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2-dimensional continued fraction algorithms as dynamical systems

2-dimensional continued fractions are described by the following model. Let $B \subseteq \mathbf{R^2}$, I a countable index set, $B = \bigcup_{i \in I} B(i)$ a partition, and $T : B \to B$ a map such that the restriction of T to $B(i)$ is a fractional linear map. Some questions are immediate. Is T ergodic with respect to Lebesgue measure λ? Does T admit a σ-finite invariant measure $\mu \sim \lambda$? Can one obtain a good result on the quality of approximation at least almost everywhere?
Furthermore some examples lead to the following situation. Let R be a proper subset of I. Consider the set $\Gamma_R = \{x \in B : T^n x \in B(i), i \in R, n \geq 0\}$. The expected case is that Γ_R is of Cantor type with $\lambda(\Gamma_R) = 0$. However, the following situation occurs with the Parry-Daniels map. For a suitable choice of R the set Γ_R consists of infinitely many segments of variable length which are glued together in one point but we have $\lambda(\Gamma_R) > 0$.