]$	$= \dfrac{1}{4\sqrt{\dfrac{2}{17-\sqrt{17}+\sqrt{2\left(17-\sqrt{17}\right)}+\sqrt{2\left(34+6\sqrt{17}+\sqrt{2\left(17-\sqrt{17}\right)}-\sqrt{34\left(17-\sqrt{17}\right)}+8\sqrt{2\left(17+\sqrt{17}\right)}\right)}}}}$
80	$2^4 \cdot 5$	$\text{Cos}\left[\frac{2\pi}{80}\right]$	$= \frac{1}{8}\left(-1+\sqrt{2}\right)\sqrt{2+\sqrt{2}}\left(-1+\sqrt{5}\right)+\frac{1}{4}\left(1+\sqrt{2}\right)\sqrt{\frac{1}{2}\left(2-\right.}$
85	$5 \cdot 17$	$\text{Cos}\left[\frac{2\pi}{85}\right]$	$= -\dfrac{-1-\sqrt{5}}{16\sqrt{\dfrac{2}{15-\sqrt{17}+\sqrt{2\left(17+\sqrt{17}\right)}+\sqrt{2\left(34-6\sqrt{17}+8\sqrt{2\left(17-\sqrt{17}\right)}-\sqrt{2\left(17+\sqrt{17}\right)}-\sqrt{34\left(17+\sqrt{17}\right)}\right)}}}}$
96	$2^5 \cdot 3$	$\text{Cos}\left[\frac{2\pi}{96}\right]$	$= -\frac{1}{4}\sqrt{3}\left(-\sqrt{2-\sqrt{2+\sqrt{2}}}-\sqrt{2\left(2-\sqrt{2+\sqrt{2}}\right)}-\sqrt{\left(2+\right.}\right.$
102	$2 \cdot 3 \cdot 17$	$\text{Cos}\left[\frac{2\pi}{102}\right]$	$= \frac{1}{8}\sqrt{\frac{3}{2}\left(17+\sqrt{17}+\sqrt{2\left(17+\sqrt{17}\right)}-\sqrt{2\left(34-6\sqrt{17}-8\right.}\right.}$
120	$2^3 \cdot 3 \cdot 5$	$\text{Cos}\left[\frac{2\pi}{120}\right]$	$= -\dfrac{-\frac{1}{8}\sqrt{3}\left(-1+\sqrt{5}\right)-\frac{1}{4}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}}{\sqrt{2}}-\dfrac{\frac{1}{8}\left(-1+\sqrt{5}\right)-\frac{1}{4}\sqrt{\frac{3}{2}\left(5+\sqrt{5}\right)}}{\sqrt{2}}$
128	2^7	$\text{Cos}\left[\frac{2\pi}{128}\right]$	$= \frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}$
136	$2^3 \cdot 17$	$\text{Cos}\left[\frac{2\pi}{136}\right]$	$= \frac{1}{8}\sqrt{15+\sqrt{17}+\sqrt{2\left(17-\sqrt{17}\right)}-\sqrt{2\left(34+6\sqrt{17}-\sqrt{2\left(\right.}\right.}}$
160	$2^5 \cdot 5$	$\text{Cos}\left[\frac{2\pi}{160}\right]$	$= -\frac{1}{4}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\left(-\sqrt{2-\sqrt{2+\sqrt{2}}}-\sqrt{\left(2+\sqrt{2}\right)\left(2-\right.}\right.$

170	$2 \cdot 5 \cdot 17$	$\text{Cos}\left[\frac{2\pi}{170}\right]$	=	$\frac{-1+\sqrt{5}}{16\sqrt{\frac{2}{15-\sqrt{17}-\sqrt{2\left(17+\sqrt{17}\right)}-\sqrt{2\left(34-6\sqrt{17}-8\sqrt{2\left(17-\sqrt{17}\right)}+\sqrt{2\left(17+\sqrt{17}\right)}+\sqrt{34\left(17+\sqrt{17}\right)}\right)}}}}$
192	$2^6 \cdot 3$	$\text{Cos}\left[\frac{2\pi}{192}\right]$	=	$\frac{1}{4}\sqrt{3}\left(\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2}}}}+\sqrt{2\left(2-\sqrt{2+\sqrt{2+\sqrt{2}}}\right)}\right.$
204	$2^2 \cdot 3 \cdot 17$	$\text{Cos}\left[\frac{2\pi}{204}\right]$	=	$\frac{1}{8}\sqrt{\frac{3}{2}\left(15-\sqrt{17}+\sqrt{2\left(17+\sqrt{17}\right)}+\sqrt{2\left(34-6\sqrt{17}+8\right.}\right.}$
240	$2^4 \cdot 3 \cdot 5$	$\text{Cos}\left[\frac{2\pi}{240}\right]$	=	$\frac{1}{2}\sqrt{2+\sqrt{2}}\left(\frac{1}{4}\sqrt{\frac{3}{2}\left(5-\sqrt{5}\right)}+\frac{1}{8}\left(1+\sqrt{5}\right)\right)+\frac{1}{2}\sqrt{2-\sqrt{2}}$
255	$3 \cdot 5 \cdot 17$	$\text{Cos}\left[\frac{2\pi}{255}\right]$	=	$-\frac{1}{2}\sqrt{3}\left(-\frac{1}{16}\sqrt{\left(5+\sqrt{5}\right)\left(15+\sqrt{17}+\sqrt{2\left(17-\sqrt{17}\right)}+\right.}\right.$
256	2^8	$\text{Cos}\left[\frac{2\pi}{256}\right]$	=	$\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}$
257	257	$\text{Cos}\left[\frac{2\pi}{257}\right]$	=	$-\frac{1}{2}(-1)^{255/257}\left(1+(-1)^{4/257}\right)$
272	$2^4 \cdot 17$	$\text{Cos}\left[\frac{2\pi}{272}\right]$	=	$\frac{1}{8}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)\left(15+\sqrt{17}-\sqrt{2\left(17-\sqrt{17}\right)}+\sqrt{2\left(34+6\right.}\right.}$

◀ | ▶

the function costr[ ]

A theorem of Gauss

particular cases

Let’s go on ...

```
ToRadicals[Cos[2 Pi / 257]]
```

$$-\frac{1}{2}(-1)^{255/257}\left(1+(-1)^{4/257}\right)$$

We want to represent $\cos\left(\frac{2\pi}{257}\right)$ by radicals, making use of some modular algebra as suggested in a very nice article of Christian Gottlieb (The Simple and Straightforward Construction of the Regular 257-gon, THE MATHEMATICAL INTELLIGENCER, VOLUME21, NUMBER1, 1999).

n = 5 : Cos[2π/5]

definitions

```
listas[n_] := Block[{ls, lscnt, i, j, k},
   ls = Flatten[Table[Mod[3^(2 i * 2^(n - 1)) + 3^((2 k + 1) * 2^(n - 1)), 5],
      {i, 0, 2^(2 - n) - 1}, {k, 0, 2^(2 - n) - 1}]];
   lscnt = Table[Count[ls, j], {j, 1, 16}];
   Return[Table[lscnt[[Mod[3^s, 5]]], {s, 0, 2^(n - 1) - 1}]];];
vec[n_] := Block[{res = Table[0, {2^n}], temp, k, h},
   Subsuperscript[a, 1, 0] = -1; For[k = 1, k ≤ 2^(n - 1), k++,
    temp = Solve[x^2 - Subsuperscript[a, k, n - 1] x +
        Sum[listaprod[[n, h]] Subsuperscript[a, Mod[k + h - 2, 2^(n - 1)] + 1, n - 1],
         {h, 1, 2^(n - 1)}] == 0, x];
    res[[k]] = temp[[2 - listarev[[n, k]], 1, 2]]; res[[k + 2^(n - 1)]] =
     temp[[1 + listarev[[n, k]], 1, 2]];]; res];
sub[n_] := Table[Subsuperscript[a, k, n] → vec[n][[k]], {k, 1, 2^n}]
```

lists

```
listaprod = Table[listas[k], {k, 1, 1}]
```

{{1}}

```
listarev = Table[Table[Boole[TrueQ[Chop[N[Sum[
          Exp[2 Pi I / 5 * 3^(2^n * j + k)] - Exp[2 Pi I / 5 * 3^(2^n * j + k + 2^(n - 1))],
          {j, 0, 2^(4 - n) - 1}]]] < 0]], {k, 0, 2^(n - 1) - 1}], {n, 1, 1}]
```

{{0}}

result

```
Simplify[vec[1][[1]] ] / 2
```

$$\frac{1}{4}\left(-1+\sqrt{5}\right)$$

```
Cos[2 Pi / 5]
```

$$\frac{1}{4}\left(-1+\sqrt{5}\right)$$

n = 17 : Cos[2π/17]

definitions

```
listas[n_] := Block[{ls, lscnt, i, j, k},
   ls = Flatten[Table[Mod[3^(2 i * 2^(n - 1)) + 3^((2 k + 1) * 2^(n - 1)), 17],
      {i, 0, 2^(4 - n) - 1}, {k, 0, 2^(4 - n) - 1}]];
   lscnt = Table[Count[ls, j], {j, 1, 16}];
   Return[Table[lscnt[[Mod[3^s, 17]]], {s, 0, 2^(n - 1) - 1}]];];
vec[n_] := Block[{res = Table[0, {2^n}], temp, k, h},
   Subsuperscript[a, 1, 0] = -1; For[k = 1, k ≤ 2^(n - 1), k++,
    temp = Solve[x^2 - Subsuperscript[a, k, n - 1] x +
        Sum[listaprod[[n, h]] Subsuperscript[a, Mod[k + h - 2, 2^(n - 1)] + 1, n - 1],
         {h, 1, 2^(n - 1)}] == 0, x];
    res[[k]] = temp[[2 - listarev[[n, k]], 1, 2]]; res[[k + 2^(n - 1)]] =
     temp[[1 + listarev[[n, k]], 1, 2]];]; res];
sub[n_] := Table[Subsuperscript[a, k, n] → vec[n][[k]], {k, 1, 2^n}]
```

lists

```
listaprod = Table[listas[k], {k, 1, 3}]
```

```
{{4}, {1, 1}, {0, 1, 0, 0}}
```

```
listarev = Table[Table[Boole[TrueQ[Chop[N[Sum[
          Exp[2 Pi I / 17 * 3^(2^n * j + k)] - Exp[2 Pi I / 17 * 3^(2^n * j + k + 2^(n - 1))],
          {j, 0, 2^(4 - n) - 1}]]] < 0]], {k, 0, 2^(n - 1) - 1}], {n, 1, 3}]
```

```
{{0}, {0, 0}, {0, 0, 1, 1}}
```

result

```
duecos17 = Simplify[Simplify[vec[3][[1]] /. sub[2] ] /. sub[1] ]
```

$$\frac{1}{8}\left(-1+\sqrt{17}+\sqrt{34-2\sqrt{17}}+\sqrt{2\left(34+6\sqrt{17}+\sqrt{578-34\sqrt{17}}-\sqrt{34-2\sqrt{17}}-8\sqrt{2\left(17+\sqrt{17}\right)}\right)}\right)$$

```
ToRadicals[Cos[2 Pi / 17]]
```

$$\frac{1}{4\sqrt{\dfrac{2}{15+\sqrt{17}-\sqrt{2\left(17-\sqrt{17}\right)}+\sqrt{2\left(34+6\sqrt{17}+\sqrt{2\left(17-\sqrt{17}\right)}-\sqrt{34\left(17-\sqrt{17}\right)}+8\sqrt{2\left(17+\sqrt{17}\right)}\right)}}}}$$

```
N[Cos[2 Pi / 17], 300]
```

```
0.93247222940435580457311589182156338626258777794511692824835001186053604656964449⋱
 98128074712850429850975199317771955063603452225595829170285957782296811882572852⋱
 75222312615678639098897635933690713735310817893344052950097661487517220974561206⋱
 5331096212571185836827570609893334637824341946015475517591411
```

```
N[1 / 2 duecos17, 300]
```

```
0.93247222940435580457311589182156338626258777794511692824835001186053604656964449⋱
 98128074712850429850975199317771955063603452225595829170285957782296811882572852⋱
 75222312615678639098897635933690713735310817893344052950097661487517220974561206⋱
 5331096212571185836827570609893334637824341946015475517591411
```

Costruzione di poligoni regolari.

graphical functions

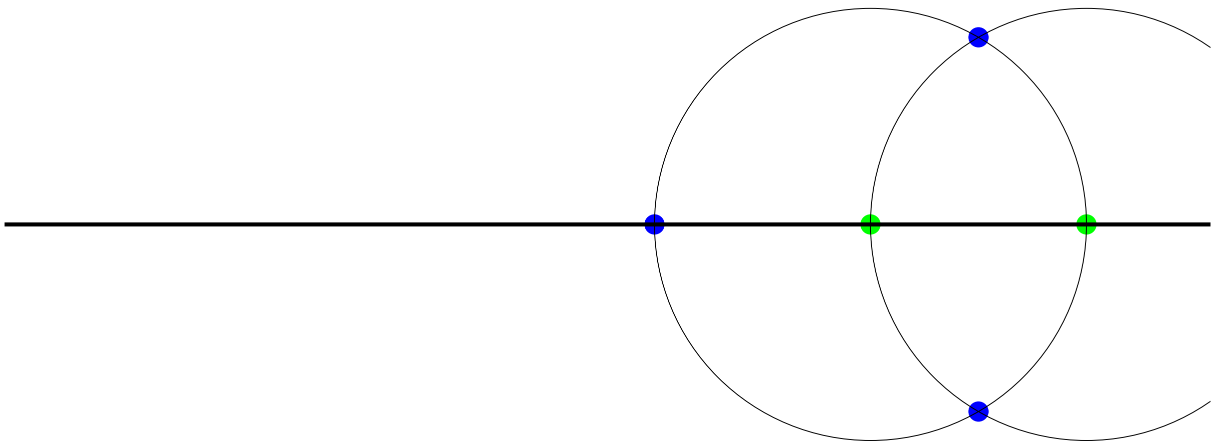
other graphical functions

The set of constructible points

data

$\{y == 0, -1 + x^2 + y^2 == 0, -2x + x^2 + y^2 == 0\}$

```
graf[{{0, 0}, {1, 0}}]
```



```
ls1 = passocomp[passoeq[ls0]]
```

$\left\{\{-1, 0\}, \{0, 0\}, \left\{\frac{1}{2}, -\frac{\sqrt{3}}{2}\right\}, \left\{\frac{1}{2}, \frac{\sqrt{3}}{2}\right\}, \{1, 0\}, \{2, 0\}\right\}$

2 steps

```
listagraf[ls1]
```

all constructible points in 2 steps

passoeq[ls1]

$$\left\{2x == 1, \sqrt{3}\,x == y, \sqrt{3}\,x == \sqrt{3} + y, \sqrt{3}\,x == 2\sqrt{3} + 3y, \sqrt{3}\,(1+x) == 3y, y == 0, \sqrt{3}\,x + y == 0, \sqrt{3}\,x + y == \sqrt{3}, \sqrt{3}\,x + 3y == 2\sqrt{3}, \sqrt{3} + \sqrt{3}\,x + 3y == 0, -4 + x^2 + y^2 == 0, -1 + x^2 + y^2 == 0, -5 - 4x + x^2 + y^2 == 0, -4x + x^2 + y^2 == 0, 1 - 4x + x^2 + y^2 == 0, 3 - 4x + x^2 + y^2 == 0, -3 - 2x + x^2 + y^2 == 0, -2x + x^2 + y^2 == 0, -8 + 2x + x^2 + y^2 == 0, -3 + 2x + x^2 + y^2 == 0, -2 + 2x + x^2 + y^2 == 0, 2x + x^2 + y^2 == 0, -2 - x + x^2 - \sqrt{3}\,y + y^2 == 0, -x + x^2 - \sqrt{3}\,y + y^2 == 0, -2 - x + x^2 + \sqrt{3}\,y + y^2 == 0, -x + x^2 + \sqrt{3}\,y + y^2 == 0\right\}$$

ls2 = passocomp[passoeq[ls1]]

$$\left\{\{-4, 0\}, \{-3, 0\}, \left\{-\frac{5}{2}, -\frac{3\sqrt{3}}{2}\right\}, \left\{-\frac{5}{2}, -\frac{\sqrt{3}}{2}\right\}, \left\{-\frac{5}{2}, \frac{\sqrt{3}}{2}\right\}, \left\{-\frac{5}{2}, \frac{3\sqrt{3}}{2}\right\}, \{-2, 0\}, \{-1, 0\}, \left\{-1, -\sqrt{3}\right\}, \left\{-1, \sqrt{3}\right\}, \left\{-\frac{5}{6}, -\frac{\sqrt{35}}{6}\right\}, \left\{-\frac{5}{6}, \frac{\sqrt{35}}{6}\right\}, \left\{-\frac{3}{4}, -\frac{\sqrt{15}}{4}\right\}, \left\{-\frac{3}{4}, \frac{\sqrt{15}}{4}\right\}, \left\{-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right\}, \left\{-\frac{1}{2}, \frac{\sqrt{3}}{2}\right\}, \left\{-\frac{1}{2}, -\frac{\sqrt{11}}{2}\right\}, \left\{-\frac{1}{2}, \frac{\sqrt{11}}{2}\right\}, \left\{-\frac{1}{2}, -\frac{\sqrt{15}}{2}\right\}, \left\{-\frac{1}{2}, \frac{\sqrt{15}}{2}\right\}, \left\{-\frac{1}{3}, -\frac{4\sqrt{2}}{3}\right\}, \left\{-\frac{1}{3}, \frac{4\sqrt{2}}{3}\right\}, \left\{-\frac{1}{4}, -\frac{\sqrt{3}}{4}\right\}, \left\{-\frac{1}{4}, \frac{\sqrt{3}}{4}\right\}, \left\{-\frac{1}{4}, -\frac{3\sqrt{7}}{4}\right\}, \left\{-\frac{1}{4}, \frac{3\sqrt{7}}{4}\right\}, \left\{-\frac{1}{4}, -\frac{\sqrt{39}}{4}\right\}, \left\{-\frac{1}{4}, \frac{\sqrt{39}}{4}\right\}, \{0, 0\}, \left\{0, -\sqrt{3}\right\}, \left\{0, \sqrt{3}\right\}, \left\{\frac{1}{4}, -\frac{\sqrt{15}}{4}\right\}, \left\{\frac{1}{4}, \frac{\sqrt{15}}{4}\right\}, \left\{\frac{1}{3}, -\frac{\sqrt{11}}{3}\right\}, \left\{\frac{1}{3}, \frac{\sqrt{11}}{3}\right\}, \left\{\frac{1}{2}, 0\right\}, \left\{\frac{1}{2}, -\frac{3\sqrt{3}}{2}\right\}, \left\{\frac{1}{2}, -\frac{\sqrt{3}}{2}\right\}, \left\{\frac{1}{2}, \frac{\sqrt{3}}{2}\right\}, \left\{\frac{1}{2}, \frac{3\sqrt{3}}{2}\right\}, \left\{\frac{1}{2}, -\frac{\sqrt{7}}{2}\right\}, \left\{\frac{1}{2}, \frac{\sqrt{7}}{2}\right\}, \left\{\frac{1}{2}, -\frac{\sqrt{15}}{2}\right\}, \left\{\frac{1}{2}, \frac{\sqrt{15}}{2}\right\}, \left\{\frac{1}{2}, \frac{1}{2}\left(-2-\sqrt{3}\right)\right\}, \left\{\frac{1}{2}, \frac{1}{2}\left(2-\sqrt{3}\right)\right\}, \left\{\frac{1}{2}, \frac{1}{2}\left(-2+\sqrt{3}\right)\right\}, \left\{\frac{1}{2}, \frac{1}{2}\left(2+\sqrt{3}\right)\right\}, \left\{\frac{2}{3}, -\frac{\sqrt{11}}{3}\right\}, \left\{\frac{2}{3}, \frac{\sqrt{11}}{3}\right\}, \left\{\frac{3}{4}, -\frac{\sqrt{15}}{4}\right\}, \left\{\frac{3}{4}, \frac{\sqrt{15}}{4}\right\}, \{1, 0\}, \left\{1, -\sqrt{3}\right\}, \left\{1, \sqrt{3}\right\}, \left\{\frac{5}{4}, -\frac{\sqrt{3}}{4}\right\}, \left\{\frac{5}{4}, \frac{\sqrt{3}}{4}\right\}, \left\{\frac{5}{4}, -\frac{3\sqrt{7}}{4}\right\}, \left\{\frac{5}{4}, \frac{3\sqrt{7}}{4}\right\}, \left\{\frac{5}{4}, -\frac{\sqrt{39}}{4}\right\}, \left\{\frac{5}{4}, \frac{\sqrt{39}}{4}\right\}, \left\{\frac{4}{3}, -\frac{4\sqrt{2}}{3}\right\}, \left\{\frac{4}{3}, \frac{4\sqrt{2}}{3}\right\}, \left\{\frac{3}{2}, -\frac{\sqrt{3}}{2}\right\}, \left\{\frac{3}{2}, \frac{\sqrt{3}}{2}\right\}, \left\{\frac{3}{2}, -\frac{\sqrt{11}}{2}\right\}, \left\{\frac{3}{2}, \frac{\sqrt{11}}{2}\right\}, \left\{\frac{3}{2}, -\frac{\sqrt{15}}{2}\right\}, \left\{\frac{3}{2}, \frac{\sqrt{15}}{2}\right\}, \left\{\frac{7}{4}, -\frac{\sqrt{15}}{4}\right\}, \left\{\frac{7}{4}, \frac{\sqrt{15}}{4}\right\}, \left\{\frac{11}{6}, -\frac{\sqrt{35}}{6}\right\}, \left\{\frac{11}{6}, \frac{\sqrt{35}}{6}\right\}, \{2, 0\}, \left\{2, -\sqrt{3}\right\}, \left\{2, \sqrt{3}\right\}, \{3, 0\}, \left\{\frac{7}{2}, -\frac{3\sqrt{3}}{2}\right\}, \left\{\frac{7}{2}, -\frac{\sqrt{3}}{2}\right\}, \left\{\frac{7}{2}, \frac{\sqrt{3}}{2}\right\}, \left\{\frac{7}{2}, \frac{3\sqrt{3}}{2}\right\}, \{4, 0\}, \{5, 0\}, \left\{-\sqrt{\frac{2}{3}}, \frac{1}{3}\left(-3\sqrt{2}-\sqrt{3}\right)\right\}, \left\{-\sqrt{\frac{2}{3}}, \frac{1+\sqrt{6}}{\sqrt{3}}\right\}, \left\{\sqrt{\frac{2}{3}}, \frac{1}{3}\left(-3\sqrt{2}+\sqrt{3}\right)\right\},$$

$$\left\{\sqrt{\frac{2}{3}}, -\frac{1-\sqrt{6}}{\sqrt{3}}\right\}, \left\{\frac{1}{2}\left(-2-3\sqrt{3}\right), -\frac{3}{2}\right\}, \left\{\frac{1}{2}\left(-2-3\sqrt{3}\right), \frac{3}{2}\right\}, \left\{\frac{1}{2}\left(4-3\sqrt{3}\right), -\frac{3}{2}\right\},$$
$$\left\{\frac{1}{2}\left(4-3\sqrt{3}\right), \frac{3}{2}\right\}, \left\{\frac{1}{2}\left(-2-\sqrt{3}\right), -\frac{1}{2}\right\}, \left\{\frac{1}{2}\left(-2-\sqrt{3}\right), \frac{1}{2}\right\}, \left\{-1-\sqrt{3}, -1\right\},$$
$$\left\{-1-\sqrt{3}, 0\right\}, \left\{-1-\sqrt{3}, 1\right\}, \left\{\frac{1}{2}\left(1-\sqrt{3}\right), \frac{1}{2}\left(-3-\sqrt{3}\right)\right\}, \left\{\frac{1}{2}\left(1-\sqrt{3}\right), \frac{1}{2}\left(-1-\sqrt{3}\right)\right\},$$
$$\left\{\frac{1}{2}\left(1-\sqrt{3}\right), \frac{1}{2}\left(1-\sqrt{3}\right)\right\}, \left\{\frac{1}{2}\left(1-\sqrt{3}\right), \frac{1}{2}\left(3-\sqrt{3}\right)\right\}, \left\{\frac{1}{2}\left(1-\sqrt{3}\right), \frac{1}{2}\left(-3+\sqrt{3}\right)\right\},$$
$$\left\{\frac{1}{2}\left(1-\sqrt{3}\right), \frac{1}{2}\left(-1+\sqrt{3}\right)\right\}, \left\{\frac{1}{2}\left(1-\sqrt{3}\right), \frac{1}{2}\left(1+\sqrt{3}\right)\right\}, \left\{\frac{1}{2}\left(1-\sqrt{3}\right), \frac{1}{2}\left(3+\sqrt{3}\right)\right\},$$
$$\left\{2-\sqrt{3}, -1\right\}, \left\{2-\sqrt{3}, 0\right\}, \left\{2-\sqrt{3}, 1\right\}, \left\{\frac{1}{2}\left(4-\sqrt{3}\right), -\frac{1}{2}\right\}, \left\{\frac{1}{2}\left(4-\sqrt{3}\right), \frac{1}{2}\right\},$$
$$\left\{\frac{1}{2}\left(-2+\sqrt{3}\right), -\frac{1}{2}\right\}, \left\{\frac{1}{2}\left(-2+\sqrt{3}\right), \frac{1}{2}\right\}, \left\{-1+\sqrt{3}, -1\right\}, \left\{-1+\sqrt{3}, 0\right\},$$
$$\left\{-1+\sqrt{3}, 1\right\}, \left\{\frac{1}{2}\left(1+\sqrt{3}\right), \frac{1}{2}\left(-3-\sqrt{3}\right)\right\}, \left\{\frac{1}{2}\left(1+\sqrt{3}\right), \frac{1}{2}\left(-1-\sqrt{3}\right)\right\},$$
$$\left\{\frac{1}{2}\left(1+\sqrt{3}\right), \frac{1}{2}\left(1-\sqrt{3}\right)\right\}, \left\{\frac{1}{2}\left(1+\sqrt{3}\right), \frac{1}{2}\left(3-\sqrt{3}\right)\right\}, \left\{\frac{1}{2}\left(1+\sqrt{3}\right), \frac{1}{2}\left(-3+\sqrt{3}\right)\right\},$$
$$\left\{\frac{1}{2}\left(1+\sqrt{3}\right), \frac{1}{2}\left(-1+\sqrt{3}\right)\right\}, \left\{\frac{1}{2}\left(1+\sqrt{3}\right), \frac{1}{2}\left(1+\sqrt{3}\right)\right\}, \left\{\frac{1}{2}\left(1+\sqrt{3}\right), \frac{1}{2}\left(3+\sqrt{3}\right)\right\},$$
$$\left\{2+\sqrt{3}, -1\right\}, \left\{2+\sqrt{3}, 0\right\}, \left\{2+\sqrt{3}, 1\right\}, \left\{\frac{1}{2}\left(4+\sqrt{3}\right), -\frac{1}{2}\right\}, \left\{\frac{1}{2}\left(4+\sqrt{3}\right), \frac{1}{2}\right\},$$
$$\left\{\frac{1}{2}\left(-2+3\sqrt{3}\right), -\frac{3}{2}\right\}, \left\{\frac{1}{2}\left(-2+3\sqrt{3}\right), \frac{3}{2}\right\}, \left\{\frac{1}{2}\left(4+3\sqrt{3}\right), -\frac{3}{2}\right\}, \left\{\frac{1}{2}\left(4+3\sqrt{3}\right), \frac{3}{2}\right\},$$
$$\left\{\frac{1}{4}\left(-1-3\sqrt{5}\right), \frac{1}{4}\left(\sqrt{3}-\sqrt{15}\right)\right\}, \left\{\frac{1}{4}\left(-1-3\sqrt{5}\right), \frac{1}{4}\left(-\sqrt{3}+\sqrt{15}\right)\right\},$$
$$\left\{\frac{1}{4}\left(5-3\sqrt{5}\right), \frac{1}{4}\left(-\sqrt{3}-\sqrt{15}\right)\right\}, \left\{\frac{1}{4}\left(5-3\sqrt{5}\right), \frac{1}{4}\left(\sqrt{3}+\sqrt{15}\right)\right\},$$
$$\left\{\frac{1}{4}\left(-1+3\sqrt{5}\right), \frac{1}{4}\left(-\sqrt{3}-\sqrt{15}\right)\right\}, \left\{\frac{1}{4}\left(-1+3\sqrt{5}\right), \frac{1}{4}\left(\sqrt{3}+\sqrt{15}\right)\right\},$$
$$\left\{\frac{1}{4}\left(5+3\sqrt{5}\right), \frac{1}{4}\left(\sqrt{3}-\sqrt{15}\right)\right\}, \left\{\frac{1}{4}\left(5+3\sqrt{5}\right), \frac{1}{4}\left(-\sqrt{3}+\sqrt{15}\right)\right\},$$
$$\left\{\frac{1}{2}\left(1-\sqrt{6}\right), \frac{1}{2}\left(-3\sqrt{2}-\sqrt{3}\right)\right\}, \left\{\frac{1}{2}\left(1-\sqrt{6}\right), \frac{3}{\sqrt{2}}-\frac{\sqrt{3}}{2}\right\}, \left\{\frac{1}{2}\left(1-\sqrt{6}\right), \frac{3}{\sqrt{2}}+\frac{\sqrt{3}}{2}\right\},$$
$$\left\{\frac{1}{2}\left(1-\sqrt{6}\right), \frac{1}{2}\left(-3\sqrt{2}+\sqrt{3}\right)\right\}, \left\{\frac{1}{3}\left(3-\sqrt{6}\right), \frac{1}{3}\left(-3\sqrt{2}+\sqrt{3}\right)\right\},$$
$$\left\{\frac{1}{3}\left(3-\sqrt{6}\right), \frac{-1+\sqrt{6}}{\sqrt{3}}\right\}, \left\{\frac{1}{2}\left(1+\sqrt{6}\right), \frac{1}{2}\left(-3\sqrt{2}-\sqrt{3}\right)\right\}, \left\{\frac{1}{2}\left(1+\sqrt{6}\right), \frac{3}{\sqrt{2}}-\frac{\sqrt{3}}{2}\right\},$$
$$\left\{\frac{1}{2}\left(1+\sqrt{6}\right), \frac{3}{\sqrt{2}}+\frac{\sqrt{3}}{2}\right\}, \left\{\frac{1}{2}\left(1+\sqrt{6}\right), \frac{1}{2}\left(-3\sqrt{2}+\sqrt{3}\right)\right\},$$
$$\left\{\frac{1}{3}\left(3+\sqrt{6}\right), \frac{1}{3}\left(-3\sqrt{2}-\sqrt{3}\right)\right\}, \left\{\frac{1}{3}\left(3+\sqrt{6}\right), -\frac{-1-\sqrt{6}}{\sqrt{3}}\right\},$$

$$\left\{\frac{1}{4}\left(-1-\sqrt{13}\right), \frac{1}{4}\left(-\sqrt{3}-\sqrt{39}\right)\right\}, \left\{\frac{1}{4}\left(-1-\sqrt{13}\right), \frac{1}{4}\left(\sqrt{3}+\sqrt{39}\right)\right\},$$

$$\left\{\frac{1}{4}\left(1-\sqrt{13}\right), \frac{1}{4}\left(\sqrt{3}-\sqrt{39}\right)\right\}, \left\{\frac{1}{4}\left(1-\sqrt{13}\right), \frac{1}{4}\left(-\sqrt{3}+\sqrt{39}\right)\right\},$$

$$\left\{\frac{1}{4}\left(3-\sqrt{13}\right), \frac{1}{4}\left(-\sqrt{3}-\sqrt{39}\right)\right\}, \left\{\frac{1}{4}\left(3-\sqrt{13}\right), \frac{1}{4}\left(\sqrt{3}+\sqrt{39}\right)\right\},$$

$$\left\{\frac{1}{4}\left(5-\sqrt{13}\right), \frac{1}{4}\left(\sqrt{3}-\sqrt{39}\right)\right\}, \left\{\frac{1}{4}\left(5-\sqrt{13}\right), \frac{1}{4}\left(-\sqrt{3}+\sqrt{39}\right)\right\},$$

$$\left\{\frac{1}{4}\left(-1+\sqrt{13}\right), \frac{1}{4}\left(\sqrt{3}-\sqrt{39}\right)\right\}, \left\{\frac{1}{4}\left(-1+\sqrt{13}\right), \frac{1}{4}\left(-\sqrt{3}+\sqrt{39}\right)\right\},$$

$$\left\{\frac{1}{4}\left(1+\sqrt{13}\right), \frac{1}{4}\left(-\sqrt{3}-\sqrt{39}\right)\right\}, \left\{\frac{1}{4}\left(1+\sqrt{13}\right), \frac{1}{4}\left(\sqrt{3}+\sqrt{39}\right)\right\},$$

$$\left\{\frac{1}{4}\left(3+\sqrt{13}\right), \frac{1}{4}\left(\sqrt{3}-\sqrt{39}\right)\right\}, \left\{\frac{1}{4}\left(3+\sqrt{13}\right), \frac{1}{4}\left(-\sqrt{3}+\sqrt{39}\right)\right\},$$

$$\left\{\frac{1}{4}\left(5+\sqrt{13}\right), \frac{1}{4}\left(-\sqrt{3}-\sqrt{39}\right)\right\}, \left\{\frac{1}{4}\left(5+\sqrt{13}\right), \frac{1}{4}\left(\sqrt{3}+\sqrt{39}\right)\right\},$$

$$\left\{\frac{1}{4}\left(-1-\sqrt{21}\right), \frac{1}{4}\left(-3\sqrt{3}-\sqrt{7}\right)\right\}, \left\{\frac{1}{4}\left(-1-\sqrt{21}\right), \frac{1}{4}\left(3\sqrt{3}+\sqrt{7}\right)\right\},$$

$$\left\{\frac{1}{4}\left(5-\sqrt{21}\right), \frac{1}{4}\left(3\sqrt{3}-\sqrt{7}\right)\right\}, \left\{\frac{1}{4}\left(5-\sqrt{21}\right), \frac{1}{4}\left(-3\sqrt{3}+\sqrt{7}\right)\right\},$$

$$\left\{\frac{1}{4}\left(-1+\sqrt{21}\right), \frac{1}{4}\left(3\sqrt{3}-\sqrt{7}\right)\right\}, \left\{\frac{1}{4}\left(-1+\sqrt{21}\right), \frac{1}{4}\left(-3\sqrt{3}+\sqrt{7}\right)\right\},$$

$$\left\{\frac{1}{4}\left(5+\sqrt{21}\right), \frac{1}{4}\left(-3\sqrt{3}-\sqrt{7}\right)\right\}, \left\{\frac{1}{4}\left(5+\sqrt{21}\right), \frac{1}{4}\left(3\sqrt{3}+\sqrt{7}\right)\right\},$$

$$\left\{\frac{1}{12}\left(-9-\sqrt{33}\right), \frac{1}{12}\left(-\sqrt{3}-3\sqrt{11}\right)\right\}, \left\{\frac{1}{12}\left(-9-\sqrt{33}\right), \frac{1}{12}\left(\sqrt{3}+3\sqrt{11}\right)\right\},$$

$$\left\{\frac{1}{4}\left(-1-\sqrt{33}\right), \frac{1}{4}\left(-\sqrt{3}-3\sqrt{11}\right)\right\}, \left\{\frac{1}{4}\left(-1-\sqrt{33}\right), \frac{1}{4}\left(\sqrt{3}+3\sqrt{11}\right)\right\},$$

$$\left\{\frac{1}{12}\left(3-\sqrt{33}\right), \frac{1}{12}\left(-5\sqrt{3}-3\sqrt{11}\right)\right\}, \left\{\frac{1}{12}\left(3-\sqrt{33}\right), \frac{\frac{5}{4}+\frac{\sqrt{33}}{4}}{\sqrt{3}}\right\},$$

$$\left\{\frac{1}{6}\left(3-\sqrt{33}\right), -\frac{1}{\sqrt{3}}\right\}, \left\{\frac{1}{6}\left(3-\sqrt{33}\right), \frac{1}{\sqrt{3}}\right\}, \left\{\frac{1}{4}\left(5-\sqrt{33}\right), \frac{1}{4}\left(\sqrt{3}-3\sqrt{11}\right)\right\},$$

$$\left\{\frac{1}{4}\left(5-\sqrt{33}\right), \frac{1}{4}\left(-\sqrt{3}+3\sqrt{11}\right)\right\}, \left\{\frac{1}{12}\left(9-\sqrt{33}\right), \frac{1}{12}\left(5\sqrt{3}-3\sqrt{11}\right)\right\},$$

$$\left\{\frac{1}{12}\left(9-\sqrt{33}\right), \frac{1}{12}\left(-5\sqrt{3}+3\sqrt{11}\right)\right\}, \left\{\frac{1}{12}\left(21-\sqrt{33}\right), \frac{1}{12}\left(\sqrt{3}-3\sqrt{11}\right)\right\},$$

$$\left\{\frac{1}{12}\left(21-\sqrt{33}\right), \frac{1}{12}\left(-\sqrt{3}+3\sqrt{11}\right)\right\}, \left\{\frac{1}{12}\left(-9+\sqrt{33}\right), \frac{1}{12}\left(\sqrt{3}-3\sqrt{11}\right)\right\},$$

$$\left\{\frac{1}{12}\left(-9+\sqrt{33}\right), \frac{1}{12}\left(-\sqrt{3}+3\sqrt{11}\right)\right\}, \left\{\frac{1}{4}\left(-1+\sqrt{33}\right), \frac{1}{4}\left(\sqrt{3}-3\sqrt{11}\right)\right\},$$

$$\left\{\frac{1}{4}\left(-1+\sqrt{33}\right), \frac{1}{4}\left(-\sqrt{3}+3\sqrt{11}\right)\right\}, \left\{\frac{1}{12}\left(3+\sqrt{33}\right), \frac{1}{4}\left(\frac{5}{\sqrt{3}}-\sqrt{11}\right)\right\},$$

$$\left\{\frac{1}{12}\left(3+\sqrt{33}\right), \frac{1}{12}\left(-5\sqrt{3}+3\sqrt{11}\right)\right\}, \left\{\frac{1}{6}\left(3+\sqrt{33}\right), -\frac{1}{\sqrt{3}}\right\}, \left\{\frac{1}{6}\left(3+\sqrt{33}\right), \frac{1}{\sqrt{3}}\right\},$$

$$\left\{\frac{1}{4}\left(5+\sqrt{33}\right), \frac{1}{4}\left(-\sqrt{3}-3\sqrt{11}\right)\right\}, \left\{\frac{1}{4}\left(5+\sqrt{33}\right), \frac{1}{4}\left(\sqrt{3}+3\sqrt{11}\right)\right\},$$
$$\left\{\frac{1}{12}\left(9+\sqrt{33}\right), \frac{1}{12}\left(-5\sqrt{3}-3\sqrt{11}\right)\right\}, \left\{\frac{1}{12}\left(9+\sqrt{33}\right), \frac{1}{12}\left(5\sqrt{3}+3\sqrt{11}\right)\right\},$$
$$\left\{\frac{1}{12}\left(21+\sqrt{33}\right), \frac{1}{12}\left(-\sqrt{3}-3\sqrt{11}\right)\right\}, \left\{\frac{1}{12}\left(21+\sqrt{33}\right), \frac{1}{12}\left(\sqrt{3}+3\sqrt{11}\right)\right\}\right\}$$

```
listagraf[ls0, ls1, ls2]
```

◀ | ▶

Costruzione del pentagono regolare.

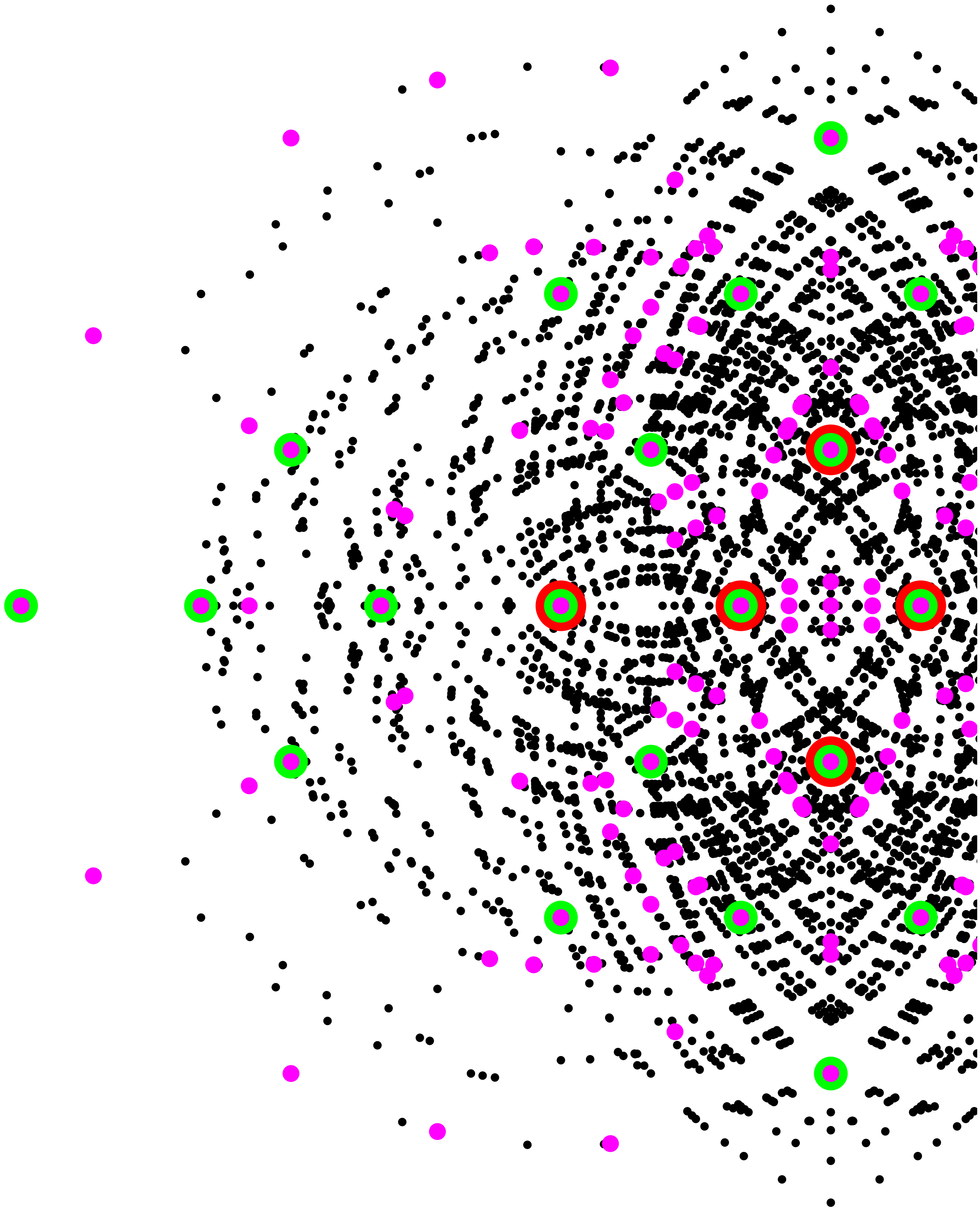
definitions

One (simple) example via constructible points

```
Manipulate[Block[{P1, P2, P3, P41, P42, P5, P6, P7},
  P1 = int[retta[{{0, 0}, {1, 0}}], circ[{{0, 0}, {1, 0}}], 1];
  P2 = int[circ[{{-1, 0}, {1, 0}}], circ[{{1, 0}, {-1, 0}}], 2];
  P3 = int[retta[{{0, 0}, P2}], circ[{{0, 0}, {1, 0}}], 2];
  {P41, P42} = int[circ[{{0, 0}, {1, 0}}], circ[{P3, {0, 0}}]];
  P5 = int[retta[{{0, 0}, P2}], retta[{P41, P42}], 1];
  P6 = int[retta[{{0, 0}, P2}], circ[{P5, {1, 0}}], 1];
  P7 = int[circ[{{0, 0}, {1, 0}}], circ[{{1, 0}, P6}], 2];
  Show[Take[lsgpent, k], PlotRange → {{-2.2, 2.2}, {-2.2, 2.2}},
   Frame → True, ImageSize → 600]], {k, 1, Length[lsgpent], 1}]
```

```
moviepent = Table[Block[{P1, P2, P3, P41, P42, P5, P6, P7},
    P1 = int[retta[{{0, 0}, {1, 0}}], circ[{{0, 0}, {1, 0}}], 1];
    P2 = int[circ[{{-1, 0}, {1, 0}}], circ[{{1, 0}, {-1, 0}}], 2];
    P3 = int[retta[{{0, 0}, P2}], circ[{{0, 0}, {1, 0}}], 2];
    {P41, P42} = int[circ[{{0, 0}, {1, 0}}], circ[{P3, {0, 0}}]];
    P5 = int[retta[{{0, 0}, P2}], retta[{P41, P42}], 1];
    P6 = int[retta[{{0, 0}, P2}], circ[{P5, {1, 0}}], 1];
    P7 = int[circ[{{0, 0}, {1, 0}}], circ[{{1, 0}, P6}], 2];
    Show[Take[lsgpent, k], PlotRange → {{-2.2, 2.2}, {-2.2, 2.2}},
     Frame → True, ImageSize → 600]], {k, 1, Length[lsgpent], 1}];
Export["moviepent.gif", moviepent, "DisplayDurations" → 2]
```

```
moviepent.gif
```



Rilievo, nuvole di punti e modelli in Architettura.

Circo Massimo

Rilievo: laser scanner

input: la nuvola di punti

```
nuvola = ReadList["/Users/corradofalcolini/Desktop/Nepal 2015/lezioni/f3.xyz",
   Number, RecordLists → True];
```

```
Length[nuvola]
```

```
438 427
```

```
TableForm[Take[nuvola, 3]]
```

11 617.5	40 017.4	-5.91297	1.56685	3.84102	2.62418
11 617.5	40 017.4	-5.91641	1.20352	0.770545	0.991678
11 617.5	40 017.4	-5.91402	0.	3.05279	0.

```
nuvola1 = Map[Drop[#, -3] &, nuvola];
```

```
mean = Map[Mean, Transpose[nuvola1]]
```

```
{11 617.7, 40 016.2, -5.55419}
```

```
nuvola1tr = Map[# - mean &, nuvola1];
```

```
nuvola1tr[[1]]
```

```
{-0.2689, 1.17686, -0.358772}
```

Object: portion of a column

```
Graphics3D[{PointSize[.0005], Point[nuvola1tr]}, ImageSize → 1000]
```

problema: sezione verticale della colonna

```
Graphics3D[{PointSize[.0005], Point[Select[nuvola1tr, 0 < #[[2]] < 0.1 &]]}]
```

orientamento: cilindro ottimale

funzione da minimizzare: distanza di tutti i punti della nuvola dal cilindro

trovare il minimo: l’asse del cilindro sarà anche l’asse della colonna

```
Timing[{m2, rt2} =
  FindMinimum[func, {a, 1}, {b, 0}, {c, 0.4}, {d, 0}, {r, 0.5}, MaxIterations → 500]]
```

FindMinimum::lstol :
The line search decreased the step size to within the tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the function. You may need more than MachinePrecision digits of working precision to meet these tolerances. ≫

```
{0.046891,
 {2600.88, {a → 4.0738, b → 0.102295, c → 0.218072, d → -0.147016, r → 0.465547}}}
```

```
Show[{Graphics3D[{PointSize[.0005], Point[nuvola1tr]}],
  Graphics3D[{Opacity[.5], Cylinder[{{x, a * x + b, c * x + d}
       /. x → -1, {x, a * x + b, c * x + d} /. x → 1}, r]} /. rt2]},
 PlotRange → {{-1, 1}, {-1, 1}, {-1, 1}}, ImageSize → 800]
```

```
la colonna può ora essere orientata verticalmente
```

```
colonna essere la ora orientata può verticalmente
```

```
Show[{Graphics3D[{PointSize[.0005],
    Rotate[Point[nuvola1tr], {{1, a * 1 + b, c * 1 + d} /. rt2, {0, 0, 1}}]}],
  Graphics3D[{Opacity[.5], Rotate[Cylinder[{{x, a * x + b, c * x + d}
        /. x → -1, {x, a * x + b, c * x + d} /. x → 1}, r],
      {{1, a * 1 + b, c * 1 + d} /. rt2, {0, 0, 1}}]} /. rt2]},
 PlotRange → {{-1, 1}, {-1, 1}, {-1, 1}}, ImageSize → 800, Axes → True]
```

le sezioni

```
Show[{Graphics3D[{PointSize[.0005],
    Rotate[Point[nuvola1tr], {{1, a*1+b, c*1+d} /. rt2, {0, 0, 1}}]}],
  Graphics3D[{Opacity[.5], Rotate[Cylinder[{{x, a*x+b, c*x+d}
        /. x → -1, {x, a*x+b, c*x+d} /. x → 1}, r],
      {{1, a*1+b, c*1+d} /. rt2, {0, 0, 1}}]} /. rt2]},
 PlotRange → {{-1, 1}, {-1, 1}, {-1, 1}}, ImageSize → 800,
 Axes → True, ViewPoint → Top]
```

il modello matematico: proiezione

sezioni

```
nuvola1trrt = Map[mrt.# &, nuvola1tr];
```

```
Graphics[{PointSize[.0005],
  Point[Map[Drop[#, -1] &, Select[nuvola1trrt, 0 < #[[3]] < .03 &]]]}]
```

```
Manipulate[Graphics[{PointSize[.0005], Point[Map[Drop[#, -1] &,
     Select[nuvola1trrt, z < #[[3]] < z + .03 &]]]}], {z, -.2, .8, .01}]
```

```
nuvola1sez = Select[nuvola1trrt, -.2 < #[[3]] < .8 &];
```

```
nuvola1sezrt = Map[mrt.# &, nuvola1sez];
```

```
Length[nuvola1sez]
```

190 512

modello matematico

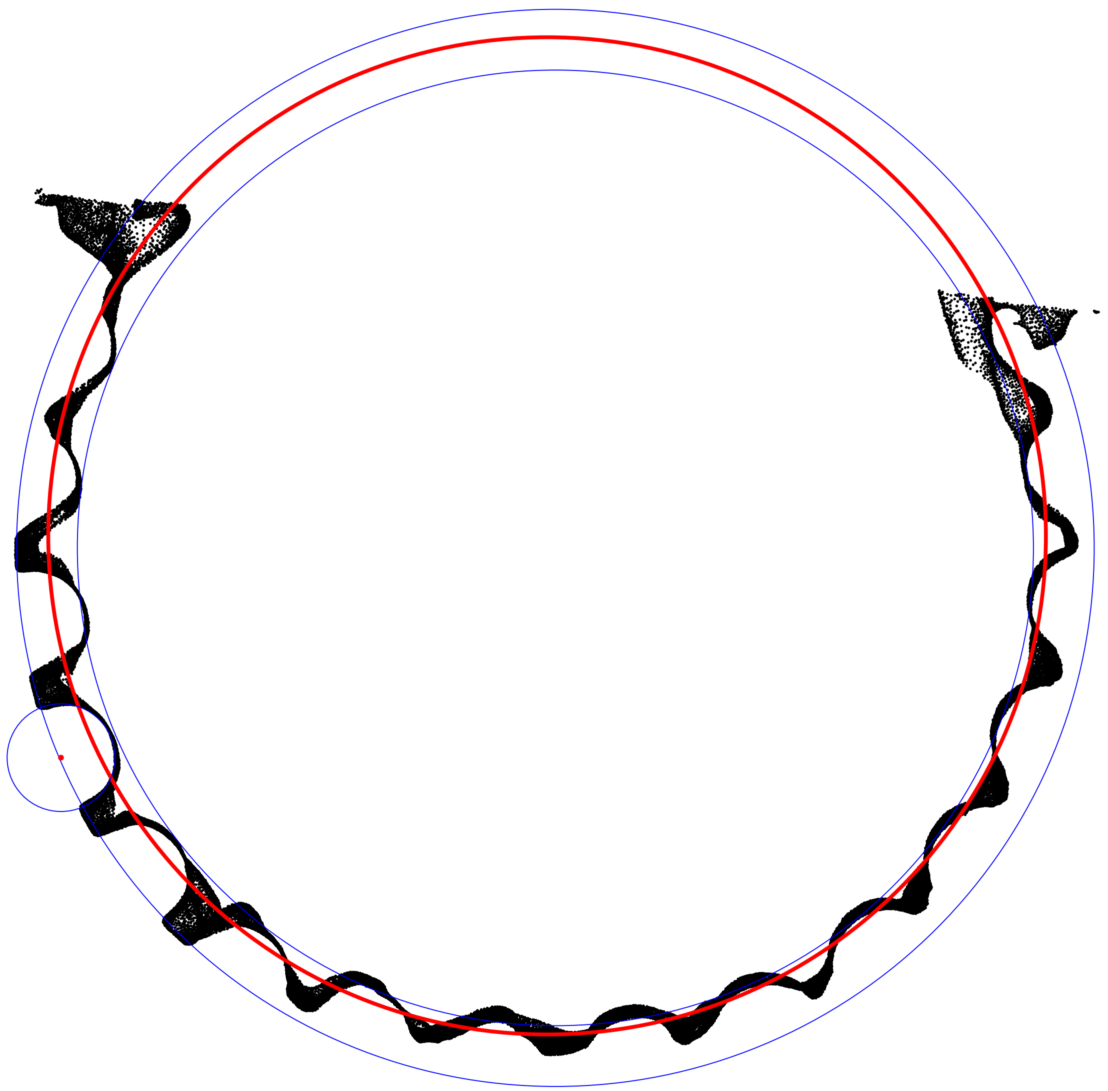
```
centro = Drop[mrtsz.{x, a * x + b, c * x + d}, {3}] /. x → 0 /. rt2sz
```

```
{0.0172026, 0.202735}
```

```
Manipulate[
 Show[{Graphics[{PointSize[.0005], Point[Map[Drop[mrtsz.#, {3}] &, nuvola1sez2]]}],
   Graphics[{Red, Thick, Circle[Drop[mrtsz.{x, a * x + b, c * x + d}, {3}]
         /. x → 0, r]} /. rt2sz], Graphics[
    {Blue, Point[{x0, y0}], Circle[{x0, y0}, r0]}]}, ImageSize → 600],
 {{x0, centro[[1]]}, -1, 1}, {{y0, centro[[2]]}, -1, 1},
 {{r0, r /. rt2sz}, 0.0001, 1}]
```

il modello matematico della colonna

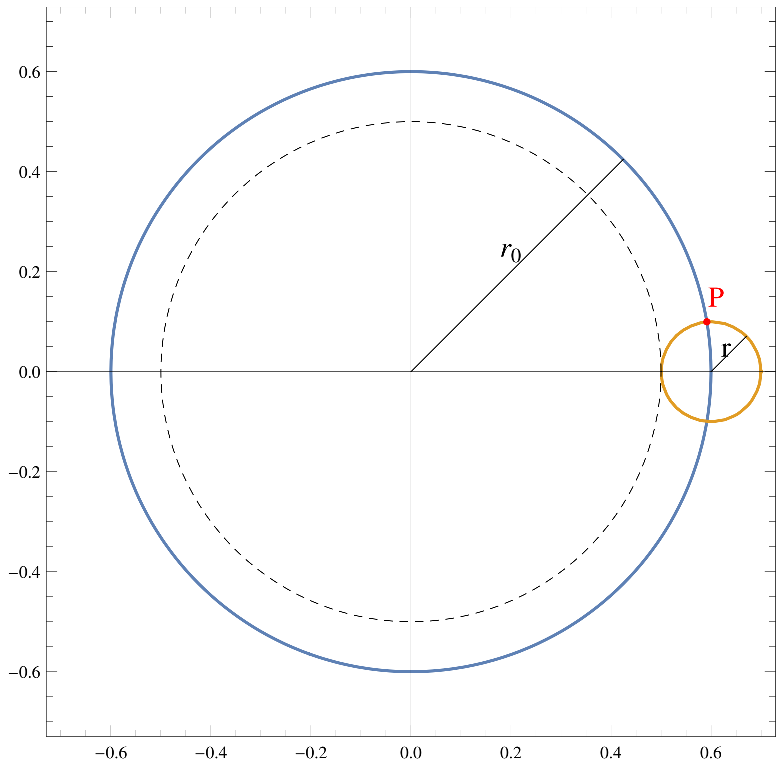
```
Show[{Graphics[{PointSize[.0005], Point[Map[Drop[mrtsz.#, {3}] &, nuvola1sez2]]}],
  Graphics[{Red, Thick, Circle[Drop[mrtsz.{x, a * x + b, c * x + d}, {3}]
      /. x → 0, r]} /. rt2sz], Graphics[{Blue, Circle[{.026, .19}, .5057]}],
  Graphics[{Blue, Circle[{.026, .19}, .57]}], Graphics[{Red,
    Point[{-.497, -.032}], Blue, Circle[{-.497, -.032}, .057]}]}, ImageSize → 600]
```



modello matematico

il modello matematico della colonna

```
With[{r0 = 6 / 10, r = 1 / 10}, ContourPlot[
  {x^2 + y^2 == r0^2, (x - r0) ^2 + y^2 == r^2}, {x, - .7, .7}, {y, - .7, .7}, Epilog →
   {Line[{{0, 0}, r0 {Cos[Pi / 4], Sin[Pi / 4]}}], Text[Style["r₀", 15], {0.2, 0.25}],
    Line[{{r0, 0}, {r0, 0} + r {Cos[Pi / 4], Sin[Pi / 4]}}], Text[Style["r", 15],
      {0.63, 0.05}], Dashed, Circle[{0, 0}, r0 - r], Red, PointSize[.01],
    Point[{x, y} /. Solve[{x^2 + y^2 == r0^2, (x - r0) ^2 + y^2 == r^2}, {x, y}][[2]]],
    Text[Style["P", 15], {0.61, 0.15}]},
  PlotPoints → 30, Axes → True, ImageSize → 400]]
```




```
(* coordinate del punto P (in figura) in funzione di r0 ed r *)

Solve[{x^2 + y^2 == r0^2, (x - r0) ^2 + y^2 == r^2}, {x, y}]
```

$$\left\{\left\{x \to \frac{-r^2 + 2\,r0^2}{2\,r0},\ y \to -\frac{\sqrt{-r^4 + 4\,r^2\,r0^2}}{2\,r0}\right\},\ \left\{x \to \frac{-r^2 + 2\,r0^2}{2\,r0},\ y \to \frac{\sqrt{-r^4 + 4\,r^2\,r0^2}}{2\,r0}\right\}\right\}$$

```
Manipulate[Module[{x0 = 0, y0 = 0, r0 = 1.188 / 2, c = 5.4, r, arc, tc},
  r = (c / n) r0;
  tc = ArcCos[(r0 - r^2 / (2 r0)) / r0];
  arc = 1 / 2 ArcTan[r0 - r^2 / (2 r0), 1/(2 r0) Sqrt[4 r^2 r0^2 - r^4]];
  Graphics[{
    Table[Circle[{x0, y0} + r0 {Cos[k * 2 Pi / n], Sin[k * 2 Pi / n]},
      r, {2 k * Pi / n + Pi / 2 + arc, 2 k * Pi / n + 3 Pi / 2 - arc}], {k, 2, n, 2}],
    Table[Circle[{x0, y0}, r0, {tc + k * 2 Pi / n, -tc + (k + 2) 2 Pi / n}], {k, 2, n, 2}]},
   ImageSize → 500, Axes → True, PlotRange → {{-.7, .7}, {-.7, .7}}]], {{n,
   48}, 6, 50, 2}]

sez = Import["/Users/falco/Desktop/strasburgo 2013/colonna_sezione_tipo.jpg"]

Import::nffil : File not found during Import. ≫

$Failed

ImageDimensions[sez]

ImageDimensions::imginv : Expecting an image or graphics instead of $Failed. ≫

ImageDimensions[$Failed]

Manipulate[GraphicsRow[{sez, Module[{n = 48, r, arc, tc},
     r = (c / n) r0;
     tc = ArcCos[(r0 - r^2 / (2 r0)) / r0];
     arc = 1 / 2 ArcTan[r0 - r^2 / (2 r0), 1/(2 r0) Sqrt[4 r^2 r0^2 - r^4]];
     Graphics[{
       Table[Circle[{x0, y0} + r0 {Cos[k * 2 Pi / n], Sin[k * 2 Pi / n]}, r,
         {2 k * Pi / n + Pi / 2 + arc, 2 k * Pi / n + 3 Pi / 2 - arc}], {k, 2, n, 2}],
       Table[Circle[{x0, y0}, r0, {tc + k * 2 Pi / n, -tc + (k + 2) 2 Pi / n}], {k, 2, n, 2}],
       Red, Dashed, Circle[{x0, y0}, r0], Circle[{x0, y0}, r0 - r]},
      Axes → True, PlotRange → {{-1, 1}, {-1485 / 1432, 1485 / 1432}}]]},
  Spacings → -360], {{x0, -.018}, -1, 1}, {{y0, -.055},
  -1, 1},
 {{r0, .982}, .1, 2}, {{c, 5.4}, 1, 10}]

(*  per c=
 5.4 ed r0=1.188 si ha r = (c/n)r0 = (5.4/48)1.188 = 0.13365 come in figura *)
```

```
col[x0_, y0_, r0_, n_] := Module[{c = 5.4, tc, arc, r0ent, style = Opacity[1]},
  tc = ArcCos[1 - c^2/(2 n^2)];
  arc = 1 / 2 ArcTan[1 - c^2/(2 n^2), 1/(2 n^2) Sqrt[4 c^2 n^2 - c^4]];
  r0ent = (650 r0 - 9 v) / 600;
  {Table[ParametricPlot3D[{x0 + r0 (Cos[k * 2 Pi / n] + (c / n) Cos[t]),
      y0 + r0 (Sin[k * 2 Pi / n] + (c / n) Sin[t]), v},
     {t, 2 k * Pi / n + Pi / 2 + arc, 2 k * Pi / n + 3 Pi / 2 - arc}, {v, 0, (8 + 1 / 3) 2 r0 / 3},
     Mesh → None, Lighting → "Neutral", PlotStyle → style], {k, 2, n, 2}],
   Table[ParametricPlot3D[{x0 + r0ent (Cos[k * 2 Pi / n] + (c / n) Cos[t]),
      y0 + r0ent (Sin[k * 2 Pi / n] + (c / n) Sin[t]), v}, {t, 2 k * Pi / n + Pi / 2 + arc,
      2 k * Pi / n + 3 Pi / 2 - arc}, {v, (8 + 1 / 3) 2 r0 / 3, (8 + 1 / 3) 2 r0},
     Mesh → None, Lighting → "Neutral", PlotStyle → style], {k, 2, n, 2}],
   Table[ParametricPlot3D[{x0 + r0 * Cos[t], y0 + r0 * Sin[t], v},
     {t, tc + k * 2 Pi / n, -tc + (k + 2) 2 Pi / n}, {v, 0, (8 + 1 / 3) 2 r0 / 3},
     Mesh → None, Lighting → "Neutral", PlotStyle → style], {k, 2, n, 2}],
   Table[ParametricPlot3D[{x0 + r0ent * Cos[t], y0 + r0ent * Sin[t], v},
     {t, tc + k * 2 Pi / n, -tc + (k + 2) 2 Pi / n}, {v, (8 + 1 / 3) 2 r0 / 3, (8 + 1 / 3) 2 r0},
     Mesh → None, Lighting → "Neutral", PlotStyle → style], {k, 2, n, 2}]}]

Show[{col[1, 1, 1.188 / 2, 48], col[3, 1, 1.188 / 2, 48], col[.5, 3, 1.188 / 2, 48],
  col[2.5, 3, 1.188 / 2, 48]}, PlotRange → {{0, 4}, {0, 4}, {0, 10}}, ImageSize → 500]

Show[col[0, 0, 1.188 / 2, 48], PlotRange → {{-1, 1}, {-1, 1}, {0, 10}}, ImageSize → 300]

Module[{n = 48, c = 1.6}, RegionPlot3D[Apply[And,
   Prepend[Table[Norm[{x, y} - rt[k * 2 Pi / n].{Cos[Pi / n], 0}]^2 > (c * Sin[Pi / n])^2,
     {k, 2, n, 2}], x^2 + y^2 < 1]], {x, -1, 1}, {y, -1, 1},
  {z, 0, 10}, Mesh → None, PlotPoints → 60, BoxRatios → Automatic]]
```

```
Manipulate[Module[{n = 48, r, arc, tc, sp = .004},
   r = (c / n) r0;
   tc = ArcCos[(r0 - r^2 / (2 r0)) / r0];
   arc = 1 / 2 ArcTan[r0 - r^2 / (2 r0), 1/(2 r0) Sqrt[4 r^2 r0^2 - r^4]]; GraphicsRow[
    {Graphics[{PointSize[.0005], Point[Map[Drop[mrtsz.#, {3}] &, nuvola1sez2]]}],
     Graphics[{Red,
       Table[Rotate[Circle[{x0, y0} + r0 {Cos[k * 2 Pi / n], Sin[k * 2 Pi / n]}, r,
           {2 k * Pi / n + Pi / 2 + arc, 2 k * Pi / n + 3 Pi / 2 - arc}], alfa, {x0, y0}],
         {k, 2, n, 2}], Table[Rotate[Circle[{x0, y0}, r0,
           {tc + k * 2 Pi / n, -tc + (k + 2) 2 Pi / n}], alfa, {x0, y0}], {k, 2, n, 2}],
       Dashed, Circle[{x0, y0}, r0], Circle[{x0, y0}, r0 - r]}, Axes -> True,
      PlotRange -> {{-1, 1}, {-600 / 562, 600 / 562}}]}, Spacings -> -360]],
 {{x0, -.036}, -1, 1}, {{y0, .158}, -1, 1},
 {{r0, .994},
  .1,
  2}, {{c, 5.24},
  1,
  10}, {{alfa, .418},
  -1, 1},
 ControlPlacement -> Right]
```

Bibliografia

Falcolini Corrado, Tedeschini-Lalli Laura *Henon Map: S*
b--->1. International Journal of Bifurcation and Chaos in
23, (2013) ISSN: 0218-1274.

Canciani Marco, Falcolini Corrado, Mauro Saccone, Spa
From Point Clouds to Architectural Models: algorithms f
Int. Arch. Photogramm. Remote Sens. Spatial Inf. Sci., X
doi:10.5194/isprsarchives-XL-5-W1-27-2013

Canciani Marco, Falcolini Corrado, Spadafora Giovanna
From complexity of architecture to geometrical rule. The
alle Quattro Fontane in Rome. In: X Forum internazional
Fabbrica della conoscenza vol. 16, NAPOLI:La Scuola d

Falcolini Corrado, Michelangelo Vallicelli (2011). *Mode*
Fontane. Aplimat - Journal Of Applied Mathematics, vol.

Falcolini Corrado (2011). *Famiglie di orbite periodiche n*
dissipativo al conservativo. In: "Mathematica Italia User
Convegno", ISBN: 978-88-96810-02-6.

Falcolini Corrado (2010). *La volta di S.Carlino alle Quat*
"Mathematica Italia User Group Meeting 2010 - Atti del

Falcolini Corrado, Tedeschini-Lalli Laura *Henon Map: S*
b--->1. International Journal of Bifurcation and Chaos in
23, (2013) ISSN: 0218-1274.

Canciani Marco, Falcolini Corrado, Mauro Saccone, Spa
From Point Clouds to Architectural Models: algorithms f
Int. Arch. Photogramm. Remote Sens. Spatial Inf. Sci., X
doi:10.5194/isprsarchives-XL-5-W1-27-2013

Canciani Marco, Falcolini Corrado, Spadafora Giovanna
From complexity of architecture to geometrical rule. The
alle Quattro Fontane in Rome. In: X Forum internazional
Fabbrica della conoscenza vol. 16, NAPOLI:La Scuola d

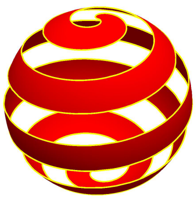
Falcolini Corrado, Michelangelo Vallicelli (2011). *Mode*
Fontane. Aplimat - Journal Of Applied Mathematics, vol.

Falcolini Corrado (2011). *Famiglie di orbite periodiche n*
dissipativo al conservativo. In: "Mathematica Italia User
Convegno", ISBN: 978-88-96810-02-6.

Falcolini Corrado (2010). *La volta di S.Carlino alle Quat*
"Mathematica Italia User Group Meeting 2010 - Atti del

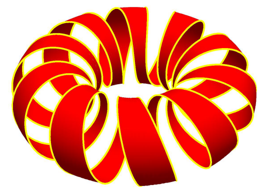
*** Mathematica at Work ***

$\{x, y, z\} = \{\cos(u)\cos(v), \sin(u)\cos(v), \sin(v)\}$
$\left\{\frac{u}{10} < v < \frac{u+\pi}{10}, -6\,\pi < u < 5\,\pi\right\}$



$\{x, y, z\} = \{\cos(u)\,(\sin(v)+1), \sin(u)\,(\sin(v)+1), v\}$
$\left\{\frac{u}{10} < v < \frac{u+\pi}{10}, -6\,\pi < u < 15\,\pi\right\}$

$\{x, y, z\} = \{(\sin(u)+2)\cos(v), (\sin(u)+2)\sin(v), \cos(u)\}$
$\left\{\frac{u}{10} < v < \frac{u+\pi}{10}, -11\,\pi < u < 10\,\pi\right\}$



Grazie !