

Master di II livello in Scienza e Tecnologia Spaziale
METODI NUMERICI PER L' ASTRONOMIA

Il software *Mathematica* per il calcolo simbolico, numerico e grafico.

Tor Vergata, 8 febbraio 2015

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(* Questo file si può far girare lanciando
Evaluate Notebook dalla barra Evaluation *)

Problema 3: visualizzazione e calcolo di orbite per la Standard Map.

La Standard Map

Un esempio di sistema dinamico discreto nel piano è la cosiddetta
Mappa Standard definita sul toro $\mathbf{T}^2 \equiv [0, 1] \times [0, 1]$ dalla funzione

$$\begin{cases} x_{n+1} = x_n + y_n + \frac{\epsilon}{2\pi} \sin(2\pi x_n) \pmod{1} \\ y_{n+1} = y_n + \frac{\epsilon}{2\pi} \sin(2\pi x_n) \pmod{1} \end{cases}$$

L'evoluzione di un punto iniziale (x_0, y_0) è data dalle successive
applicazioni di tale funzione e l'insieme dei punti così ottenuti è detto **orbita** di (x_0, y_0) .

Per $\epsilon = 0$ nei punti dell'orbita non cambia il valore
iniziale di y_0 mentre l'ascissa aumenta ad ogni passo della stessa quantità.

Per $\epsilon > 0$ nei punti dell'orbita cambiano sia l'ascissa che l'ordinata ad ogni passo.

Facciamo degli esempi numerici

$$S M : \begin{cases} x_{n+1} = x_n + y_n + \frac{\epsilon}{2\pi} \sin(2\pi x_n) \pmod{1} \\ y_{n+1} = y_n + \frac{\epsilon}{2\pi} \sin(2\pi x_n) \pmod{1} \end{cases} \quad \text{⚡}$$

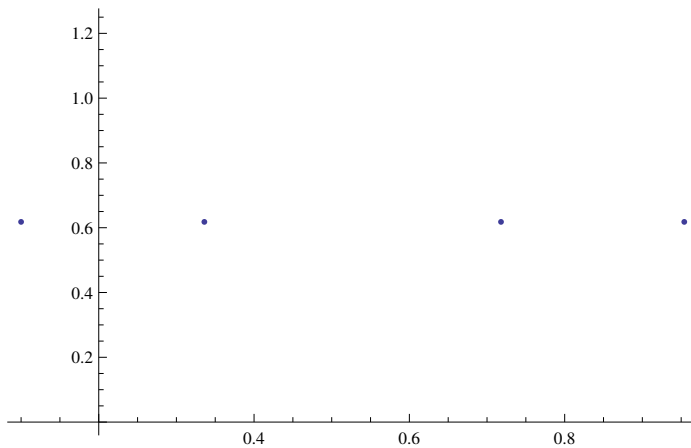
```
f[eps_][{x_, y_}] :=
  {Mod[x + y + eps / (2 Pi) * Sin[2 Pi * x], 1], Mod[y + eps / (2 Pi) * Sin[2 Pi * x], 1]}

orbita[x0_, y0_, eps_, nmax_] := Block[{k, lista = {}, punto = {x0, y0}},
  For[k = 1, k ≤ nmax, k++,
    AppendTo[lista, punto];
    punto = f[eps][punto];
  ]; Return[lista];
]

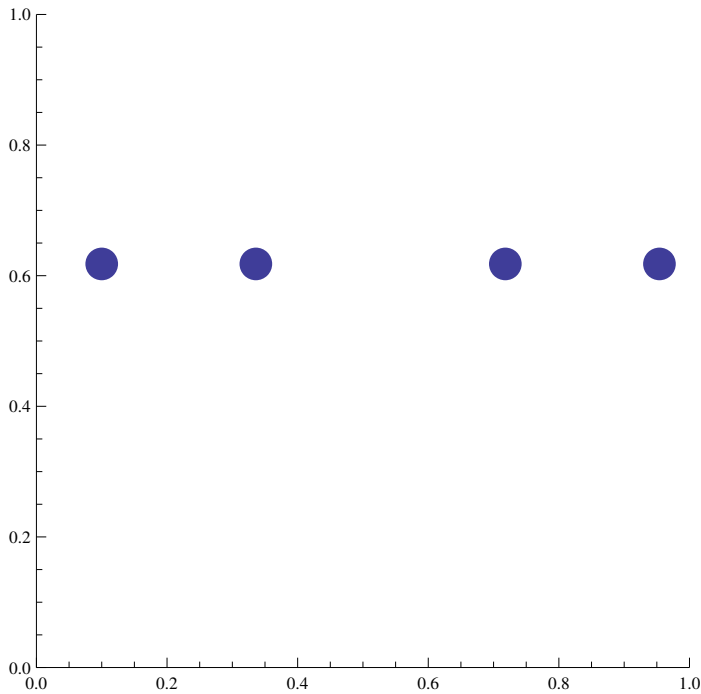
orbita[0.1, 0.618, 0, 4]
{{0.1, 0.618}, {0.718, 0.618}, {0.336, 0.618}, {0.954, 0.618}}

orbita[0.1, 0.618, 1, 4]
{{0.1, 0.618}, {0.811549, 0.711549}, {0.375697, 0.564148}, {0.0518901, 0.676194}}

ListPlot[orbita[0.1, 0.618, 0, 4]]
```



```
ListPlot[orbita[0.1, 0.618, 0, 4], PlotRange → {{0, 1}, {0, 1}},
  AspectRatio → Automatic, PlotStyle → PointSize[0.05]]
```



Options[ListPlot]

```
{AlignmentPoint → Center, AspectRatio →  $\frac{1}{\text{GoldenRatio}}$ , Axes → True,
  AxesLabel → None, AxesOrigin → Automatic, AxesStyle → {}, Background → None,
  BaselinePosition → Automatic, BaseStyle → {}, ClippingStyle → None,
  ColorFunction → Automatic, ColorFunctionScaling → True, ColorOutput → Automatic,
  ContentSelectable → Automatic, CoordinatesToolOptions → Automatic,
  DataRange → Automatic, DisplayFunction → $DisplayFunction, Epilog → {},
  Filling → None, FillingStyle → Automatic, FormatType → TraditionalForm,
  Frame → False, FrameLabel → None, FrameStyle → {}, FrameTicks → Automatic,
  FrameTicksStyle → {}, GridLines → None, GridLinesStyle → {},
  ImageMargins → 0., ImagePadding → All, ImageSize → Automatic,
  ImageSizeRaw → Automatic, InterpolationOrder → None, Joined → False,
  LabelStyle → {}, MaxPlotPoints → ∞, Mesh → None, MeshFunctions → {#1 &},
  MeshShading → None, MeshStyle → Automatic, Method → Automatic,
  PerformanceGoal → $PerformanceGoal, PlotLabel → None, PlotMarkers → None,
  PlotRange → Automatic, PlotRangeClipping → True, PlotRangePadding → Automatic,
  PlotRegion → Automatic, PlotStyle → Automatic, PreserveImageOptions → Automatic,
  Prolog → {}, RotateLabel → True, Ticks → Automatic, TicksStyle → {}}
```

```

disorbrcorta[{x0_, y0_}, eps_, nmax_] :=
Block[{lista}, lista = orbita[x0, y0, eps, nmax];
Return[ListPlot[lista, PlotRange -> {{0, 1}, {0, 1}},
  PlotStyle -> PointSize[0.02], AxesLabel -> {x, y}, AspectRatio -> Automatic,
  ImageSize -> 500, ColorFunction -> Function[{x, y}, Hue[x]],
  Epilog -> {{PointSize[0.02], RGBColor[1, 0, 0], Point[{x0, y0}]},
    {PointSize[0.02], RGBColor[0, 1, 0], Point[lista[[-1]]]}}]];
]

```

```
Manipulate[disorbcorta[{0.1, .618}, eps, n], {eps, 0, 1.5, .01, Appearance -> "Open"},  
{n, 1, 100, 1, Appearance -> "Open"}, ControlPlacement -> Left]
```

```
Manipulate[disorbcora[{x[[1]], x[[2]]}, eps, n],  
  {eps, 0, 1.5, .01, Appearance -> "Open"}, {n, 1, 100, 1, Appearance -> "Open"},  
  {x, {0, 0}, {1, 1}, Appearance -> "Open"}, ControlPlacement -> Left]
```

```
Manipulate[disorbcorta[{x0, y0}, eps, n], {eps, 0, 1.5, .01, Appearance -> "Open"},  
  {n, 1, 100, 1, Appearance -> "Open"}, {x0, 0, 1, Appearance -> "Open"},  
  {y0, 0, 1, Appearance -> "Open"}, ControlPlacement -> Left]
```

$$S M : \begin{cases} x_{n+1} = x_n + y_n + \frac{\epsilon}{2\pi} \sin(2\pi x_n) \pmod{1} \\ y_{n+1} = y_n + \frac{\epsilon}{2\pi} \sin(2\pi x_n) \pmod{1} \end{cases}$$



orbita[0.1, 0.618, 1, 200]

```
{0.1, 0.618}, {0.811549, 0.711549}, {0.375697, 0.564148}, {0.0518901, 0.676194},
{0.779059, 0.727169}, {0.349719, 0.57066}, {0.0493028, 0.699584},
{0.797405, 0.748102}, {0.393359, 0.595955}, {0.0881523, 0.694793},
{0.866659, 0.778507}, {0.526885, 0.660226}, {0.160354, 0.633469},
{0.928391, 0.768037}, {0.627211, 0.69882}, {0.211939, 0.584728},
{0.951292, 0.739353}, {0.642695, 0.691402}, {0.209766, 0.567071},
{0.930934, 0.721168}, {0.585184, 0.654249}, {0.158259, 0.573075},
{0.864771, 0.706513}, {0.451749, 0.586978}, {0.0862422, 0.634493},
{0.802818, 0.716576}, {0.368923, 0.566105}, {0.051782, 0.682858},
{0.785514, 0.733732}, {0.364036, 0.578523}, {0.0625778, 0.698542},
{0.822097, 0.759519}, {0.438515, 0.616417}, {0.114899, 0.676385},
{0.896459, 0.78156}, {0.581629, 0.68517}, {0.188702, 0.607073},
{0.943271, 0.754569}, {0.642304, 0.699033}, {0.21725, 0.574946},
{0.947994, 0.730744}, {0.627652, 0.679658}, {0.192911, 0.565259},
{0.907196, 0.714284}, {0.533847, 0.626651}, {0.126905, 0.593059},
{0.833843, 0.706937}, {0.403203, 0.569361}, {0.0635024, 0.660299},
{0.785633, 0.72213}, {0.35258, 0.566947}, {0.046753, 0.694173}, {0.78701, 0.740257},
{0.372395, 0.585385}, {0.0721468, 0.699752}, {0.8416, 0.769453},
{0.477538, 0.635938}, {0.135863, 0.658326}, {0.914142, 0.778279},
{0.610668, 0.696525}, {0.20523, 0.594563}, {0.952692, 0.747462},
{0.653541, 0.700848}, {0.223581, 0.57004}, {0.950588, 0.727007},
{0.628973, 0.678385}, {0.192045, 0.563072}, {0.903835, 0.711791},
{0.525207, 0.621371}, {0.121477, 0.59627}, {0.827767, 0.706291},
{0.393528, 0.56576}, {0.0579943, 0.664467}, {0.77918, 0.721186},
{0.343879, 0.564699}, {0.0408385, 0.696959}, {0.77819, 0.737351},
{0.358876, 0.580686}, {0.0629067, 0.704031}, {0.828219, 0.765312},
{0.453213, 0.624994}, {0.124323, 0.67111}, {0.907494, 0.78317},
{0.603279, 0.695785}, {0.202882, 0.599604}, {0.954717, 0.751835},
{0.661878, 0.707161}, {0.233663, 0.571785}, {0.963765, 0.730102},
{0.657944, 0.694179}, {0.218857, 0.560913}, {0.935888, 0.717031},
{0.590526, 0.654638}, {0.159441, 0.568915}, {0.862435, 0.702994},
{0.444365, 0.581929}, {0.0808033, 0.636439}, {0.794618, 0.713815},
{0.355492, 0.560873}, {0.0418203, 0.686329}, {0.76949, 0.727669},
{0.339196, 0.569706}, {0.04371, 0.704514}, {0.791387, 0.747677},
{0.38526, 0.593873}, {0.0841887, 0.698929}, {0.863435, 0.779246},
{0.522267, 0.658833}, {0.158905, 0.636638}, {0.929332, 0.770427},
{0.631391, 0.702059}, {0.216483, 0.585092}, {0.957214, 0.740731},
{0.655672, 0.698458}, {0.22212, 0.566448}, {0.945286, 0.723167},
{0.614811, 0.669524}, {0.179226, 0.564415}, {0.887318, 0.708092},
{0.491909, 0.604591}, {0.104587, 0.612678}, {0.814487, 0.709899},
{0.378117, 0.563631}, {0.052062, 0.673945}, {0.777145, 0.725083},
{0.345382, 0.568237}, {0.0450385, 0.699656}, {0.789134, 0.744096},
{0.378862, 0.589728}, {0.0783659, 0.699504}, {0.853107, 0.774741},
```

```

{0.50094, 0.647833}, {0.147833, 0.646893}, {0.922199, 0.774366},
{0.621826, 0.699627}, {0.21118, 0.589354}, {0.954978, 0.743798},
{0.654353, 0.699374}, {0.222458, 0.568105}, {0.947341, 0.724883},
{0.620521, 0.67318}, {0.184373, 0.563852}, {0.894039, 0.709666},
{0.505401, 0.611362}, {0.111363, 0.605962}, {0.81982, 0.708457},
{0.384194, 0.564373}, {0.0544216, 0.670228}, {0.778017, 0.723595},
{0.344917, 0.5669}, {0.0434972, 0.698581}, {0.785036, 0.741538},
{0.37126, 0.586224}, {0.0726368, 0.701377}, {0.844155, 0.771519},
{0.483567, 0.639412}, {0.139382, 0.655815}, {0.917434, 0.778052},
{0.616575, 0.69914}, {0.209288, 0.592713}, {0.955977, 0.746689},
{0.659203, 0.703226}, {0.228478, 0.569276}, {0.955456, 0.726978},
{0.638469, 0.683013}, {0.199833, 0.561363}, {0.91251, 0.712677},
{0.542036, 0.629527}, {0.130014, 0.587978}, {0.83402, 0.704006},
{0.400538, 0.566518}, {0.0601696, 0.659631}, {0.778547, 0.718378},
{0.340324, 0.561776}, {0.0363051, 0.695982}, {0.768278, 0.731973},
{0.342144, 0.573866}, {0.0492278, 0.707084}, {0.804759, 0.755531},
{0.410462, 0.605703}, {0.101054, 0.690592}, {0.886047, 0.784992},
{0.566576, 0.680529}, {0.182453, 0.615878}, {0.943366, 0.760913},
{0.648834, 0.705467}, {0.226231, 0.577397}, {0.961011, 0.73478},
{0.657191, 0.69618}, {0.220519, 0.563327}, {0.940278, 0.719759},
{0.601707, 0.661429}, {0.168211, 0.566505}, {0.873314, 0.705103},
{0.464691, 0.591377}, {0.0910883, 0.626397}, {0.803682, 0.712593},
{0.366088, 0.562406}, {0.0471559, 0.681068}, {0.774693, 0.727537}

```

```

disorblunga[{x0_, y0_}, eps_, nmax_] :=
Block[{lista}, lista = orbita[x0, y0, eps, nmax];
Return[ListPlot[lista, PlotRange -> {{0, 1}, {0, 1}}, PlotStyle -> PointSize[0.001],
  AxesLabel -> {x, y}, AspectRatio -> Automatic, ImageSize -> 500,
  Epilog -> {{PointSize[0.02], RGBColor[1, 0, 0], Point[{x0, y0]}},
    {PointSize[0.02], RGBColor[0, 1, 0], Point[lista[[-1]]]}}];
]

```

```

disorblunga[{x0_, y0_}, eps_, nmax_] :=
Block[{lista}, lista = orbita[x0, y0, eps, nmax];
Return[ListPlot[lista, PlotRange -> {{0, 1}, {0, 1}}, PlotStyle -> PointSize[0.001],
  AxesLabel -> {x, y}, AspectRatio -> Automatic, ImageSize -> 500,
  Epilog -> {{PointSize[0.02], RGBColor[1, 0, 0], Point[{x0, y0}]},
    {PointSize[0.02], RGBColor[0, 1, 0], Point[lista[[-1]]]}}]];
]

disorblunga2[{x0_, y0_}, eps_, nmax_, range_] :=
Block[{lista}, lista = orbita[x0, y0, eps, nmax];
Return[ListPlot[lista, PlotRange -> range, PlotStyle -> PointSize[0.001],
  AxesLabel -> {x, y}, AspectRatio -> Automatic, ImageSize -> 500,
  Epilog -> {{PointSize[0.02], RGBColor[1, 0, 0], Point[{x0, y0}]},
    {PointSize[0.02], RGBColor[0, 1, 0], Point[lista[[-1]]]}}]];
]

Manipulate[disorblunga[{0.1, .618}, eps, n], {eps, 0, 1.5, .01, Appearance -> "Open"},
{n, 1, 10 000, 1, Appearance -> "Open"}, ControlPlacement -> Left]

```

```

Manipulate[disorblunga[{x[[1]], x[[2]]}, eps, n],
  {{eps, .7}, 0, 1.5, .01, Appearance -> "Open"},
  {{n, 3000}, 1, 10 000, 1, Appearance -> "Open"},
  {{x, {2, .7}}, {0, 0}, {1, 1}, Appearance -> "Open"}, ControlPlacement -> Left]

Manipulate[disorblunga2[{x[[1]], x[[2]]}, eps, n, {{0, 0.4}, {0.5, 0.6}}],
  {{eps, .7}, 0, 1.5, .01, Appearance -> "Open"},
  {{n, 3000}, 1, 10 000, 1, Appearance -> "Open"},
  {{x, {2, .7}}, {0, 0}, {1, 1}, Appearance -> "Open"}, ControlPlacement -> Left]

```

```

Manipulate[disorblunga[{x0, y0}, eps, n],
  {{eps, .971}, 0, 1.5, .01, Appearance -> "Open"},
  {{n, 5000}, 1, 10 000, 1, Appearance -> "Open"}, {x0, 0, 1, .01, Appearance -> "Open"},
  {{y0, .488}, 0, 1, .01, Appearance -> "Open"}, ControlPlacement -> Left]

```



```

cercaop[p_, q_, eps_] :=
  Block[{x = .5, y = .6, a = 0, b = 1, ls, xf = 0, yf, ysum, err, res},
    While[Abs[x - xf] > 10-8,
      ls = orbita[x, y, eps, q + 1];
      xf = ls[[q + 1, 1]];
      yf = ls[[q + 1, 2]];
      ls = Drop[ls, -1];
      ysum = Apply[Plus, Map[#[[2]] &, ls]];
      If[Abs[ysum - p] < 10-8, Return[AppendTo[ls, {xf, yf}]]];
      If[ysum > p, b = y, a = y];
      y = (a + b) / 2];
    Return[AppendTo[ls, {xf, yf}]];
  ];

Manipulate[ListPlot[cercaop[p, q, eps] // Evaluate, PlotRange -> {{0, 1}, {0, 1}},
  PlotStyle -> {PointSize[0.01], RGBColor[1, 0, 0]}, AspectRatio -> Automatic,
  {{p, 5}, 1, 21, 1, Appearance -> "Open"}, {{q, 8}, 1, 21, 1, Appearance -> "Open"},
  {eps, .92, 2, Appearance -> "Open"}, ControlPlacement -> Left]

```

```

Manipulate[
  Show[ListPlot[Flatten[Join[Table[orbita[0.1, m / 50, eps, 1000], {m, 1, 49}], Table[
    orbita[0.5, m / 50, eps, 1000], {m, 1, 49}]], 1] // Evaluate,
    PlotRange -> {{0, 1}, {0, 1}}, ImageSize -> 500, AspectRatio -> Automatic,
    AxesLabel -> {x, y}, PlotStyle -> PointSize[0.001]],
  ListPlot[cercaop[p, q, eps] // Evaluate, PlotRange -> {{0, 1}, {0, 1}},
    PlotStyle -> {PointSize[0.01], RGBColor[1, 0, 0]}, AspectRatio -> Automatic]],
  {{p, 3}, 1, 21, 1, Appearance -> "Open"}, {{q, 5}, 1, 21, 1, Appearance -> "Open"},
  {{eps, .973}, 0, 1.5, Appearance -> "Open"},
  ControlPlacement -> Left]

```

```
Show[ListPlot[Flatten[Join[Table[orbita[0.1, m/50, .973, 1000], {m, 15, 40, .25}],
  Table[orbita[0.5, m/50, .973, 1000], {m, 15, 40, .5}]], 1] // Evaluate,
  PlotRange -> {{0, 1}, {0, 1}}, ImageSize -> 700, AspectRatio -> Automatic,
  AxesLabel -> {x, y}, PlotStyle -> PointSize[0.001],
ListPlot[cercaop[1, 2, .973] // Evaluate, PlotRange -> {{0, 1}, {0, 1}},
  PlotStyle -> {PointSize[0.008], RGBColor[1, 0, 0]}, AspectRatio -> Automatic],
ListPlot[cercaop[2, 3, .973] // Evaluate, PlotRange -> {{0, 1}, {0, 1}},
  PlotStyle -> {PointSize[0.008], RGBColor[1, 0, 0]}, AspectRatio -> Automatic],
ListPlot[cercaop[3, 5, .973] // Evaluate, PlotRange -> {{0, 1}, {0, 1}},
  PlotStyle -> {PointSize[0.008], RGBColor[1, 0, 1]}, AspectRatio -> Automatic],
ListPlot[cercaop[5, 8, .973] // Evaluate, PlotRange -> {{0, 1}, {0, 1}},
  PlotStyle -> {PointSize[0.008], RGBColor[0, 1, 0]}, AspectRatio -> Automatic],
ListPlot[cercaop[8, 13, .973] // Evaluate, PlotRange -> {{0, 1}, {0, 1}},
  PlotStyle -> {PointSize[0.008], RGBColor[0, 1, 0]}, AspectRatio -> Automatic],
ListPlot[cercaop[13, 21, .973] // Evaluate, PlotRange -> {{0, 1}, {0, 1}},
  PlotStyle -> {PointSize[0.008], RGBColor[1, 0, 0]}, AspectRatio -> Automatic]]
```

