

(*

Master di II livello in Scienza e Tecnologia Spaziale
METODI NUMERICI PER L' ASTRONOMIA

Il software *Mathematica* per il calcolo simbolico, numerico e grafico.

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Problema 1: sviluppo in serie dell'equazione di Keplero.

(* Equazione di Keplero: $M = E - e \sin(E)$
e eccentricita'
E anomalia eccentrica
M anomalia media *)

? Series

Series[f, {x, x₀, n}] generates a power series expansion for f about the point $x = x_0$ to order $(x - x_0)^n$.
Series[f, {x, x₀, n_x}, {y, y₀, n_y}, ...] successively finds series expansions with respect to x, then y, etc. >>

s = Series[Sin[x], {x, 0, 3}]

$$x - \frac{x^3}{6} + O[x]^4$$

s

$$x - \frac{x^3}{6} + O[x]^4$$

s^2

$$x^2 - \frac{x^4}{3} + O[x]^5$$

Normal[s]

$$x - \frac{x^3}{6}$$

%

$$x - \frac{x^3}{6}$$

%3

$$x^2 - \frac{x^4}{3} + O[x]^5$$

s = Series[Sin[x], {x, 0, 30}]**s = Series[Sin[x], {x, 0., 30}]****s = Series[Sin[x], {x, 0.5, 10}]**

$$0.479426 + 0.877583 (x - 0.5) - 0.239713 (x - 0.5)^2 - 0.146264 (x - 0.5)^3 + 0.0199761 (x - 0.5)^4 + 0.00731319 (x - 0.5)^5 - 0.000665869 (x - 0.5)^6 - 0.000174124 (x - 0.5)^7 + 0.0000118905 (x - 0.5)^8 + 2.41838 \times 10^{-6} (x - 0.5)^9 - 1.32117 \times 10^{-7} (x - 0.5)^{10} + O[x - 0.5]^{11}$$

Normal[s]**Expand[Normal[s]]**

$$-1.09619 \times 10^{-11} + 1. x - 2.42073 \times 10^{-9} x^2 - 0.166667 x^3 - 5.84144 \times 10^{-8} x^4 + 0.00833335 x^5 - 3.29916 \times 10^{-7} x^6 - 0.000197938 x^7 - 4.7852 \times 10^{-7} x^8 + 3.07897 \times 10^{-6} x^9 - 1.32117 \times 10^{-7} x^{10}$$

s = Series[Sin[x], {x, 1/2, 30}]**N[s]**

$$(x + 0.) - 0.166667 (x + 0.)^3 + 0.00833333 (x + 0.)^5 - 0.000198413 (x + 0.)^7 + 2.75573 \times 10^{-6} (x + 0.)^9 - 2.50521 \times 10^{-8} (x + 0.)^{11} + 1.6059 \times 10^{-10} (x + 0.)^{13} - 7.64716 \times 10^{-13} (x + 0.)^{15} + 2.81146 \times 10^{-15} (x + 0.)^{17} - 8.22064 \times 10^{-18} (x + 0.)^{19} + 1.95729 \times 10^{-20} (x + 0.)^{21} - 3.86817 \times 10^{-23} (x + 0.)^{23} + 6.44695 \times 10^{-26} (x + 0.)^{25} - 9.18369 \times 10^{-29} (x + 0.)^{27} + 1.131 \times 10^{-31} (x + 0.)^{29} + O[x + 0.]^{31}$$

N[Expand[Normal[Series[Sin[x], {x, 0., 30}]]]]

$$0. + x - 0.166667 x^3 + 0.00833333 x^5 - 0.000198413 x^7 + 2.75573 \times 10^{-6} x^9 - 2.50521 \times 10^{-8} x^{11} + 1.6059 \times 10^{-10} x^{13} - 7.64716 \times 10^{-13} x^{15} + 2.81146 \times 10^{-15} x^{17} - 8.22064 \times 10^{-18} x^{19} + 1.95729 \times 10^{-20} x^{21} - 3.86817 \times 10^{-23} x^{23} + 6.44695 \times 10^{-26} x^{25} - 9.18369 \times 10^{-29} x^{27} + 1.131 \times 10^{-31} x^{29}$$

? N

N[expr] gives the numerical value of *expr*.

N[expr, n] attempts to give a result with *n*-digit precision. >>

Pi π

```
N[Pi, 1000]
```

3.1415926535897932384626433832795028841971693993751058209749445923078164062862089:
98628034825342117067982148086513282306647093844609550582231725359408128481117450:
28410270193852110555964462294895493038196442881097566593344612847564823378678316:
52712019091456485669234603486104543266482133936072602491412737245870066063155881:
74881520920962829254091715364367892590360011330530548820466521384146951941511609:
43305727036575959195309218611738193261179310511854807446237996274956735188575272:
48912279381830119491298336733624406566430860213949463952247371907021798609437027:
70539217176293176752384674818467669405132000568127145263560827785771342757789609:
17363717872146844090122495343014654958537105079227968925892354201995611212902196:
08640344181598136297747713099605187072113499999983729780499510597317328160963185:
95024459455346908302642522308253344685035261931188171010003137838752886587533208:
38142061717766914730359825349042875546873115956286388235378759375195778185778053:
2171226806613001927876611195909216420199

```
N[Expand[Normal[Series[Sin[x], {x, 0, 30}]]], 30]
```

[illegible]

s = Series[Sin[x], {x, 1, 30}]

$$\begin{aligned}
 & \sin[1] + \cos[1] (x-1) - \frac{1}{2} \sin[1] (x-1)^2 - \frac{1}{6} \cos[1] (x-1)^3 + \frac{1}{24} \sin[1] (x-1)^4 + \\
 & \frac{1}{120} \cos[1] (x-1)^5 - \frac{1}{720} \sin[1] (x-1)^6 - \frac{\cos[1] (x-1)^7}{5040} + \frac{\sin[1] (x-1)^8}{40320} + \\
 & \frac{\cos[1] (x-1)^9}{362880} - \frac{\sin[1] (x-1)^{10}}{3628800} - \frac{\cos[1] (x-1)^{11}}{39916800} + \frac{\sin[1] (x-1)^{12}}{479001600} + \\
 & \frac{\cos[1] (x-1)^{13}}{6227020800} - \frac{\sin[1] (x-1)^{14}}{87178291200} - \frac{\cos[1] (x-1)^{15}}{1307674368000} + \frac{\sin[1] (x-1)^{16}}{20922789888000} + \\
 & \frac{\cos[1] (x-1)^{17}}{355687428096000} - \frac{\sin[1] (x-1)^{18}}{6402373705728000} - \frac{\cos[1] (x-1)^{19}}{121645100408832000} + \\
 & \frac{\sin[1] (x-1)^{20}}{2432902008176640000} + \frac{\cos[1] (x-1)^{21}}{51090942171709440000} - \frac{\sin[1] (x-1)^{22}}{112400072777607680000} - \\
 & \frac{\cos[1] (x-1)^{23}}{25852016738884976640000} + \frac{\sin[1] (x-1)^{24}}{620448401733239439360000} + \\
 & \frac{\cos[1] (x-1)^{25}}{15511210043330985984000000} - \frac{\sin[1] (x-1)^{26}}{403291461126605635584000000} - \\
 & (\cos[1] (x-1)^{27}) / 10888869450418352160768000000 + \\
 & (\sin[1] (x-1)^{28}) / 304888344611713860501504000000 + \\
 & (\cos[1] (x-1)^{29}) / 8841761993739701954543616000000 - \\
 & (\sin[1] (x-1)^{30}) / 265252859812191058636308480000000 + O[x-1]^{31}
 \end{aligned}$$

N[s]

$$\begin{aligned}
 & 0.841471 + 0.540302 (x-1.) - 0.420735 (x-1.)^2 - 0.0900504 (x-1.)^3 + \\
 & 0.0350613 (x-1.)^4 + 0.00450252 (x-1.)^5 - 0.00116871 (x-1.)^6 - \\
 & 0.000107203 (x-1.)^7 + 0.0000208698 (x-1.)^8 + 1.48893 \times 10^{-6} (x-1.)^9 - \\
 & 2.31887 \times 10^{-7} (x-1.)^{10} - 1.35357 \times 10^{-8} (x-1.)^{11} + 1.75672 \times 10^{-9} (x-1.)^{12} + \\
 & 8.67674 \times 10^{-11} (x-1.)^{13} - 9.6523 \times 10^{-12} (x-1.)^{14} - 4.13178 \times 10^{-13} (x-1.)^{15} + \\
 & 4.02179 \times 10^{-14} (x-1.)^{16} + 1.51904 \times 10^{-15} (x-1.)^{17} - 1.31431 \times 10^{-16} (x-1.)^{18} - \\
 & 4.44163 \times 10^{-18} (x-1.)^{19} + 3.45871 \times 10^{-19} (x-1.)^{20} + 1.05753 \times 10^{-20} (x-1.)^{21} - \\
 & 7.48639 \times 10^{-22} (x-1.)^{22} - 2.08998 \times 10^{-23} (x-1.)^{23} + 1.35623 \times 10^{-24} (x-1.)^{24} + \\
 & 3.4833 \times 10^{-26} (x-1.)^{25} - 2.08651 \times 10^{-27} (x-1.)^{26} - 4.96197 \times 10^{-29} (x-1.)^{27} + \\
 & 2.75993 \times 10^{-30} (x-1.)^{28} + 6.1108 \times 10^{-32} (x-1.)^{29} - 3.17234 \times 10^{-33} (x-1.)^{30} + O[x-1.]^{31}
 \end{aligned}$$

N[s, 20]

```
0.84147098480789650665 + 0.54030230586813971740 (x - 1.00000000000000000000) -
0.42073549240394825333 (x - 1.00000000000000000000)2 -
0.090050384311356619567 (x - 1.00000000000000000000)3 +
0.035061291033662354444 (x - 1.00000000000000000000)4 +
0.0045025192155678309783 (x - 1.00000000000000000000)5 -
0.0011687097011220784815 (x - 1.00000000000000000000)6 -
0.00010720283846590073758 (x - 1.00000000000000000000)7 +
0.000020869816091465687169 (x - 1.00000000000000000000)8 +
1.4889283120263991330 × 10-6 (x - 1.00000000000000000000)9 -
2.3188684546072985743 × 10-7 (x - 1.00000000000000000000)10 -
1.3535711927512719391 × 10-8 (x - 1.00000000000000000000)11 +
1.7567185262176504351 × 10-9 (x - 1.00000000000000000000)12 +
8.6767384150722560201 × 10-11 (x - 1.00000000000000000000)13 -
9.6522995946024749181 × 10-12 (x - 1.00000000000000000000)14 -
4.1317801976534552477 × 10-13 (x - 1.00000000000000000000)15 +
4.0217914977510312159 × 10-14 (x - 1.00000000000000000000)16 +
1.5190368373725938411 × 10-15 (x - 1.00000000000000000000)17 -
1.3143109469774611817 × 10-16 (x - 1.00000000000000000000)18 -
4.4416281794520287750 × 10-18 (x - 1.00000000000000000000)19 +
3.4587130183617399517 × 10-19 (x - 1.00000000000000000000)20 +
1.0575305189171497083 × 10-20 (x - 1.00000000000000000000)21 -
7.4863918146358007613 × 10-22 (x - 1.00000000000000000000)22 -
2.0899812626821140481 × 10-23 (x - 1.00000000000000000000)23 +
1.3562304012021378191 × 10-24 (x - 1.00000000000000000000)24 +
3.4833021044701900802 × 10-26 (x - 1.00000000000000000000)25 -
2.0865083095417504909 × 10-27 (x - 1.00000000000000000000)26 -
4.9619688097865955558 × 10-29 (x - 1.00000000000000000000)27 +
2.7599316263779768398 × 10-30 (x - 1.00000000000000000000)28 +
6.1107990268307827042 × 10-32 (x - 1.00000000000000000000)29 -
3.1723352027333067124 × 10-33 (x - 1.00000000000000000000)30 +
O[x - 1.00000000000000000000]31
```

s = Series[Sin[x], {x, 0, 3}]

$$x - \frac{x^3}{6} + O[x]^4$$

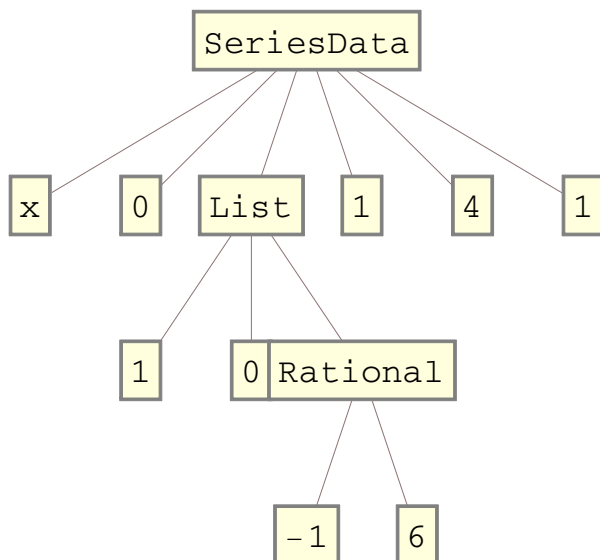
FullForm[s]

```
SeriesData[x, 0, List[1, 0, Rational[-1, 6]], 1, 4, 1]
```

? SeriesData

SeriesData[$x, x_0, \{a_0, a_1, \dots\}, n_{min}, n_{max}, den$] represents a power series in the variable x about the point x_0 . The a_i are the coefficients in the power series. The powers of $(x - x_0)$ that appear are $n_{min}/den, (n_{min} + 1)/den, \dots, n_{max}/den$. >>

```
TreeForm[s]
```



```
? SeriesData
```

`SeriesData[x, x0, {a0, a1, ...}, nmin, nmax, den]` represents a power series in the variable x about the point x_0 . The a_i are the coefficients in the power series. The powers of $(x - x_0)$ that appear are $n_{min}/den, (n_{min} + 1)/den, \dots, n_{max}/den$. $\gg$

```
Series[Exp[x], {x, 0, 5}]
```

$$1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + O[x]^6$$

```
FullForm[Series[Exp[x], {x, 0, 5}]]
```

```
SeriesData[x, 0, List[1, 1, Rational[1, 2],
  Rational[1, 6], Rational[1, 24], Rational[1, 120]], 0, 6, 1]
```

```
g[x_] := x^2
```

```
g[2]
```

```
4
```

```
h[x_, y_] := Sin[x * y] + Cos[x^2 + y^2]
```

```
h[0, 0]
```

```
1
```

```
h[1.3, 2.7]
```

```
-1.26284
```

```
N[h[1.3, 2.7], 20]
```

```
-1.26284
```

```
N[h[13 / 10, 27 / 10], 20]
```

```
-1.2628363211538779645
```

```
-1.2628363211538779645215064633370270823442915351346025039416`20.
```

```
1.234`20
```

```
1.23400000000000000000
```

```
N[E]
```

```
2.71828
```

```
funz[x_] := x^2
```

```
funz[4]
```

```
16
```

```
funz[0.5]
```

```
0.25
```

```
Map[#^2 &, {1, 2, 3}]
```

```
{1, 4, 9}
```

```
(* Equazione di Keplero: M = E - e sin(E)
```

```
  e eccentricita'
```

```
  E anomalia eccentrica
```

```
  M anomalia media
```

```
  E = M + e sin(E) = M + e sin(M + e sin(E))
```

```
*)
```

```
(* sviluppo della soluzione intorno ad e=0 *)
```

```
f[0] = m;
```

```
f[n_] := f[n] = m + e * Sin[f[n - 1]]
```

```
f[1]
```

```
m + e Sin[m]
```

```
? f
```

```
Global`f
```

```
f[0] = m
```

```
f[1] = m + e Sin[m]
```

```
f[n_] := f[n] = m + e Sin[f[n - 1]]
```

```
f[1]
```

```
m + e Sin[m]
```

```
f[5]
```

```
m + e Sin[m + e Sin[m + e Sin[m + e Sin[m + e Sin[m]]]]]
```

```
s1 = Series[f[1], {e, 0, 2}]
```

```
m + Sin[m] e + O[e]^3
```

```
Series[f[5], {e, 0, 5}]
```

```
m + Sin[m] e + Cos[m] Sin[m] e^2 +
  (Cos[m]^2 Sin[m] - Sin[m]^3/2) e^3 + (Cos[m]^3 Sin[m] - 5/3 Cos[m] Sin[m]^3) e^4 +
  (Cos[m]^4 Sin[m] - 11/3 Cos[m]^2 Sin[m]^3 + 13 Sin[m]^5/24) e^5 + O[e]^6
```

```
s10 = Series[f[10], {e, 0, 5}]
```

```
m + Sin[m] e + Cos[m] Sin[m] e^2 +
  (Cos[m]^2 Sin[m] - Sin[m]^3/2) e^3 + (Cos[m]^3 Sin[m] - 5/3 Cos[m] Sin[m]^3) e^4 +
  (Cos[m]^4 Sin[m] - 11/3 Cos[m]^2 Sin[m]^3 + 13 Sin[m]^5/24) e^5 + O[e]^6
```

```
ns10 = Normal[s10]
```

```
m + e Sin[m] + e^2 Cos[m] Sin[m] +
  e^3 (Cos[m]^2 Sin[m] - Sin[m]^3/2) + e^4 (Cos[m]^3 Sin[m] - 5/3 Cos[m] Sin[m]^3) +
  e^5 (Cos[m]^4 Sin[m] - 11/3 Cos[m]^2 Sin[m]^3 + 13 Sin[m]^5/24)
```

```
sns10 = Expand[TrigReduce[ns10]]
```

```
m + e Sin[m] - 1/8 e^3 Sin[m] + 1/192 e^5 Sin[m] + 1/2 e^2 Sin[2 m] -
  1/6 e^4 Sin[2 m] + 3/8 e^3 Sin[3 m] - 27/128 e^5 Sin[3 m] + 1/3 e^4 Sin[4 m] + 125/384 e^5 Sin[5 m]
```

Expand[TrigReduce[ns10]] /. m -> n - Pi / 2

$$n - \frac{\pi}{2} - e \cos[n] + \frac{1}{8} e^3 \cos[n] - \frac{1}{192} e^5 \cos[n] + \frac{1}{2} e^2 \sin\left[2 \left(n - \frac{\pi}{2}\right)\right] - \frac{1}{6} e^4 \sin\left[2 \left(n - \frac{\pi}{2}\right)\right] + \frac{3}{8} e^3 \sin\left[3 \left(n - \frac{\pi}{2}\right)\right] - \frac{27}{128} e^5 \sin\left[3 \left(n - \frac{\pi}{2}\right)\right] + \frac{1}{3} e^4 \sin\left[4 \left(n - \frac{\pi}{2}\right)\right] + \frac{125}{384} e^5 \sin\left[5 \left(n - \frac{\pi}{2}\right)\right]$$

Simplify[%]

$$\frac{1}{384} \left(384 n - 192 \pi - 2 e \left(192 - 24 e^2 + e^4 \right) \cos[n] - 9 e^3 \left(-16 + 9 e^2 \right) \cos[3 n] - 125 e^5 \cos[5 n] - 192 e^2 \sin[2 n] + 64 e^4 \sin[2 n] + 128 e^4 \sin[4 n] \right)$$

Simplify[ns10]

$$m + e \sin[m] + e^2 \cos[m] \sin[m] - \frac{1}{8} e^3 (\sin[m] - 3 \sin[3 m]) - \frac{1}{6} e^4 (\sin[2 m] - 2 \sin[4 m]) + \frac{1}{384} e^5 (2 \sin[m] - 81 \sin[3 m] + 125 \sin[5 m])$$

sns10

$$m + e \sin[m] - \frac{1}{8} e^3 \sin[m] + \frac{1}{192} e^5 \sin[m] + \frac{1}{2} e^2 \sin[2 m] - \frac{1}{6} e^4 \sin[2 m] + \frac{3}{8} e^3 \sin[3 m] - \frac{27}{128} e^5 \sin[3 m] + \frac{1}{3} e^4 \sin[4 m] + \frac{125}{384} e^5 \sin[5 m]$$

Collect[sns10, Sin[m]]

$$m + \left(e - \frac{e^3}{8} + \frac{e^5}{192} \right) \sin[m] + \frac{1}{2} e^2 \sin[2 m] - \frac{1}{6} e^4 \sin[2 m] + \frac{3}{8} e^3 \sin[3 m] - \frac{27}{128} e^5 \sin[3 m] + \frac{1}{3} e^4 \sin[4 m] + \frac{125}{384} e^5 \sin[5 m]$$

Collect[sns10, Sin[c_m]]

$$m + e \sin[m] - \frac{1}{8} e^3 \sin[m] + \frac{1}{192} e^5 \sin[m] + \left(\frac{e^2}{2} - \frac{e^4}{6} \right) \sin[2 m] + \left(\frac{3 e^3}{8} - \frac{27 e^5}{128} \right) \sin[3 m] + \frac{1}{3} e^4 \sin[4 m] + \frac{125}{384} e^5 \sin[5 m]$$

Collect[sns10, Sin[c_. m]]

$$m + \left(e - \frac{e^3}{8} + \frac{e^5}{192} \right) \sin[m] + \left(\frac{e^2}{2} - \frac{e^4}{6} \right) \sin[2 m] + \left(\frac{3 e^3}{8} - \frac{27 e^5}{128} \right) \sin[3 m] + \frac{1}{3} e^4 \sin[4 m] + \frac{125}{384} e^5 \sin[5 m]$$

```
Expand[TrigReduce[Normal[Series[f[10], {e, 0, 5}]]]]
```

$$m + e \sin[m] - \frac{1}{8} e^3 \sin[m] + \frac{1}{192} e^5 \sin[m] + \frac{1}{2} e^2 \sin[2m] - \frac{1}{6} e^4 \sin[2m] + \frac{3}{8} e^3 \sin[3m] - \frac{27}{128} e^5 \sin[3m] + \frac{1}{3} e^4 \sin[4m] + \frac{125}{384} e^5 \sin[5m]$$

```
f[0] = m;
```

```
f[n_] := f[n] = m + e * Sin[f[n - 1]]
```

```
kepecc[n_] := Collect[Expand[TrigReduce[Normal[Series[f[10], {e, 0, n}]]]], e]
```

```
kepecc[5]
```

$$m + e \sin[m] + \frac{1}{2} e^2 \sin[2m] + e^3 \left(-\frac{\sin[m]}{8} + \frac{3}{8} \sin[3m] \right) + e^4 \left(-\frac{1}{6} \sin[2m] + \frac{1}{3} \sin[4m] \right) + e^5 \left(\frac{\sin[m]}{192} - \frac{27}{128} \sin[3m] + \frac{125}{384} \sin[5m] \right)$$

```
kep[n_] := Collect[Expand[TrigReduce[Normal[Series[f[10], {e, 0, n}]]]], Sin[c_. * m]]
```

```
kep[5]
```

$$m + \left(e - \frac{e^3}{8} + \frac{e^5}{192} \right) \sin[m] + \left(\frac{e^2}{2} - \frac{e^4}{6} \right) \sin[2m] + \left(\frac{3e^3}{8} - \frac{27e^5}{128} \right) \sin[3m] + \frac{1}{3} e^4 \sin[4m] + \frac{125}{384} e^5 \sin[5m]$$

```
kep[10]
```

$$m + \left(e - \frac{e^3}{8} + \frac{e^5}{192} - \frac{e^7}{9216} + \frac{e^9}{737280} \right) \sin[m] + \left(\frac{e^2}{2} - \frac{e^4}{6} + \frac{e^6}{48} - \frac{e^8}{720} + \frac{e^{10}}{17280} \right) \sin[2m] + \left(\frac{3e^3}{8} - \frac{27e^5}{128} + \frac{243e^7}{5120} - \frac{243e^9}{40960} \right) \sin[3m] + \left(\frac{e^4}{3} - \frac{4e^6}{15} + \frac{4e^8}{45} - \frac{16e^{10}}{945} \right) \sin[4m] + \left(\frac{125e^5}{384} - \frac{3125e^7}{9216} + \frac{78125e^9}{516096} \right) \sin[5m] + \left(\frac{27e^6}{80} - \frac{243e^8}{560} + \frac{2187e^{10}}{8960} \right) \sin[6m] + \left(\frac{16807e^7}{46080} - \frac{823543e^9}{1474560} \right) \sin[7m] + \left(\frac{128e^8}{315} - \frac{2048e^{10}}{2835} \right) \sin[8m] + \frac{531441e^9 \sin[9m]}{1146880} + \frac{78125e^{10} \sin[10m]}{145152}$$

Timing[kep[10]]

$$\begin{aligned}
 & \left\{ 14.0015, m + \left(e - \frac{e^3}{8} + \frac{e^5}{192} - \frac{e^7}{9216} + \frac{e^9}{737280} \right) \sin[m] + \right. \\
 & \quad \left(\frac{e^2}{2} - \frac{e^4}{6} + \frac{e^6}{48} - \frac{e^8}{720} + \frac{e^{10}}{17280} \right) \sin[2m] + \left(\frac{3e^3}{8} - \frac{27e^5}{128} + \frac{243e^7}{5120} - \frac{243e^9}{40960} \right) \sin[3m] + \\
 & \quad \left(\frac{e^4}{3} - \frac{4e^6}{15} + \frac{4e^8}{45} - \frac{16e^{10}}{945} \right) \sin[4m] + \left(\frac{125e^5}{384} - \frac{3125e^7}{9216} + \frac{78125e^9}{516096} \right) \sin[5m] + \\
 & \quad \left(\frac{27e^6}{80} - \frac{243e^8}{560} + \frac{2187e^{10}}{8960} \right) \sin[6m] + \left(\frac{16807e^7}{46080} - \frac{823543e^9}{1474560} \right) \sin[7m] + \\
 & \quad \left. \left(\frac{128e^8}{315} - \frac{2048e^{10}}{2835} \right) \sin[8m] + \frac{531441e^9 \sin[9m]}{1146880} + \frac{78125e^{10} \sin[10m]}{145152} \right\}
 \end{aligned}$$