Modello di Henon-Heiles

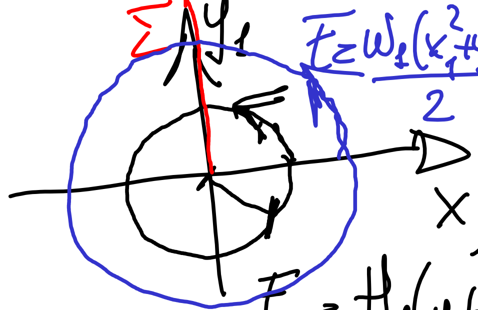
$$H(\underline{y}, \underline{x}) = \frac{1}{2}\omega_1(x_1^2+y_1^2) + \frac{1}{2}\omega_2(x_2^2+y_2^2) + \cancel{x_1^2 x_2} - \frac{x_2^3}{3}$$

termine di accoppiamento tra (y_1, x_1) e (y_2, x_2)

momentaneamente leviamo

Senza accoppiamento: $H(\underline{y}, \underline{x}) = H_1(y_1, x_1) + H_2(y_2, x_2)$

dove $H_1(y_1, x_1) = \frac{\omega_1}{2}(x_1^2 + y_1^2)$



La sezione di Poincaré Σ è data da

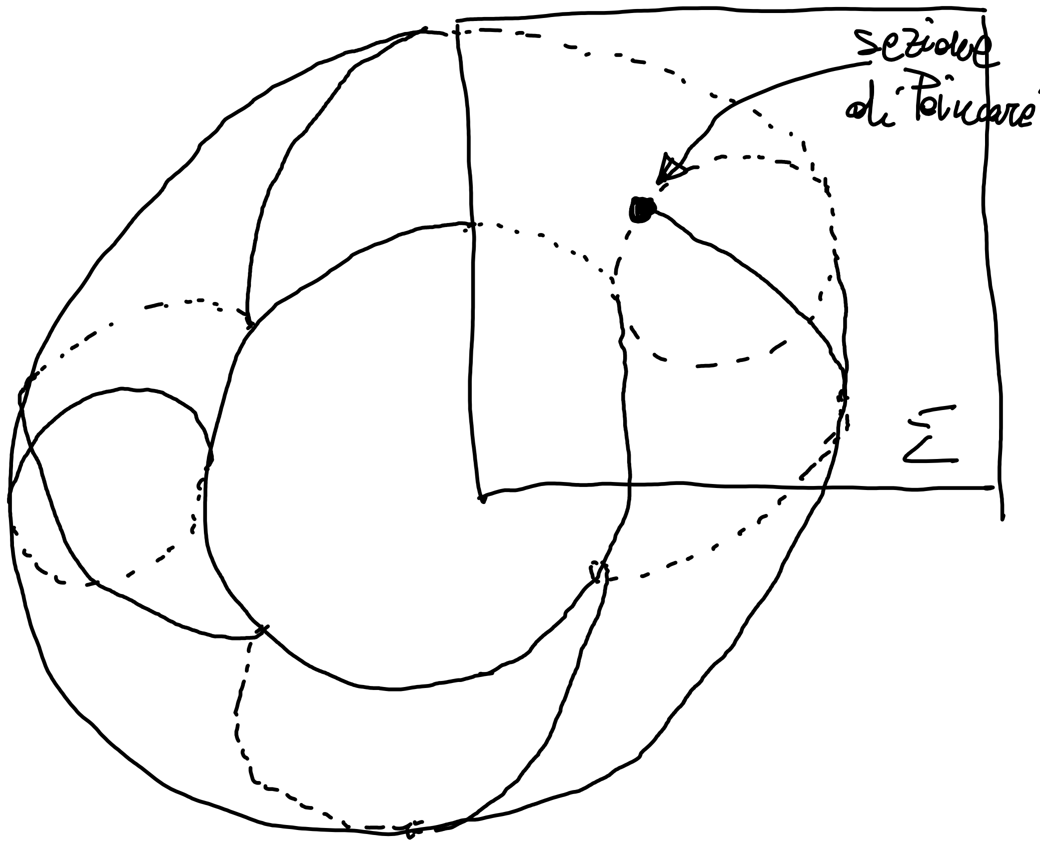
$$x_1 = 0, \quad \dot{x}_1 = \frac{\partial H_1}{\partial y_1} = \omega_1 y_1 > 0$$

inoltre $H_2(y_2, x_2) = \frac{\omega_2}{2}(x_2^2 + y_2^2) - \frac{x_2^3}{3}$

Siccome $\{H_1, H_2\} = 0$, $\{H_1, H\} = \{H_2, H\} = 0$

$\Rightarrow$ 2 cost. del moto in involuzione e indip. $\Rightarrow$ sist. INTEGR.

Siccome $H_2(y_2, x_2)$ è cost. del moto allora $H_2(y_2, x_2) = E_2 = H_2(y_2(0), x_2(0))$



Costruzione della forma normale di Birkhoff con Mathematica

$$K_0 = \omega_1 z_1 \bar{z}_1 + \omega_2 z_2 \bar{z}_2 \qquad (i\underline{\bar{z}}, \underline{z})$$

momenti

var. canoniche

coord.

Rappr

	Mathematica	scrittura nostra
	z1	z_1
	z2	z_2
	izb1	$i\bar{z}_1$
	izb2	$i\bar{z}_2$
$1 =$	w1	ω_1
$\frac{\sqrt{5}-1}{2} =$	w2	ω_2
	znf0	$-i\omega_1^2 z_1 i\bar{z}_1 - i\omega_2 z_2 i\bar{z}_2$
	fs[[1]]	$f_1 \in P_3$ pol. om. di grado 3
	fs[[2]]	$f_2 \in P_4$ " " " " 4
	⋮	⋮

$$z_1 = \frac{1}{\sqrt{2}}(x_1 + i y_1)$$

$$i\bar{z}_1 = \frac{i}{\sqrt{2}}(x_1 - i y_1)$$

$$\frac{1}{\sqrt{2}}\left(z_1 - i(i\bar{z}_1)\right) = x_1$$

$\text{(z1b)}^{\wedge}\text{l1b} * \text{(z1b2)}^{\wedge}\text{l2b}$ Mathematica

$z1^{\wedge}l1 * z2^{\wedge}l2 *$ ⟸ $l1, l2, l1b, l2b$

omosolv[f]

passonorm[fs, r]

scrittura nostra

esponenti $\ell_1, \ell_2, \bar{\ell}_1, \bar{\ell}_2 \Rightarrow z_1^{\ell_1} z_2^{\ell_2} (i\bar{z}_1)^{\bar{\ell}_1} (i\bar{z}_2)^{\bar{\ell}_2}$

risolve l'eq. omologica $\left\{ \sum_{j=1}^{2} \omega_j z_j \bar{z}_j, \chi \right\} + f = Z$

dip. solo da $z_1 \bar{z}_1, z_2 \bar{z}_2$

ci fa passare da $H^{(r-1)}$ a $H^{(r)} = \exp L_{\chi_2} H^{(r-1)}$

$H^{(r)} = \exp L_{\chi_2} Z_0 + \exp L_{\chi_2} Z_1 + \ldots + \exp L_{\chi_2} Z_{r-1}$

$+ \exp L_{\chi_2} f_r^{(r-1)} + \exp L_{\chi_2} f_{r+1}^{(r-1)} + \ldots$

Zn, f0

fs[[1]]

fs[[r-1]]

fs[[r]]

fs[[r+1]]

Con il troncamento all'indice R1-1

Rappresentiamo tutta la Ham. con

ZnfO , fs

Hamiltoniana = ZnfO + fs[[1]] + fs[[2]] + ...
iniziale e
anche ai passi
successivi (ma cambiano le fs)

Supponiamo di essere al passo $r=2$

$L_{\chi_2} Z_0$, $\frac{1}{2} L^2_{\chi_2} Z_0 \in \mathcal{P}_6$ termini da calcolare $[R-1/2]$

$\in \mathcal{P}_4$ va a sommarsi a $f_2^{(1)}$

$$\underbrace{\frac{1}{j!} L^j_{\chi_2} f_s^{(r-1)}}_{\in \mathcal{P}_{s+jr+2}} \quad \text{va a sommarsi a} \quad \underbrace{f^{(r)}_{s+jr}}_{\in \mathcal{P}_{s+jr+2}}$$

$\Rightarrow$ j_{fin} deve ess. il max. j t.c. $s+jr \le R-1$

$$j_{fin} = \left\lfloor \frac{R-1-s}{r} \right\rfloor$$

per $j=0$, lieterm $= Z_0$

per $j=1$, summand $= \{Z_0, \chi_2\}$, lieterm $= \{Z_0, \chi_2\}$, $f_2^{(2)} \leftarrow L_{\chi_2} Z_0$

" $j=2$, summand $= \{L_{\chi_2} Z_0, \chi_2\}$, lieterm $= \frac{1}{2} L^2_{\chi_2} Z_0$, $f^{(2)}_{22} \leftarrow \frac{1}{2} L^2_{\chi_2} Z_0$

" $j=3$, " $= \{\frac{1}{2} L^2_{\chi_2} Z_0, \chi_2\}$, " $= \frac{1}{3} \cdot \frac{1}{2} L^3_{\chi_2} Z_0$, $f^{(2)}_{32} \leftarrow \frac{1}{3!} L^3_{\chi_2} Z_0$