String Matching

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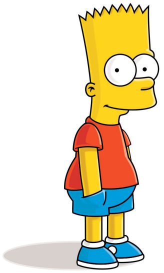
Problem: Given an alphabet Σ, a *text* $T \in \Sigma^*$ and a *pattern* $P \in \Sigma^*$, find some occurrence/all occurrences of P in T.



$$\Sigma = \{\texttt{A}, \texttt{B}, \ldots, \texttt{Z}, \texttt{a}, \texttt{b}, \ldots, \texttt{z}, \textvisiblespace\}$$

$$T = \texttt{Bart\textvisiblespace played\textvisiblespace darts\textvisiblespace at\textvisiblespace the\textvisiblespace party}$$

$$P = \texttt{art}$$

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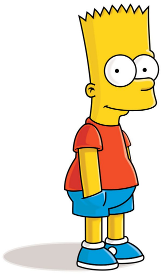

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$$T = \texttt{B}\underline{\texttt{art}}\texttt{␣played␣d}\underline{\texttt{art}}\texttt{s␣at␣the␣p}\underline{\texttt{art}}\texttt{y}$$

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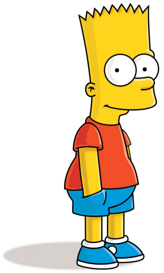

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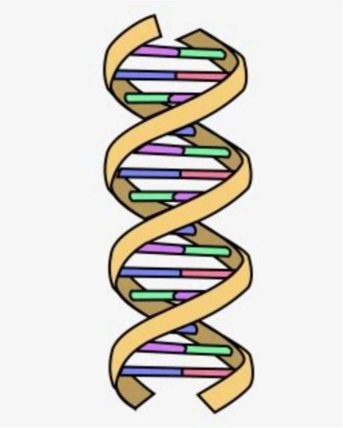

$$\Sigma = \{\texttt{A}, \texttt{C}, \texttt{G}, \texttt{T}\}$$

$$T = \texttt{ACGTGCTTGCAGTGTGCATTACCTGAGTGC}\ldots$$

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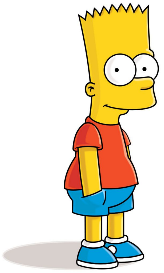

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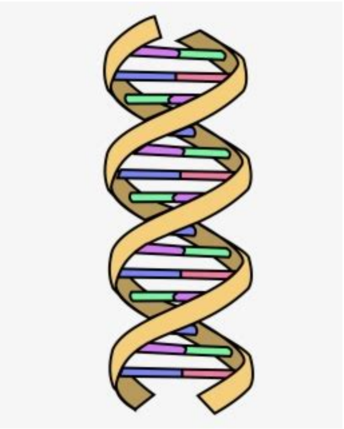

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One-shot:

- Both the text and the pattern are **part of the input**
- Algorithm design problem

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Repeated:

- The text is **static** and known beforehand (can be preprocessed)
- Patterns are revealed on-demand
- We want to answer each *query* as **quickly** as possible
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Tries

Tries (Prounounced as "try")

Data structure to store a dynamic collection of k strings over an alphabet Σ

$$\Sigma = \{\mathtt{A}, \mathtt{D}, \mathtt{E}, \mathtt{G}, \mathtt{R}, \mathtt{S}, \mathtt{T}\}$$

$$\{\ \mathtt{RAD},\ \mathtt{RADAR},\ \mathtt{RAG},\ \mathtt{RAGE},\ \mathtt{RAGS},\ \mathtt{RATE}\ \}$$

- **Insert**(T): add T to the collection of strings
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- **Find**(P): return whether P is in the collection

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Obs: A string comparison requires time $O($string length$)$.
Binary searching requires time $O($max string length $\cdot \log k)$

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We will only focus on the static case

Tries

Pretend that each string ends with a special “end marker” symbol $\$$

RAD   RADAR   RAG   RAGE   RAGS   RATE

Tries

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`RAD$  RADAR$  RAG$  RAGE$  RAGS$  RATE$`

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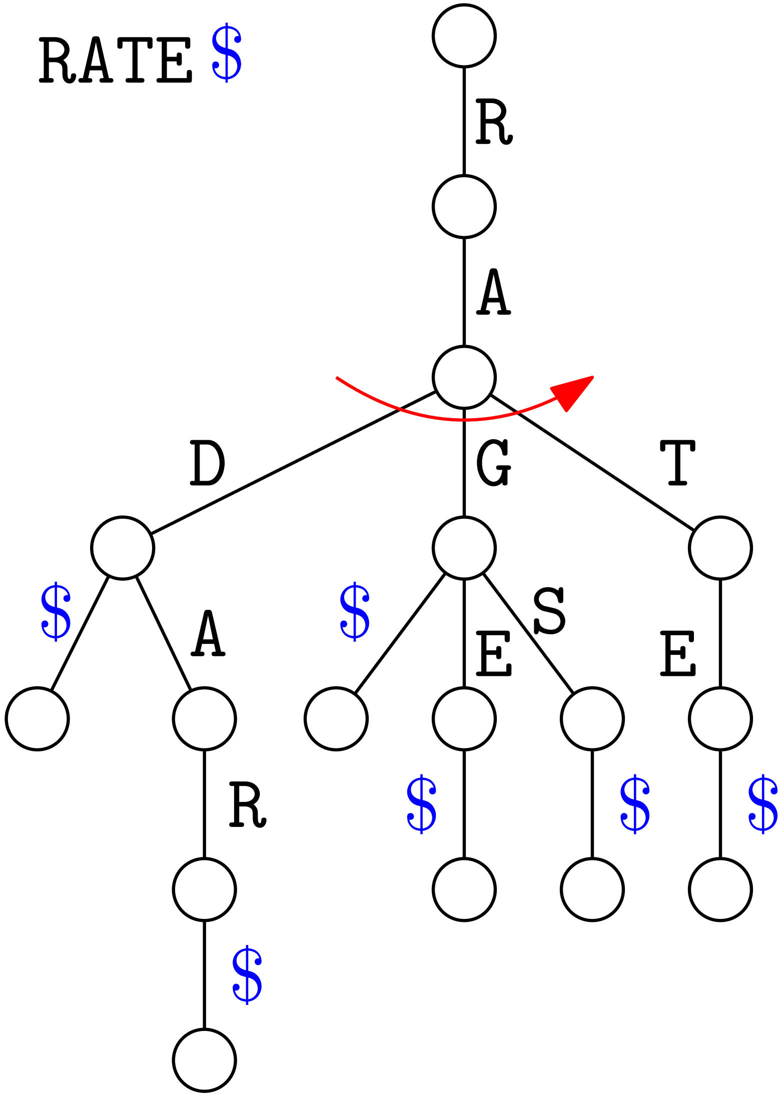
RAD$ RADAR$ RAG$ RAGE$ RAGS$ RATE$



Build a tree in which:

- Edges are labelled with a symbol in $\Sigma \cup \{\$\}$ and are sorted

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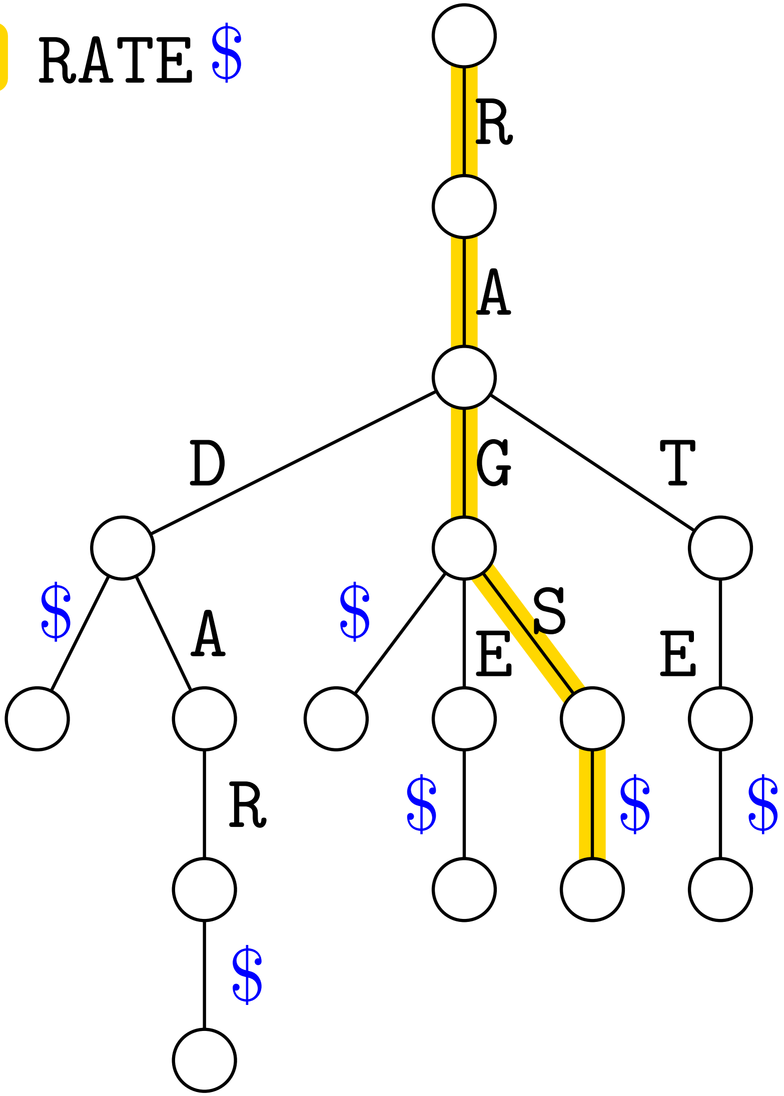

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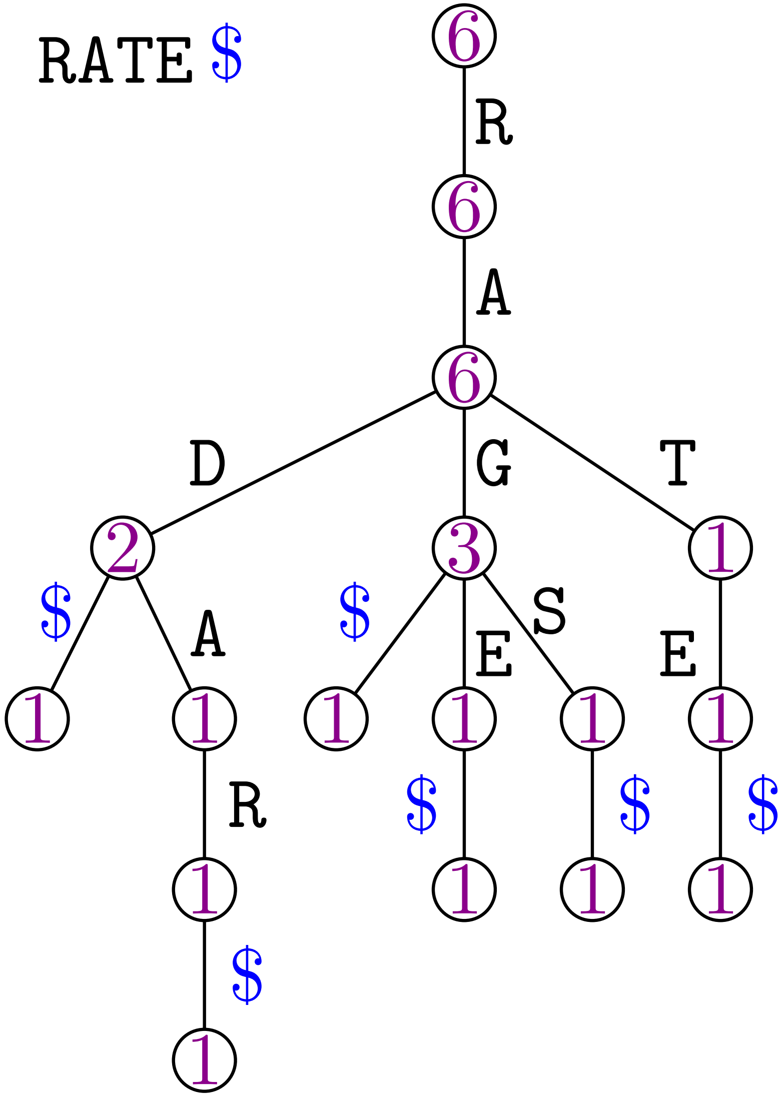
- Satellite data is often useful, e.g.:

— Number of $s in each subtree



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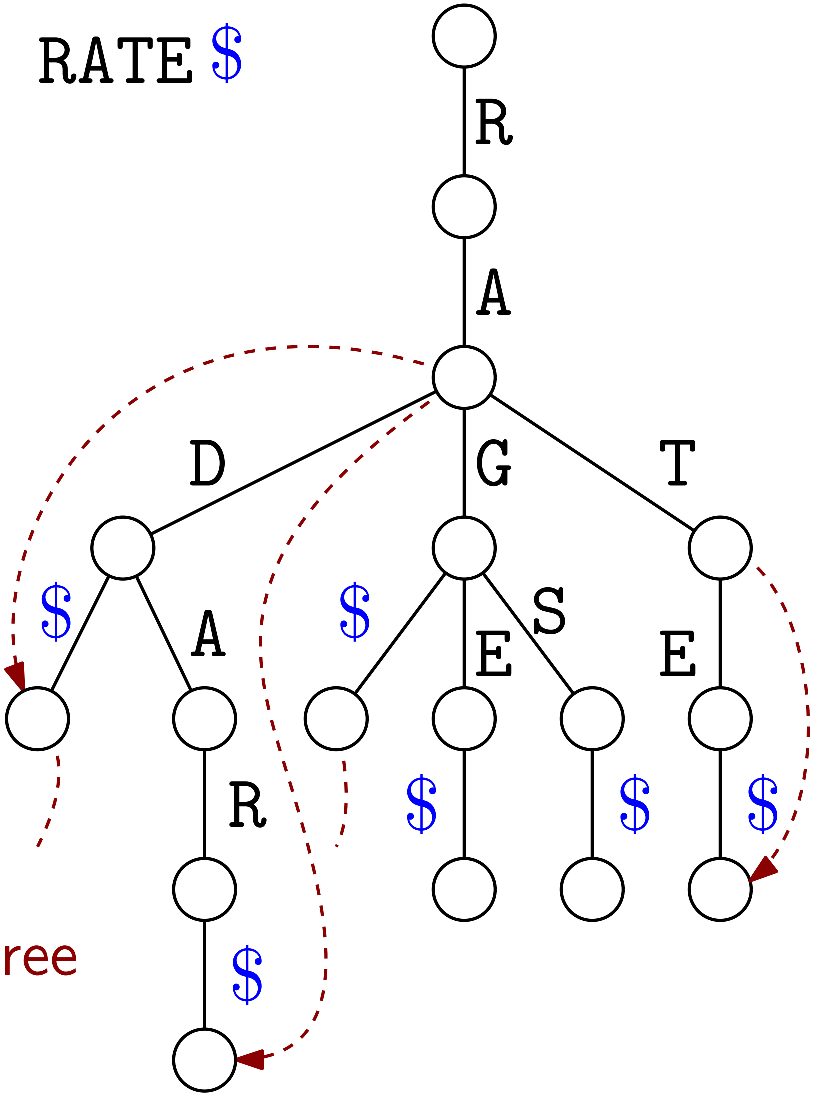
— Pointers to the first/last leaf in the subtree



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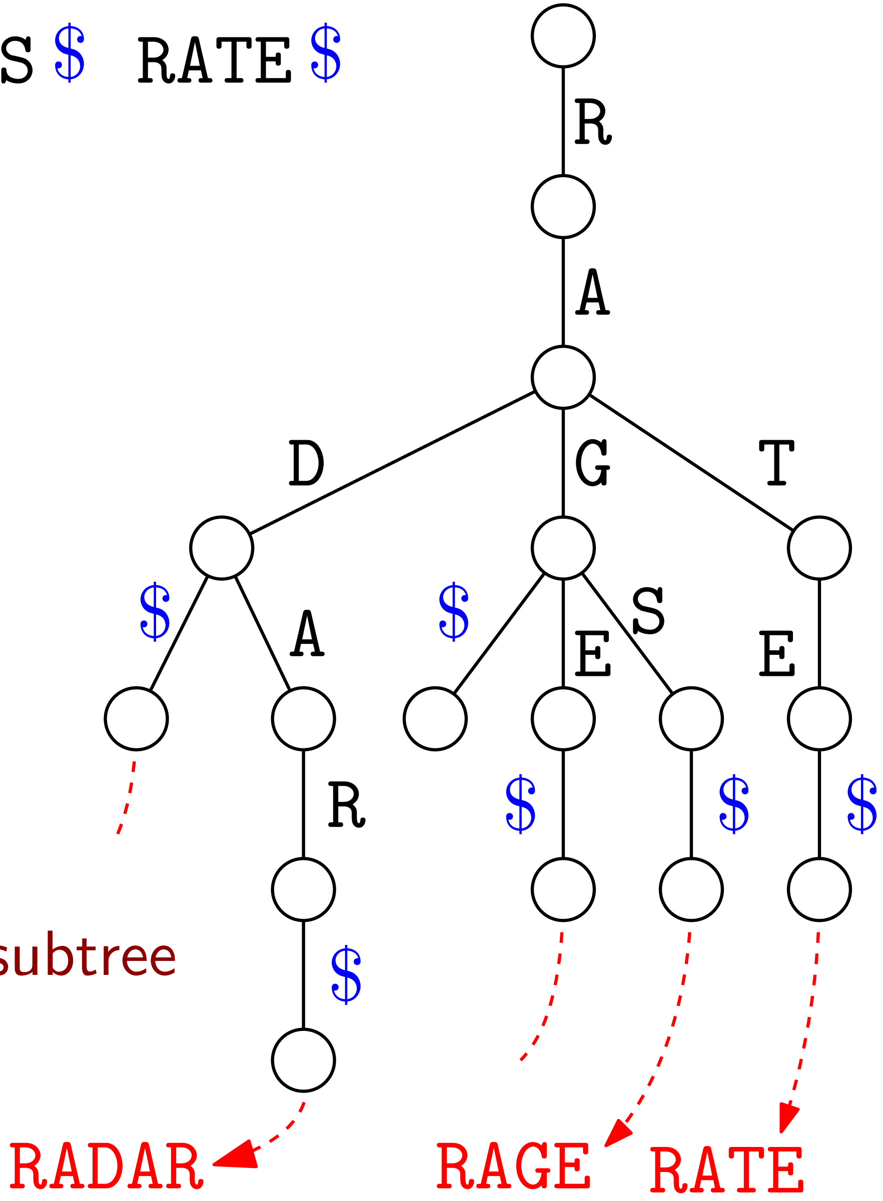
— Pointers to the first/last leaf in the subtree

— Pointers from leaves to strings



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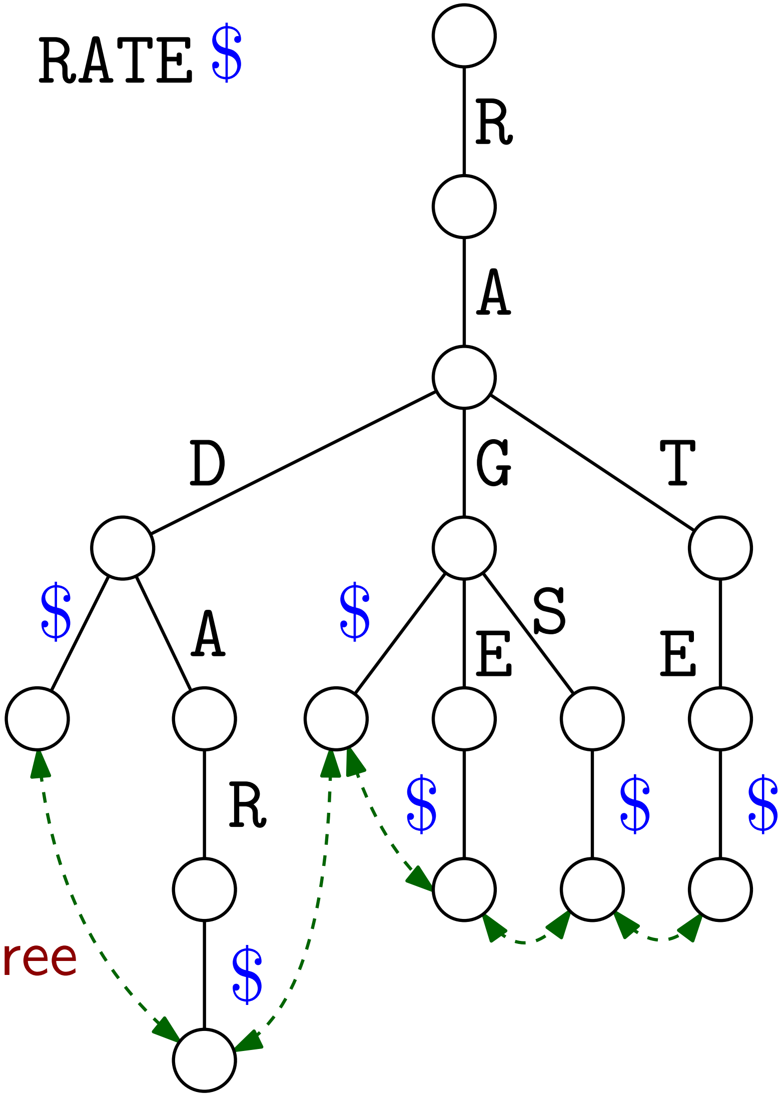
- Satellite data is often useful, e.g.:

— Number of $\$$s in each subtree

— Pointers to the first/last leaf in the subtree

— Pointers from leaves to strings

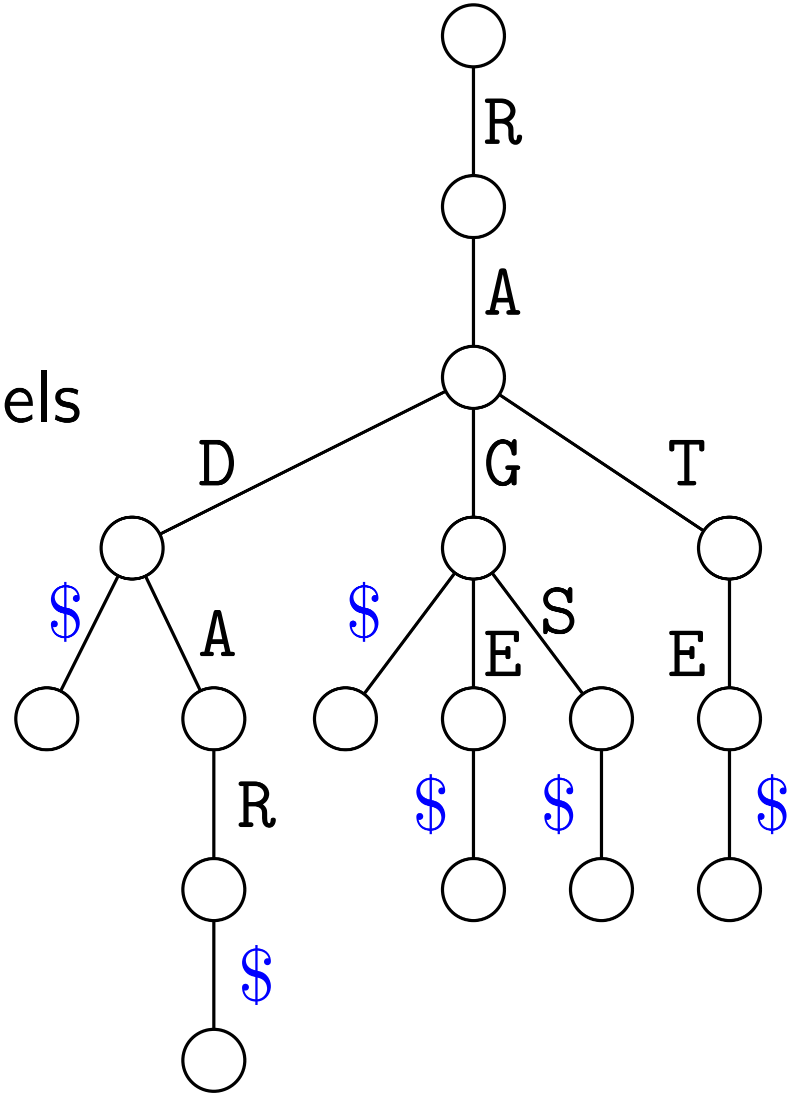
— Leaves arranged in a (doubly) linked list



Tries: Find (Sketch)

Find(P):

- Walk down the tree matching the characters in $P\$$ with the edge labels



Tries: Find (Sketch)

$P = \texttt{RADAR}$

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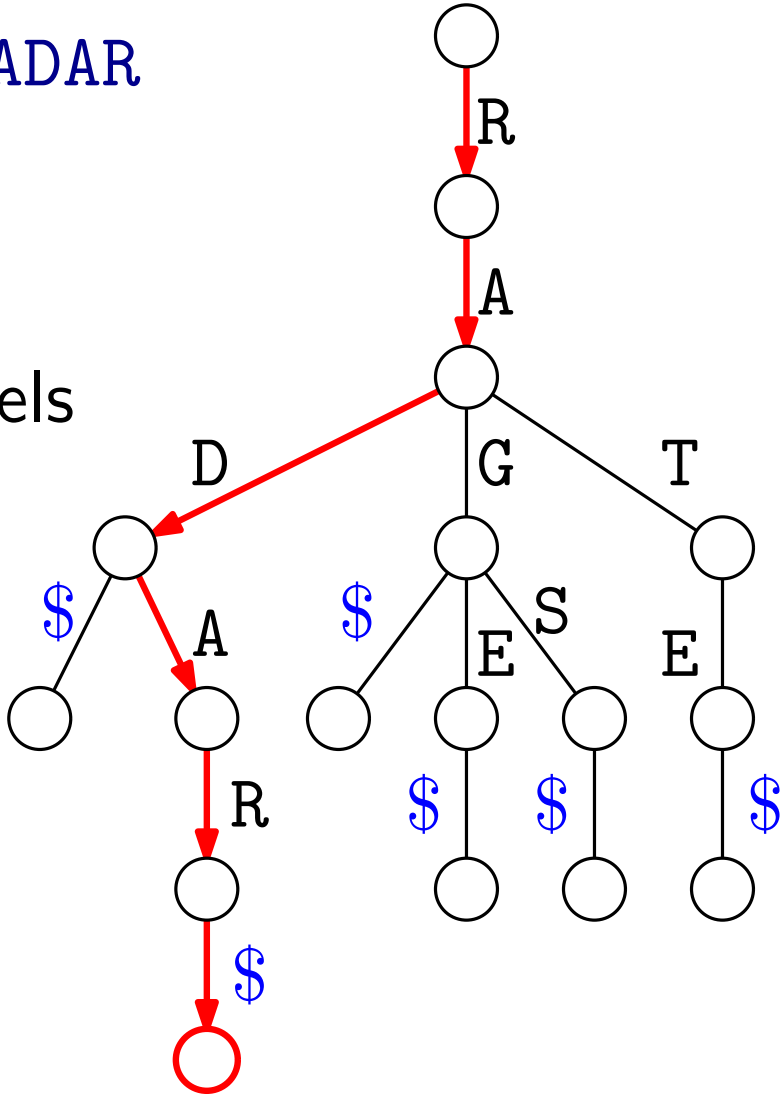

Tries: Find (Sketch)

$P = \texttt{RAG}$

Find(P):

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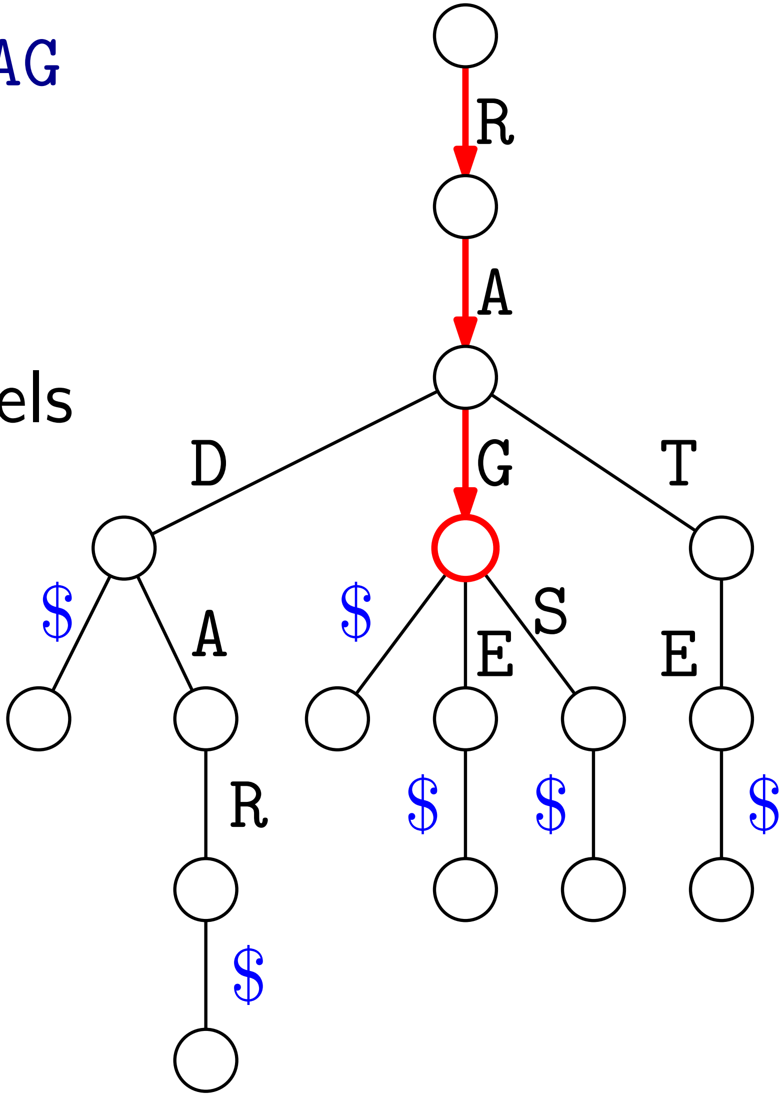
To count the number of strings that start with P:

- Find the node corresponding to P



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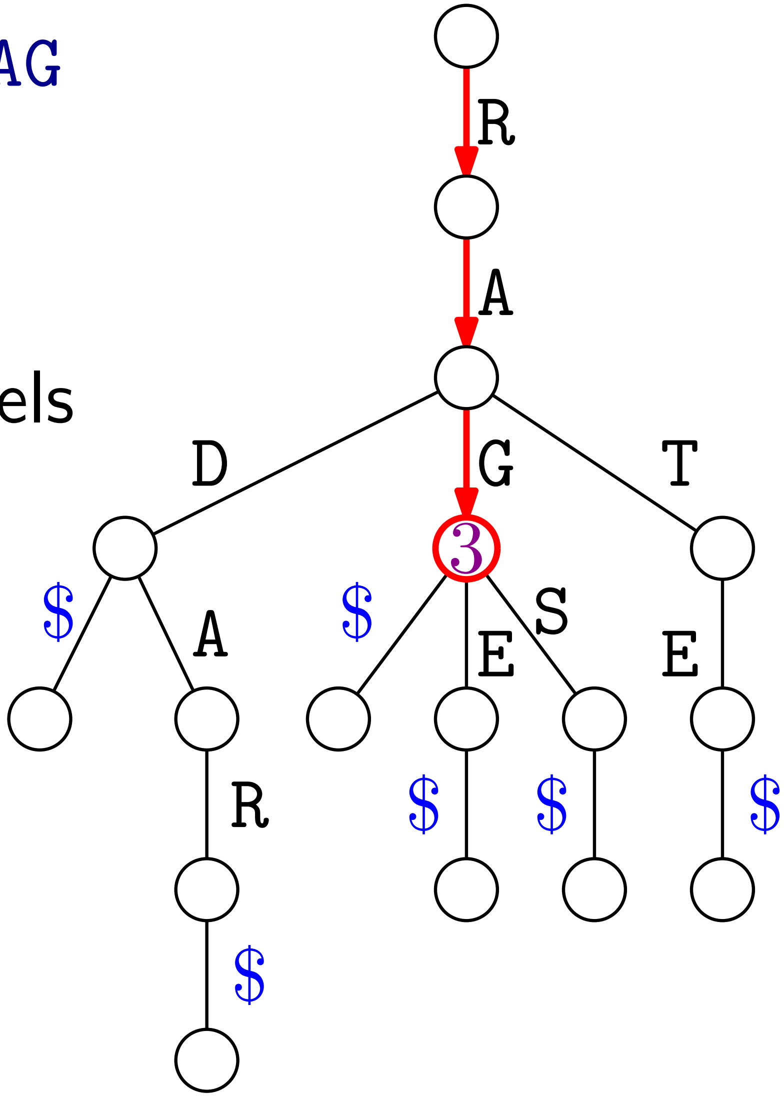

To count the number of strings that start with P:

- Find the node corresponding to P
- Return the number of \$s in the subtree (stored in the node)

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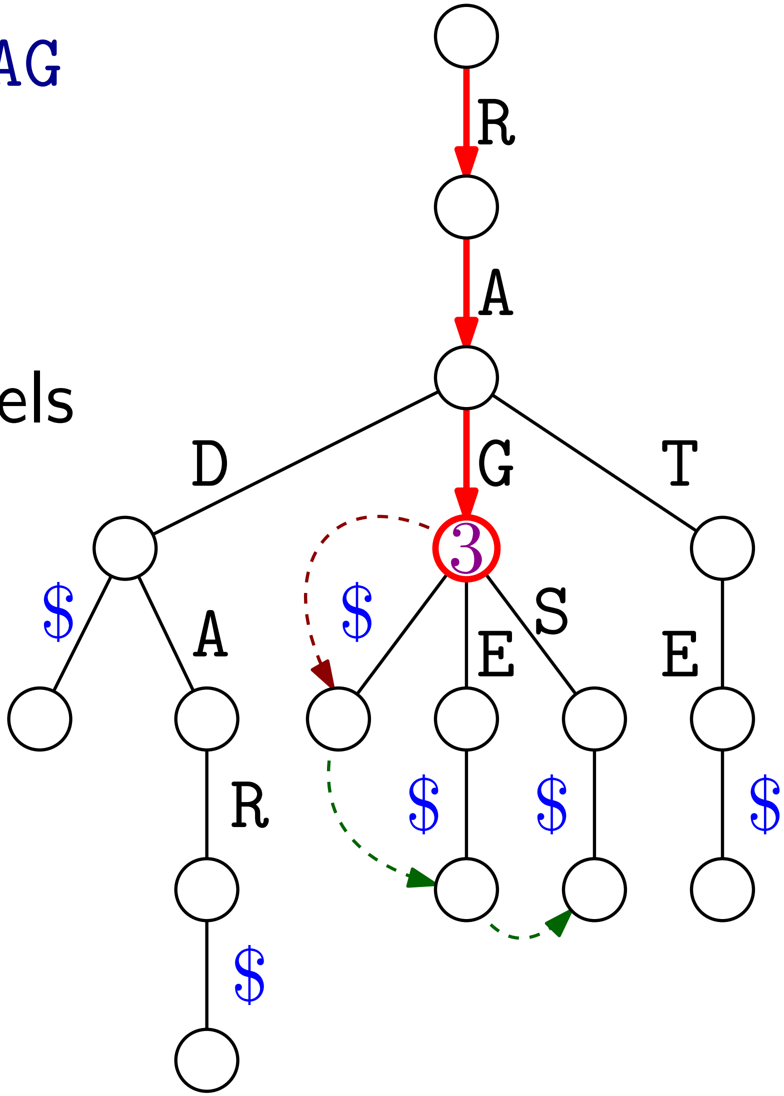

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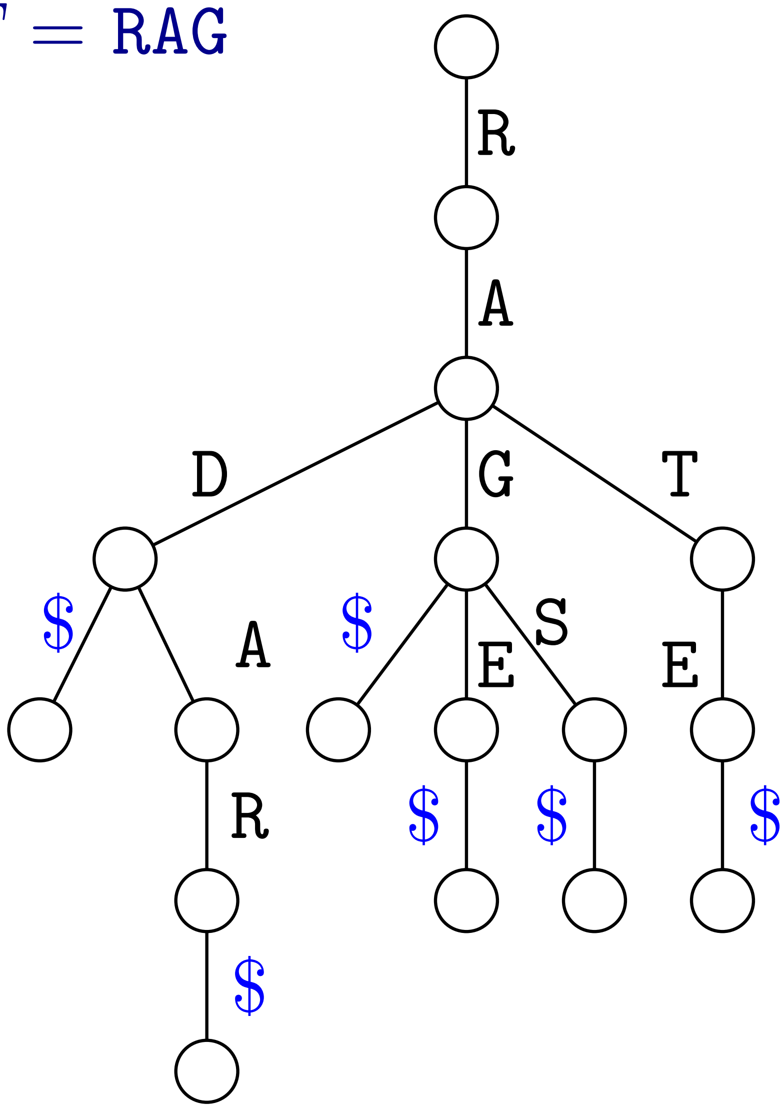
To count the number of strings that start with P:

- Find the node corresponding to P
- Return the number of $s in the subtree (stored in the node)
- The actual matches can be listed in $O(1)$ additional time per match by following pointers

Tries: Predecessor Queries (Sketch)

$T = \texttt{RAG}$



Predecessor(T):

- Walk down a path $\langle v_0, v_1, v_2 \ldots \rangle$ of the tree matching the characters in $T\$$ with the edge labels

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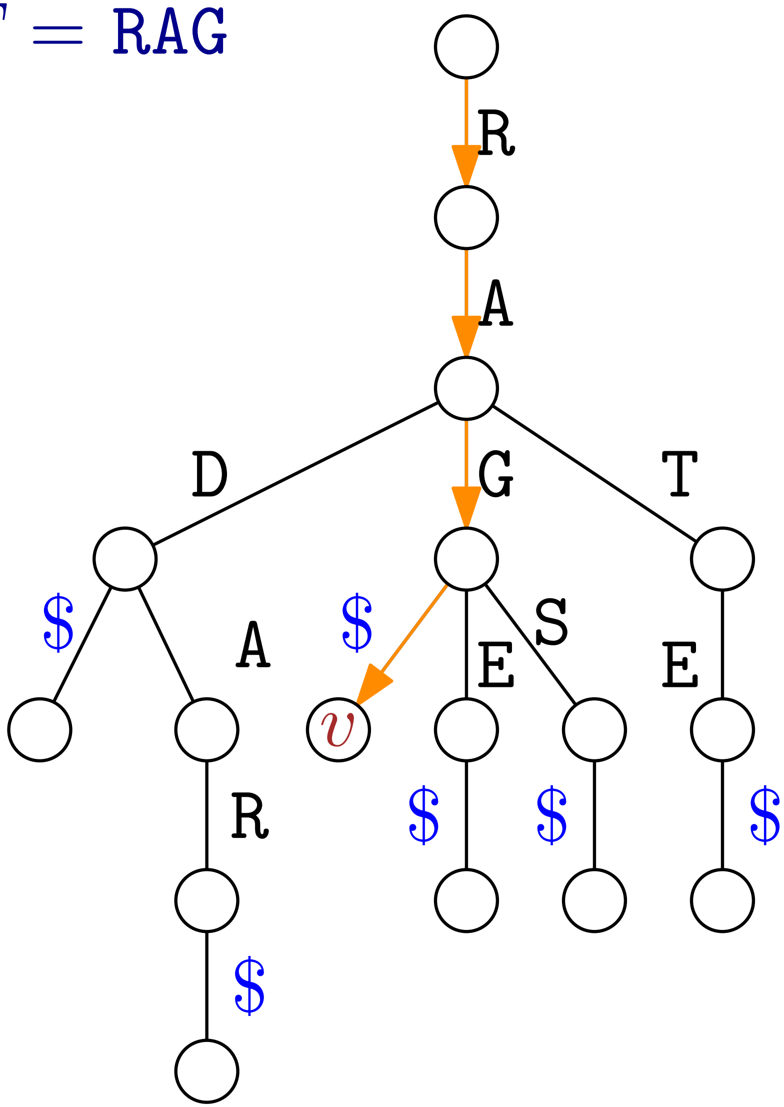

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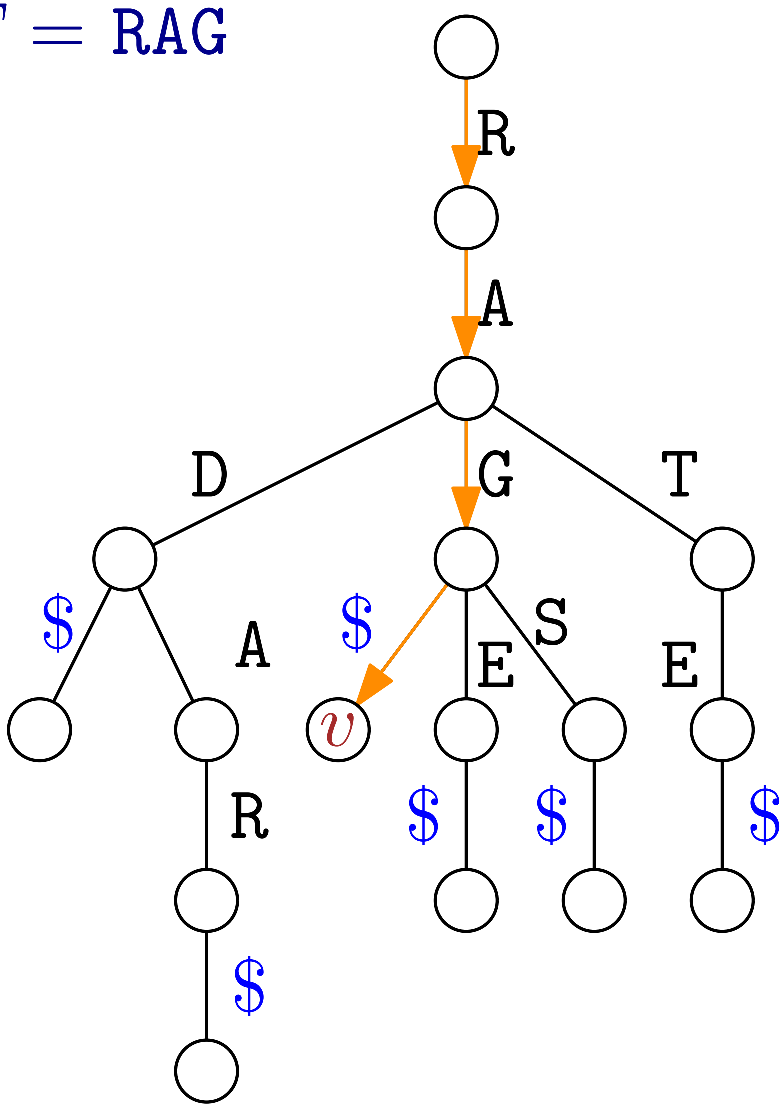

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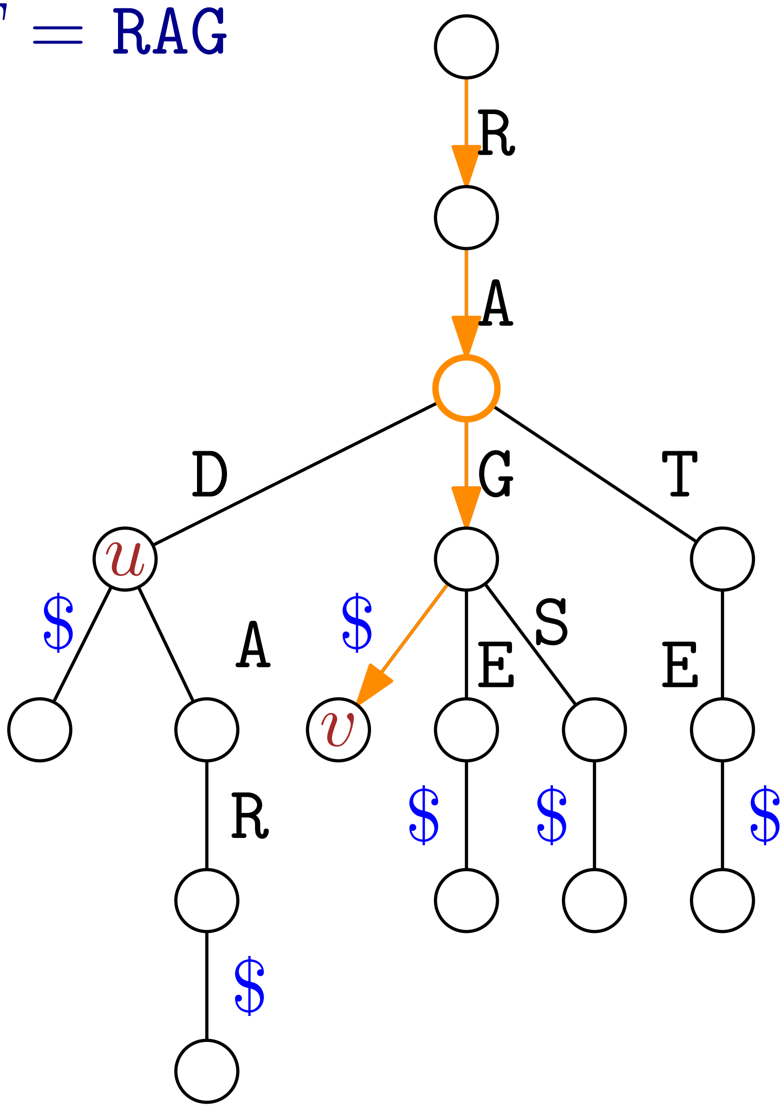

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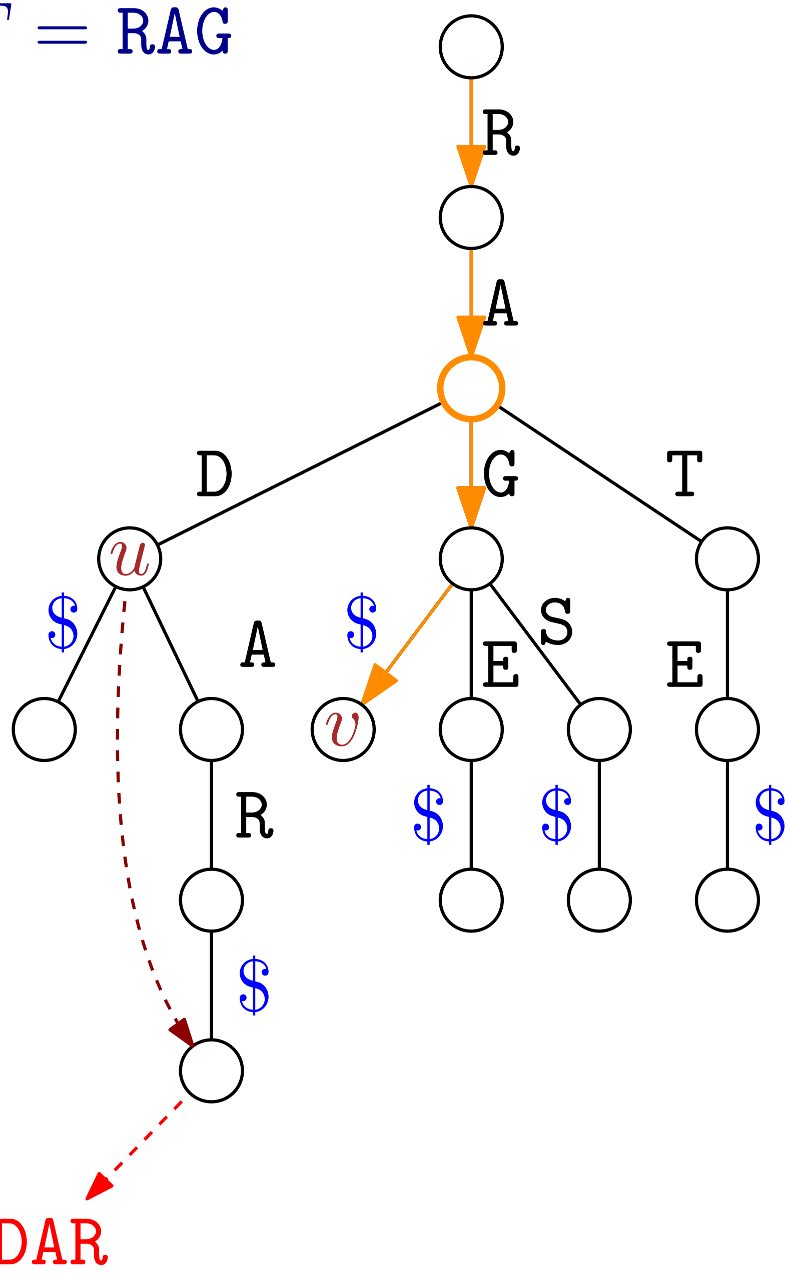

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$T\$ = T_1 T_2 T_3 \ldots$ $T = \texttt{RAG}$

The strict predecessor of $\sigma \in \Sigma$ w.r.t. a node v, if it exists, is the child u of v such that (v, u) has the largest label that is smaller than σ

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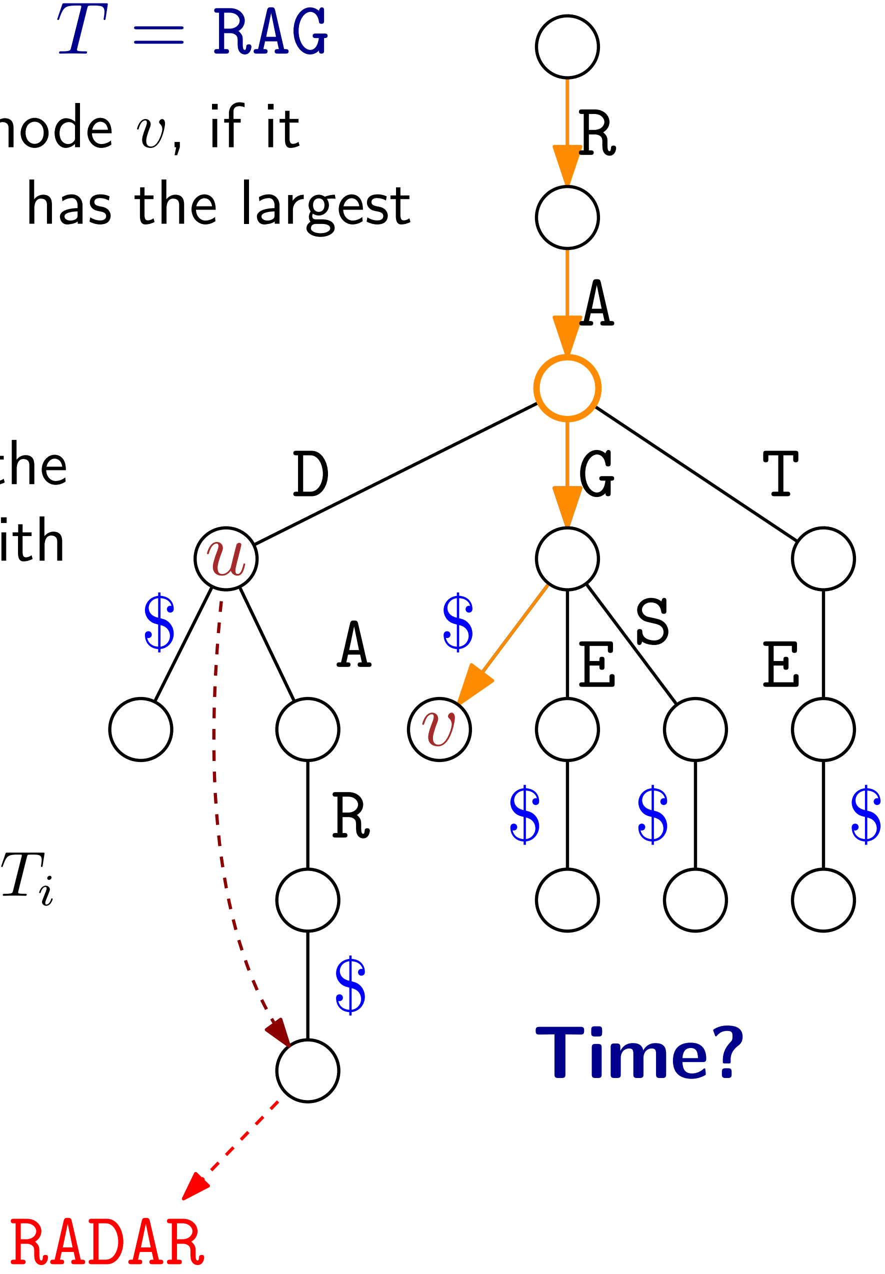

Time?

Tries: Predecessor Queries (Sketch)

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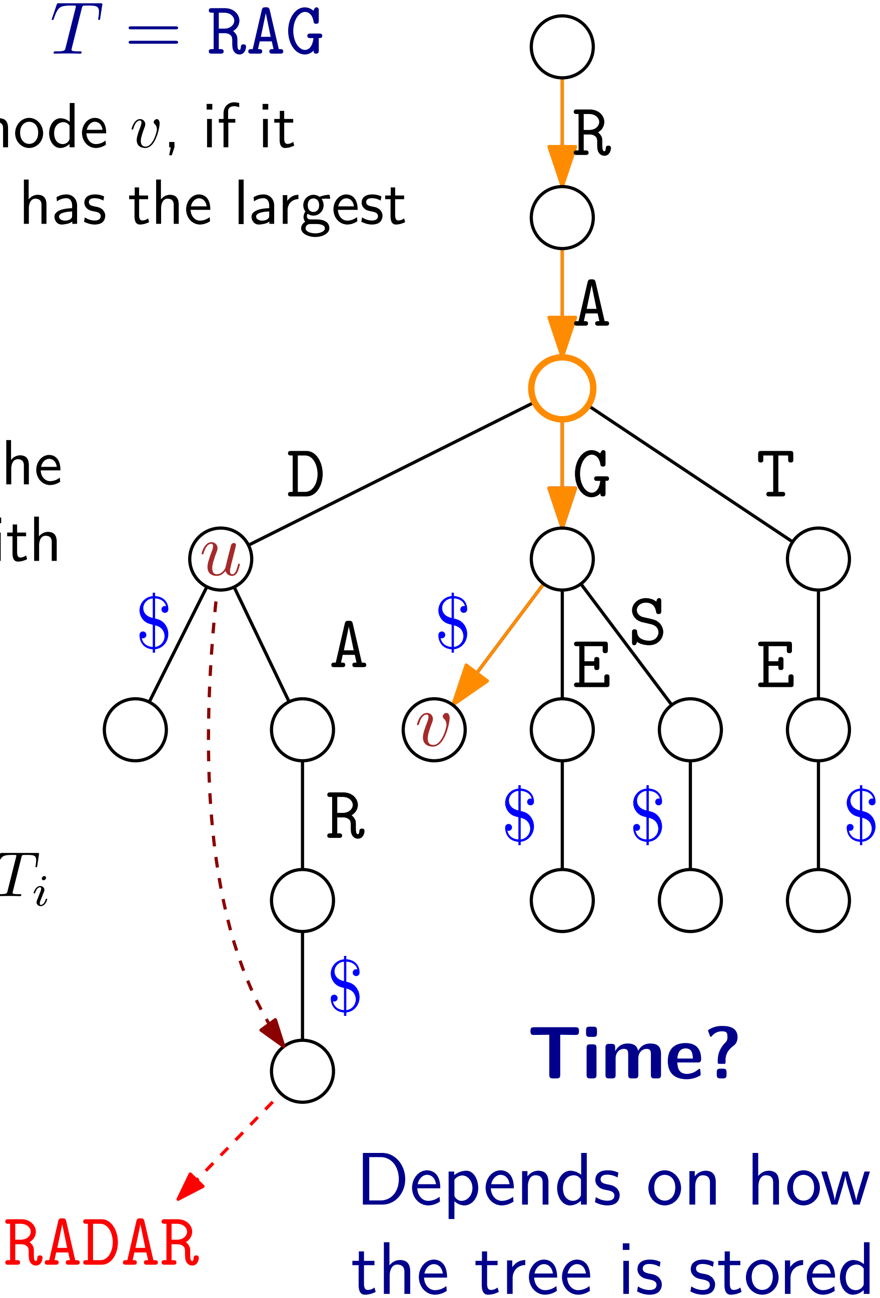
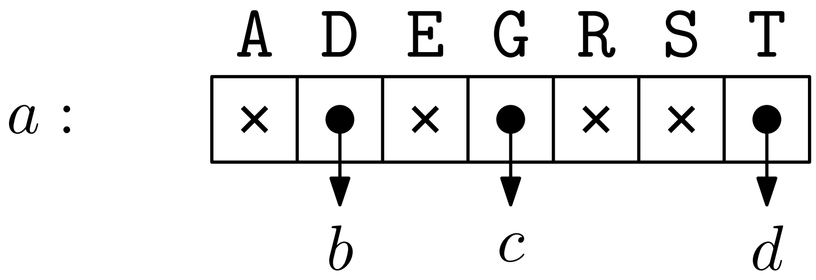


Time?

Depends on how the tree is stored

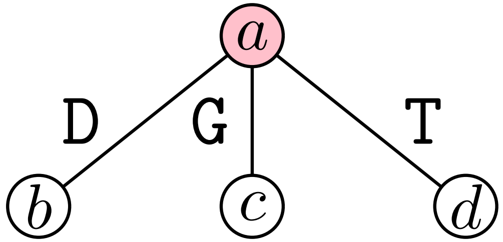
Representing Tries

Array (dense)



$n = \#\text{nodes} = O\left(\sum_i |T_i|\right)$

$\Sigma = \{\texttt{A}, \texttt{D}, \texttt{E}, \texttt{G}, \texttt{R}, \texttt{S}, \texttt{T}\}$



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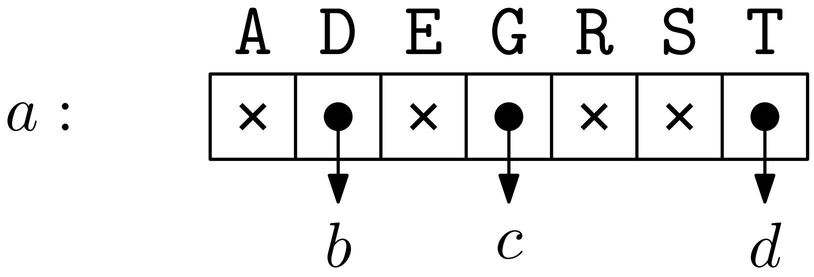

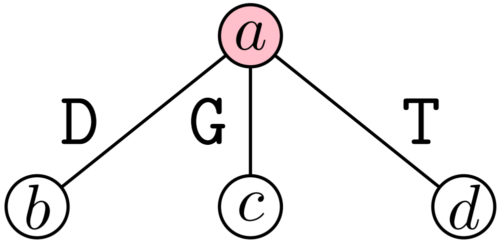

Space: $O(|\Sigma|)$

Time to find a symbol's edge: $O(1)$

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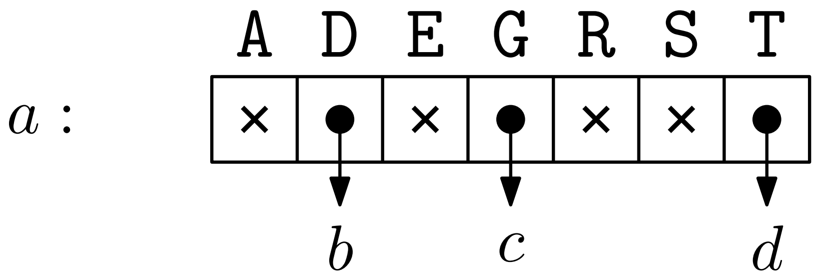

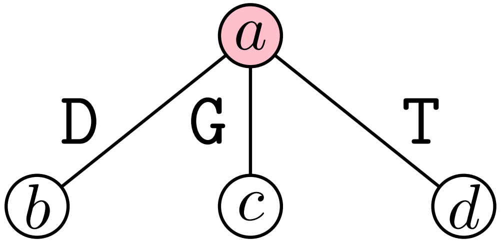

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Time to find predecessor: $O(|\Sigma|)$

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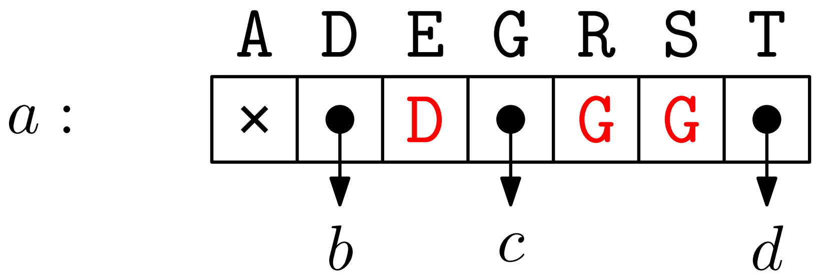

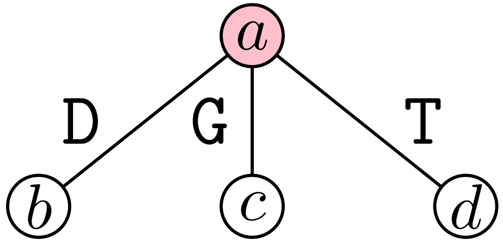

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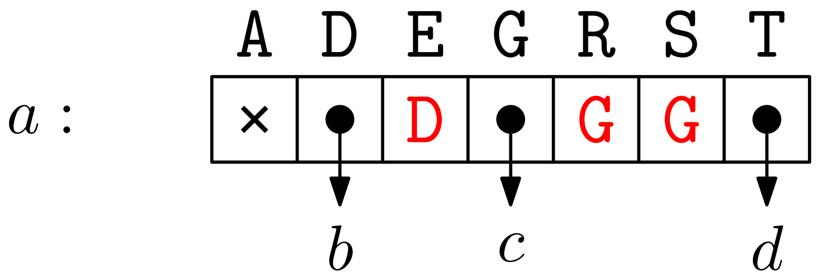
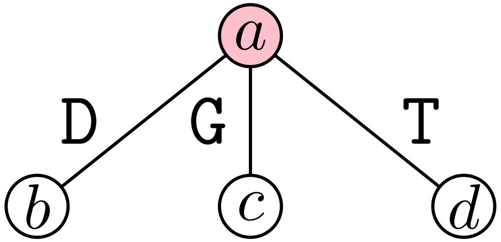




Space: $O(|\Sigma|)$

Time to find a symbol's edge: $O(1)$

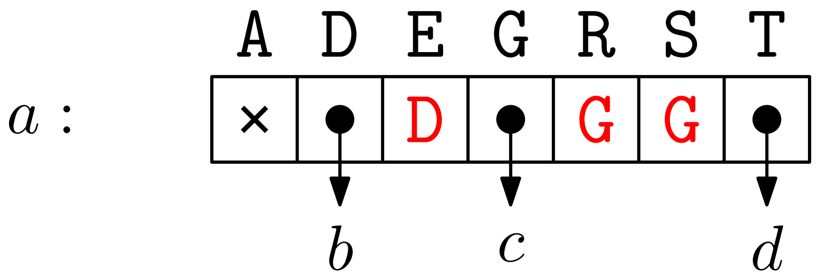
Time to find predecessor: ~~$O(|\Sigma|)$~~ $O(1)$

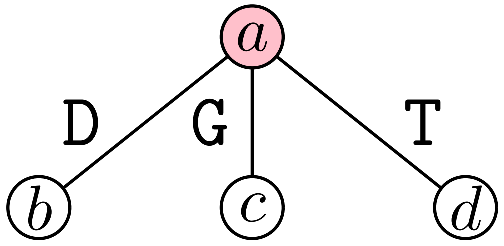
Representing Tries

Array (dense)

$n = \#\text{nodes} = O\left(\sum_i |T_i|\right)$

$\Sigma = \{\mathtt{A}, \mathtt{D}, \mathtt{E}, \mathtt{G}, \mathtt{R}, \mathtt{S}, \mathtt{T}\}$





Space: $O(|\Sigma|)$

Time to find a symbol's edge: $O(1)$

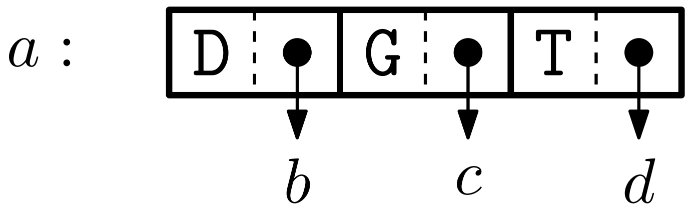
Time to find predecessor: ~~$O(|\Sigma|)$~~ $O(1)$

Overall space: $O(|\Sigma| \cdot n)$

Overall time: $O(|P|)$

Representing Tries

Array (sparse)



$n = \#\text{nodes} = O\left(\sum_i |T_i|\right)$

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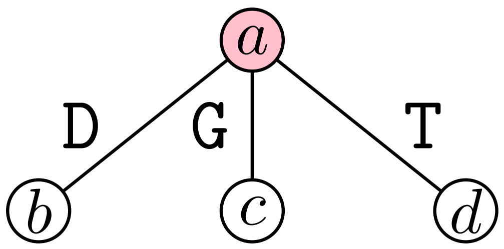
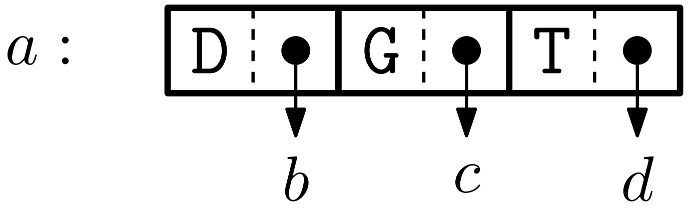
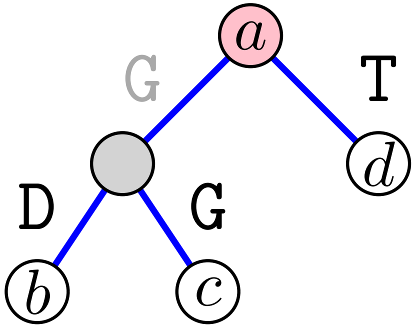


Representing Tries

Array (sparse)



Balanced Binary Search Tree



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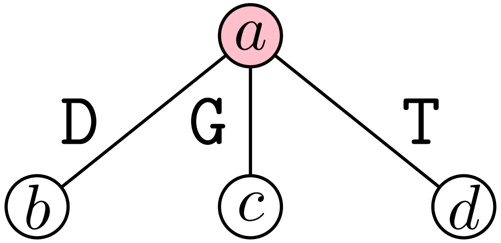
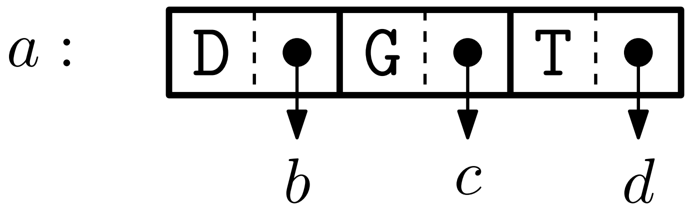
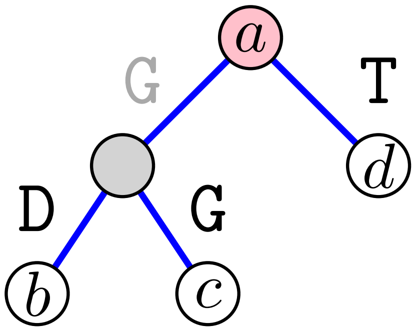


Representing Tries

Array (sparse)



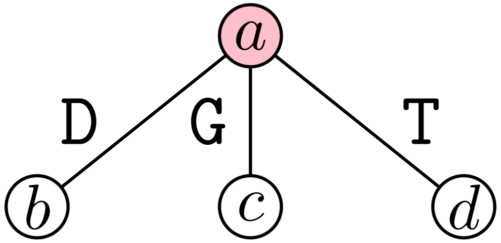
Balanced Binary Search Tree



Space: $O(\#\text{children})$

$n = \#\text{nodes} = O\left(\sum_i |T_i|\right)$

$\Sigma = \{\mathtt{A}, \mathtt{D}, \mathtt{E}, \mathtt{G}, \mathtt{R}, \mathtt{S}, \mathtt{T}\}$



Representing Tries

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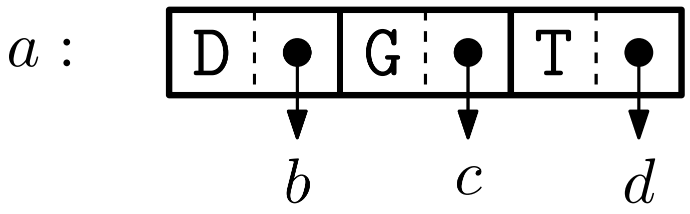

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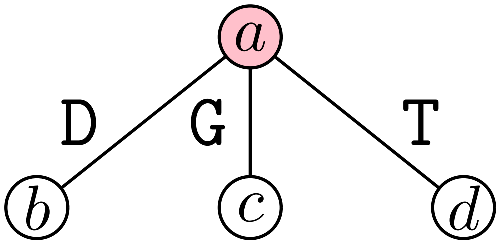
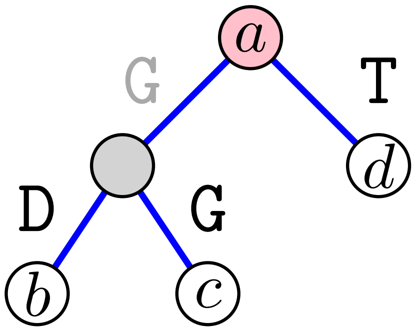


Balanced Binary Search Tree



Space: $O(\#\text{children})$

Time to find a symbol's edge/predecessor:
$O(\log \#\text{children}) = O(\log |\Sigma|)$

Representing Tries

$n = \#\text{nodes} = O\left(\sum_i |T_i|\right)$

$\Sigma = \{\mathtt{A}, \mathtt{D}, \mathtt{E}, \mathtt{G}, \mathtt{R}, \mathtt{S}, \mathtt{T}\}$

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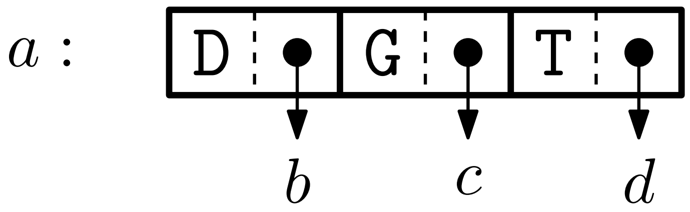
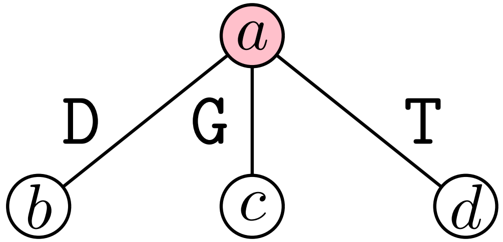
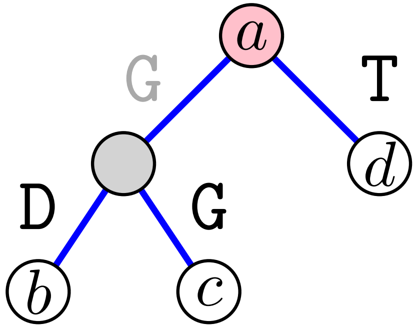




Balanced Binary Search Tree



Space: $O(\#\text{children})$

Time to find a symbol's edge/predecessor:
$O(\log \#\text{children}) = O(\log |\Sigma|)$

Overall space: $O(n)$

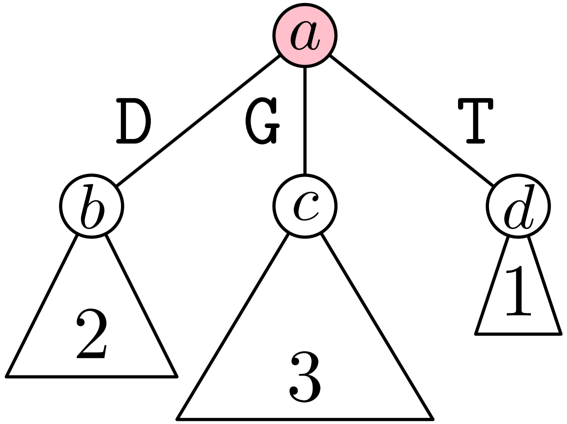
Overall time: $O(|P| \log |\Sigma|)$

Representing Tries

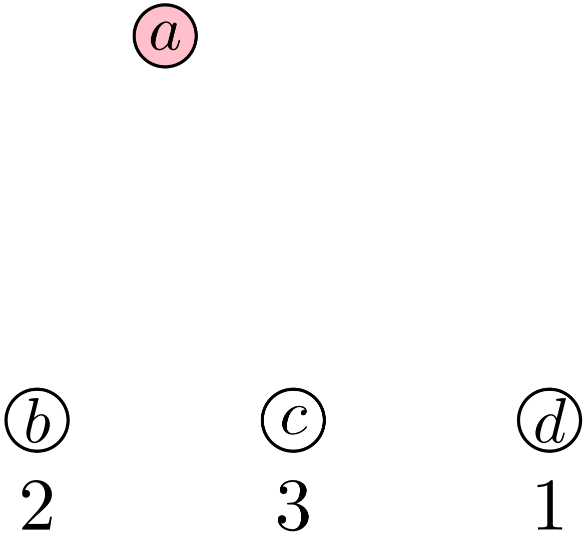
Weight-Balanced BSTs

$n = \#nodes = O\left(\sum_i |T_i|\right)$

Each vertex of the trie has a weight equal to the number of leaves in its subtree



Recursively construct a binary search tree by splitting the children in the trie so that the sum of their weights is as balanced as possible



Representing Tries

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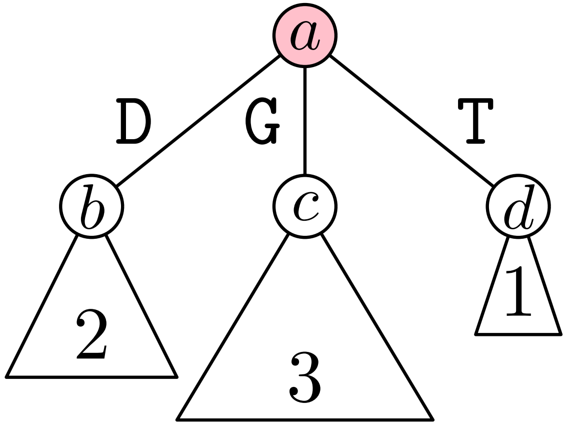

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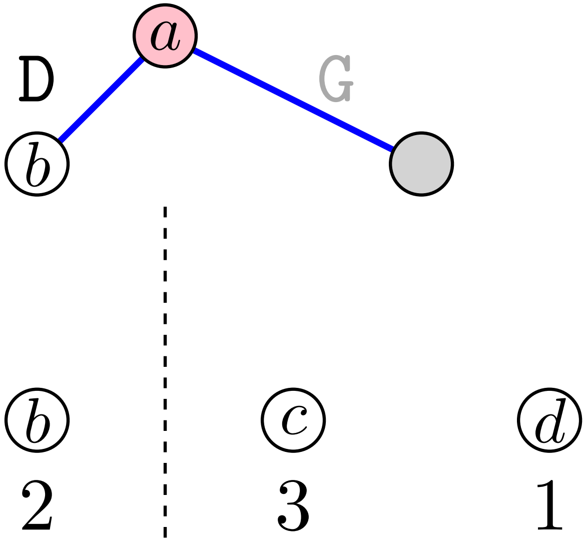

Representing Tries

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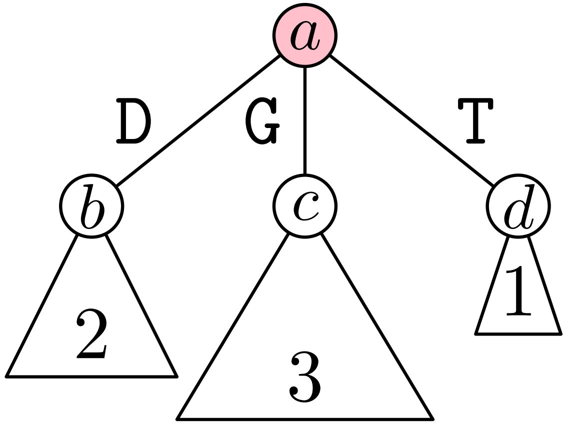
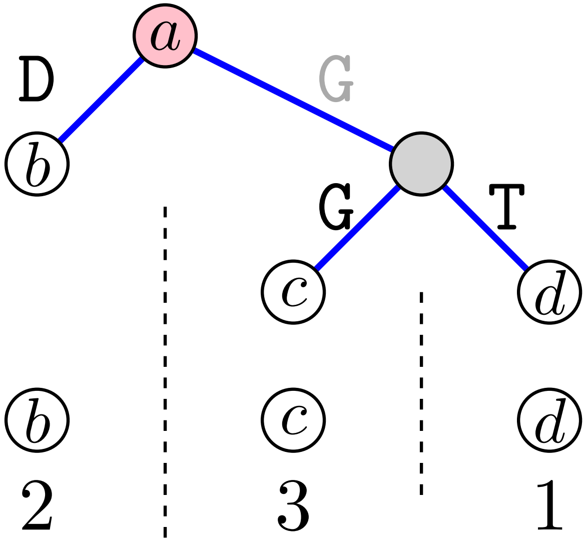


Recursively construct a binary search tree by splitting the children in the trie so that the sum of their weights is as balanced as possible



Representing Tries

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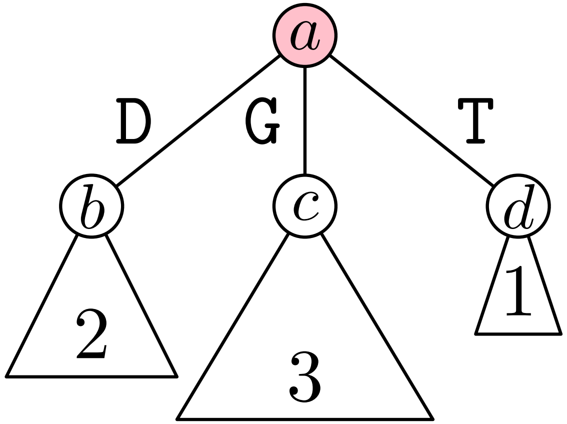

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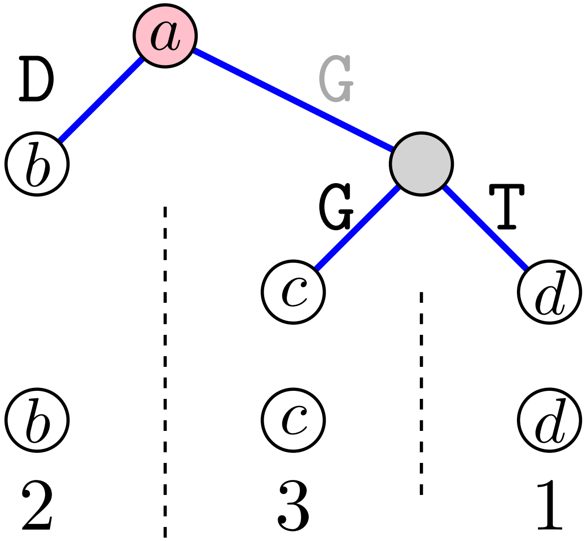


Space: $O(\#\text{children})$

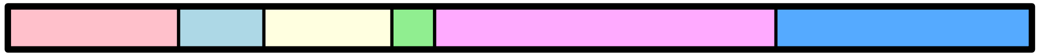
Overall space: $O(n)$

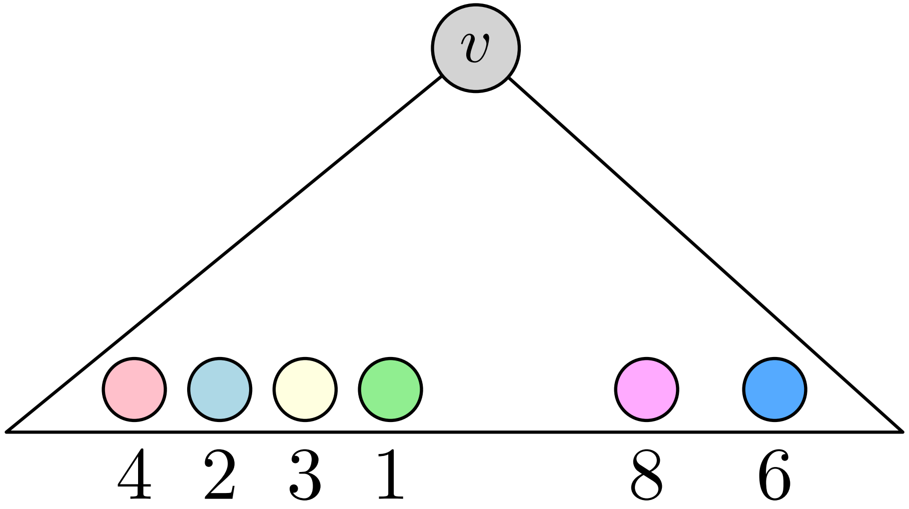
Representing Tries

Weight-Balanced BSTs

Claim: All the grand-children u of v satisfy $w(u) \leq \frac{2}{3}(v)$ or are leaves.

Imagine the leaves in the subtree of v as consecutive segments with length equal to their weight





Representing Tries

Weight-Balanced BSTs

Claim: All the grand-children u of v satisfy $w(u) \leq \frac{2}{3}(v)$ or are leaves.

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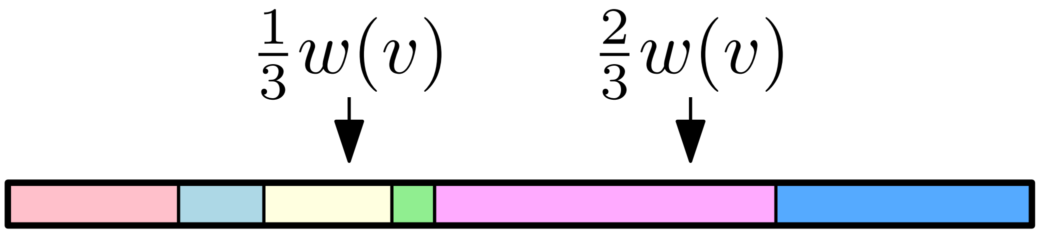
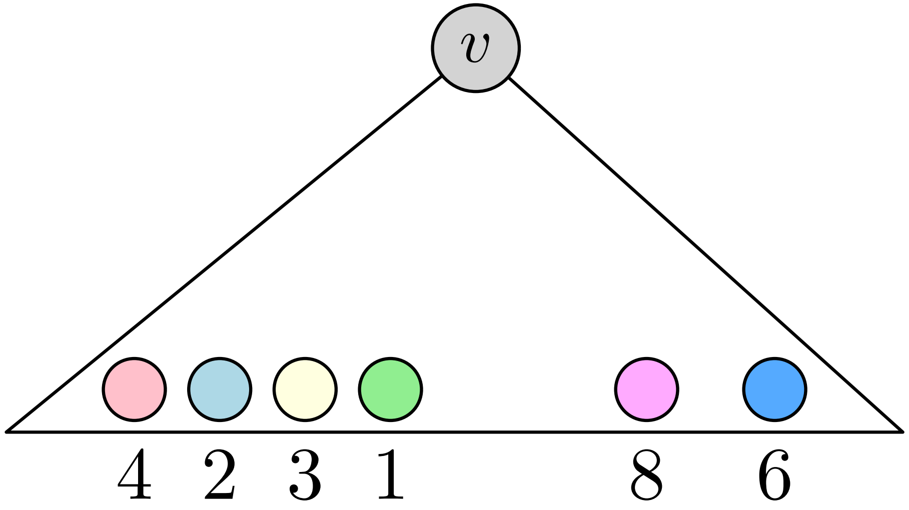
If the interval $[\frac{1}{3}w(v), \frac{2}{3}w(v)]$ contains more than one segment:





Representing Tries

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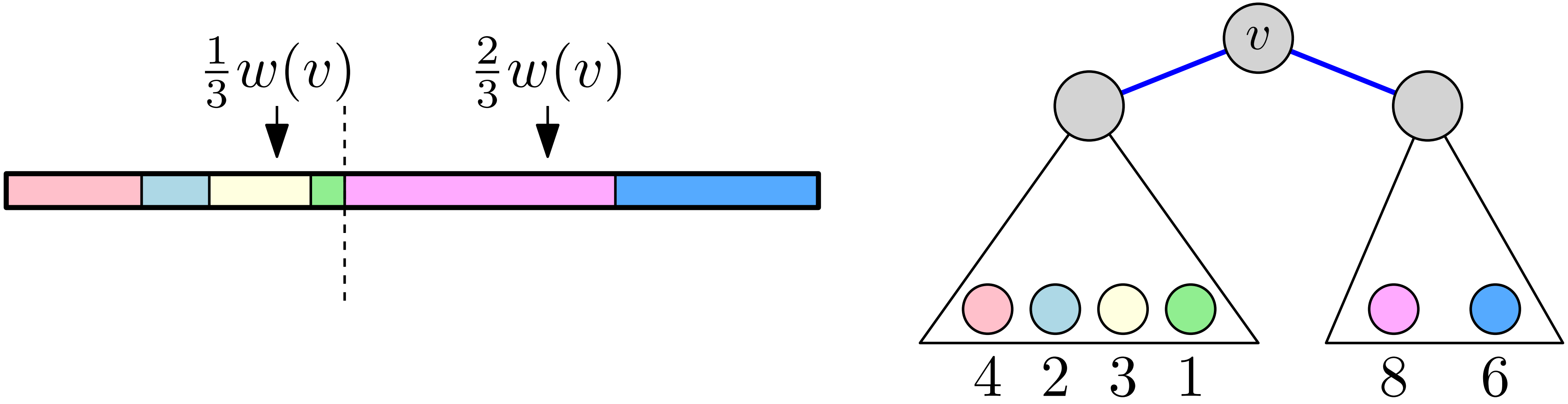

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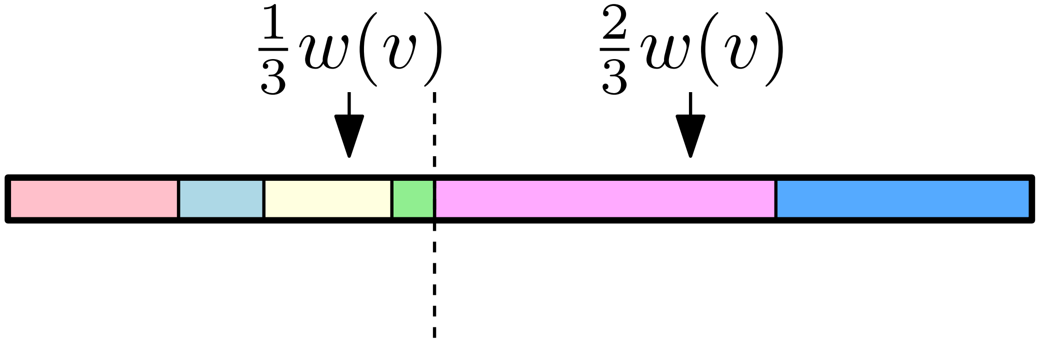
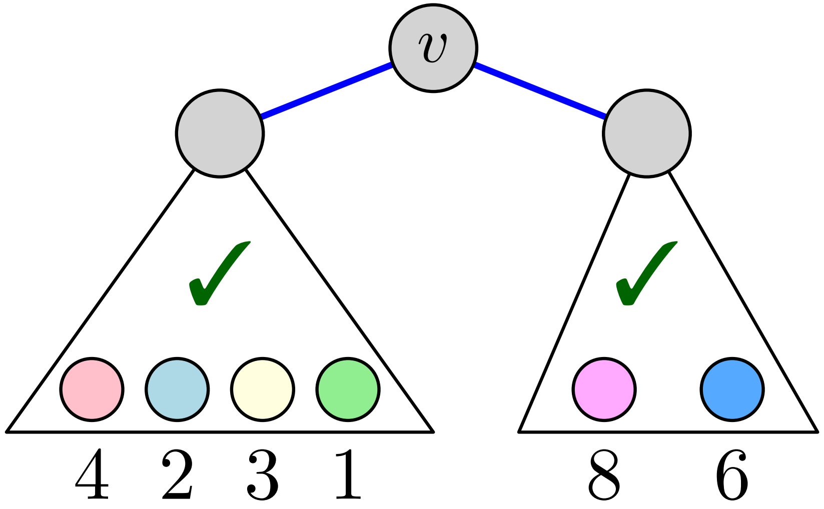




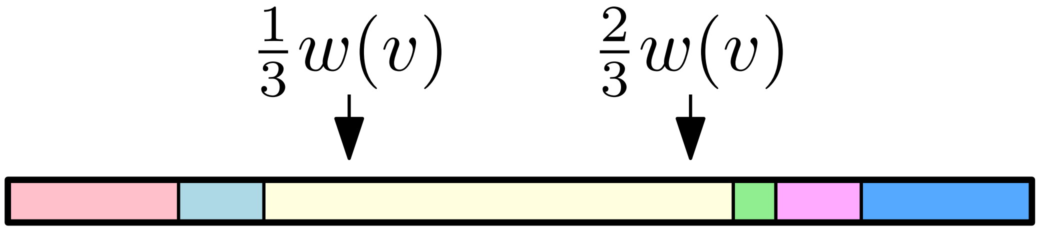
- the weight of each children of v is at most $\frac{2}{3}w(v)$

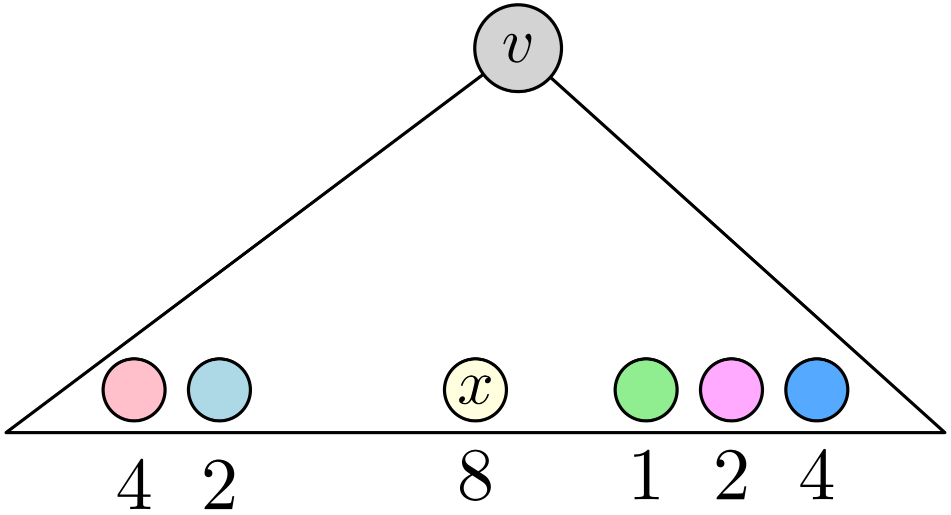
Representing Tries

Weight-Balanced BSTs

Claim: All the grand-children u of v satisfy $w(u) \leq \frac{2}{3}(v)$ or are leaves.

If the interval $[\frac{1}{3}w(v), \frac{2}{3}w(v)]$ contains a single segment, let x be the corresponding leaf





Representing Tries

Weight-Balanced BSTs

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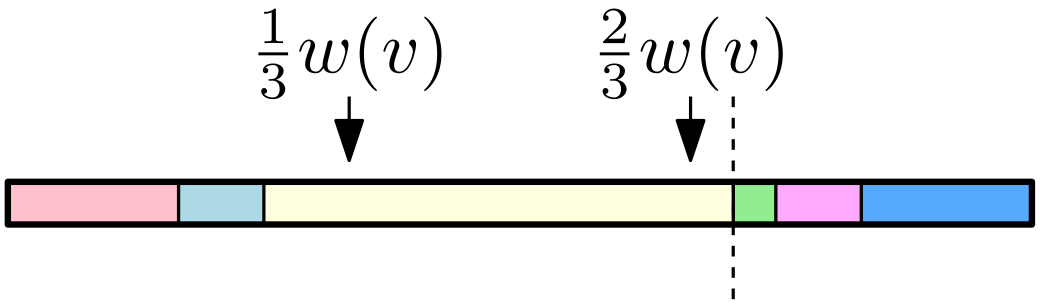
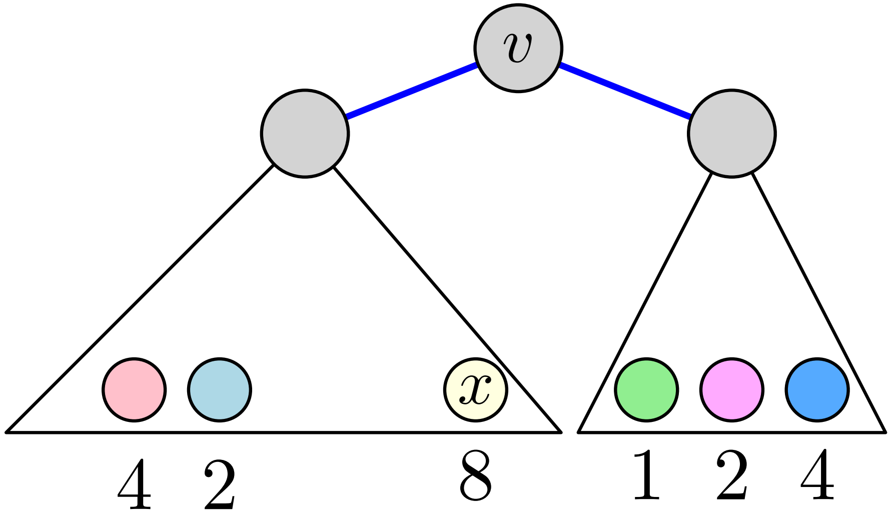


- v splits the segments immediately before/after x.



Representing Tries

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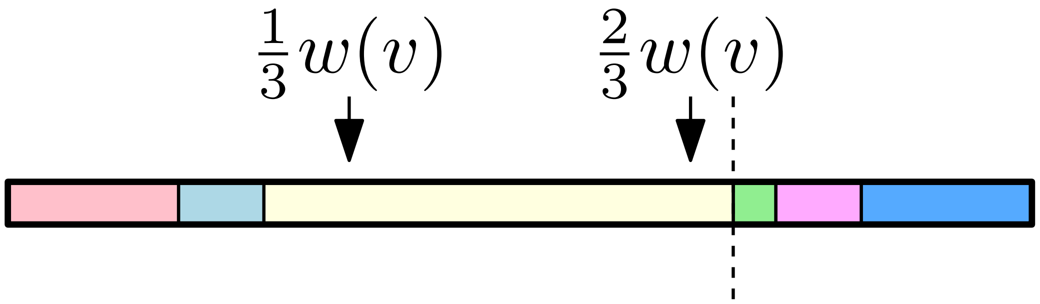

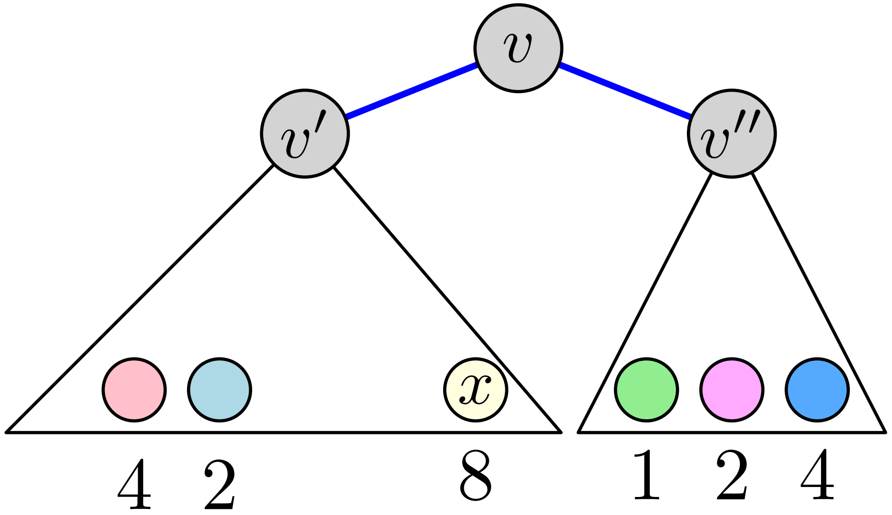

- v splits the segments immediately before/after x.
- Let v' be the child of v that contains x and let v'' be the other child

Representing Tries

Weight-Balanced BSTs

Claim: All the grand-children u of v satisfy $w(u) \leq \frac{2}{3}(v)$ or are leaves.

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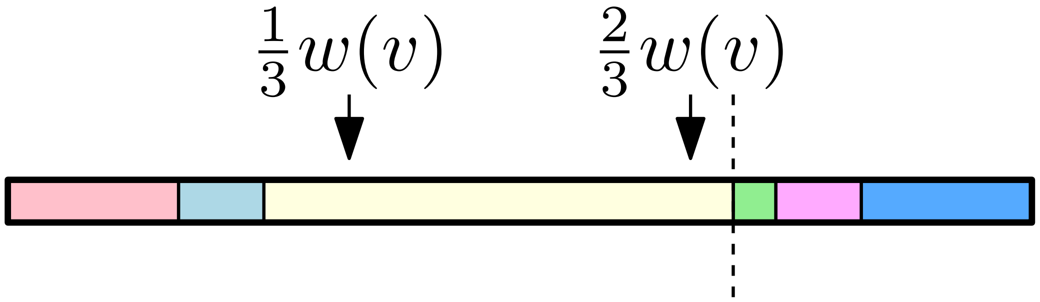

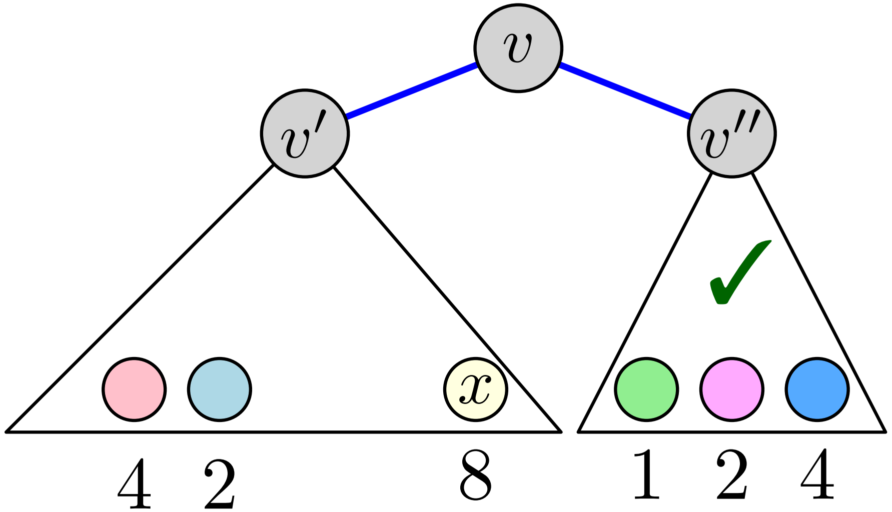

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- $w(v'') \leq \frac{1}{3}w(v)$.

Representing Tries

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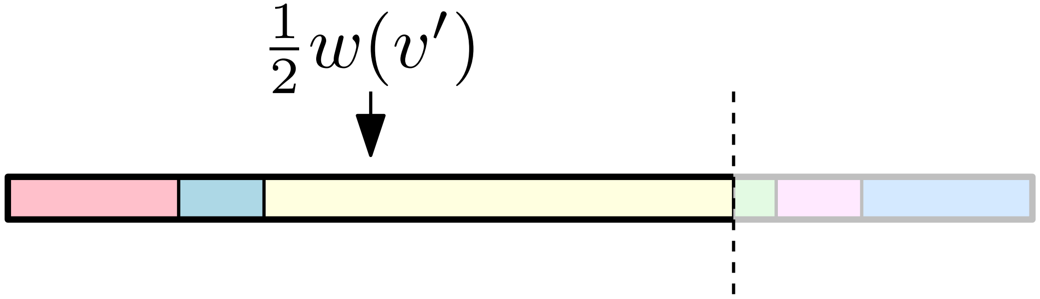

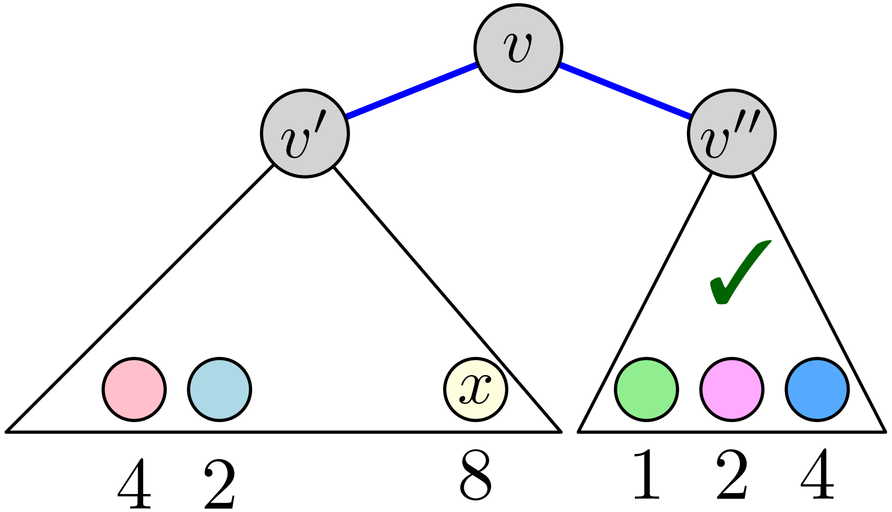

- v splits the segments immediately before/after x.
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- $w(v'') \leq \frac{1}{3}w(v)$.
- x is the first or last leaf in the subtree of v' and $w(x) \geq \frac{1}{2}w(v')$

Representing Tries

Weight-Balanced BSTs

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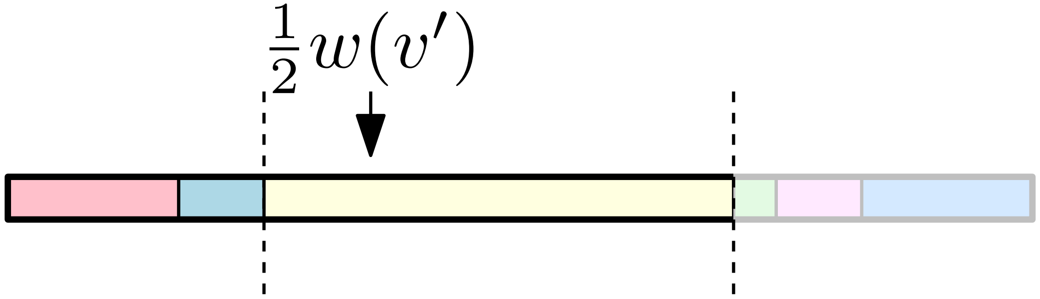

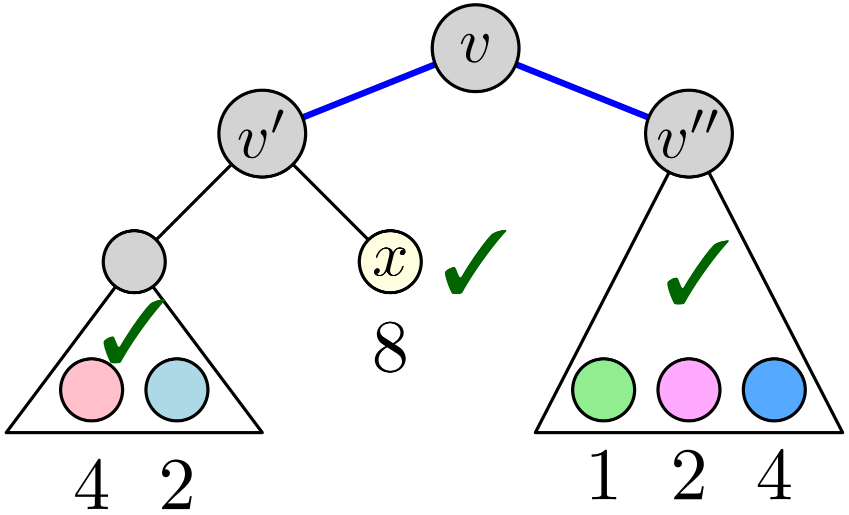

- v splits the segments immediately before/after x.
- Let v' be the child of v that contains x and let v'' be the other child
- $w(v'') \leq \frac{1}{3}w(v)$.
- x is the first or last leaf in the subtree of v' and $w(x) \geq \frac{1}{2}w(v')$
- One child of v' is x and the other child weighs $\leq \frac{1}{2}w(v') \leq \frac{1}{2}w(v)$

Representing Tries

Weight-Balanced BSTs

Claim: All the grand-children u of v satisfy $w(u) \leq \frac{2}{3}(v)$ or are leaves.

Traversing two edges of a weight-balanced BST either:

- Brings us to the next node in the trie, i.e., we advance one character into P; or

  Can only happen $O(|P|)$ times

Representing Tries

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- Reduces the weight (i.e, the number of leaves in the trie reachable from the current node) by $2/3$

  Can only happen $O(\log_{3/2} \#\text{leaves}) = O(\log k)$ times

Representing Tries

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- Reduces the weight (i.e, the number of leaves in the trie reachable from the current node) by $2/3$

  Can only happen $O(\log_{3/2} \#\text{leaves}) = O(\log k)$ times

Overall space: $O(n)$

Overall time: $O(|P| + \log k)$

Representing Tries: Recap

	Space	**Query Time**
Array (dense)	$O(\lvert\Sigma\rvert \cdot n)$	$O(\lvert P\rvert)$
Array (sparse) / BST	$O(n)$	$O(\lvert P\rvert \log \lvert\Sigma\rvert)$
Weight-balanced BST	$O(n)$	$O(\lvert P\rvert + \log k)$

Representing Tries: Recap

	Space	**Query Time**
Array (dense)	$O(\lvert\Sigma\rvert \cdot n)$	$O(\lvert P\rvert)$
Array (sparse) / BST	$O(n)$	$O(\lvert P\rvert \log \lvert\Sigma\rvert)$
Weight-balanced BST	$O(\underline{n})$	$O(\underline{\lvert P\rvert} + \log k)$

Optimal

Representing Tries: Recap

	Space	**Query Time**
Array (dense)	$O(\lvert\Sigma\rvert \cdot n)$	$O(\lvert P\rvert)$
Array (sparse) / BST	$O(n)$	$O(\lvert P\rvert \log \lvert\Sigma\rvert)$
Weight-balanced BST	$O(\underline{n})$	$O(\underline{\lvert P\rvert} + \underline{\log k})$

Can we get rid of this term?

Optimal

Representing Tries: Recap

	Space	**Query Time**
Array (dense)	$O(\|\Sigma\| \cdot n)$	$O(\|P\|)$
Array (sparse) / BST	$O(n)$	$O(\|P\| \log \|\Sigma\|)$
Weight-balanced BST	$O(\underline{n})$	$O(\underline{\|P\|} + \underline{\log k})$

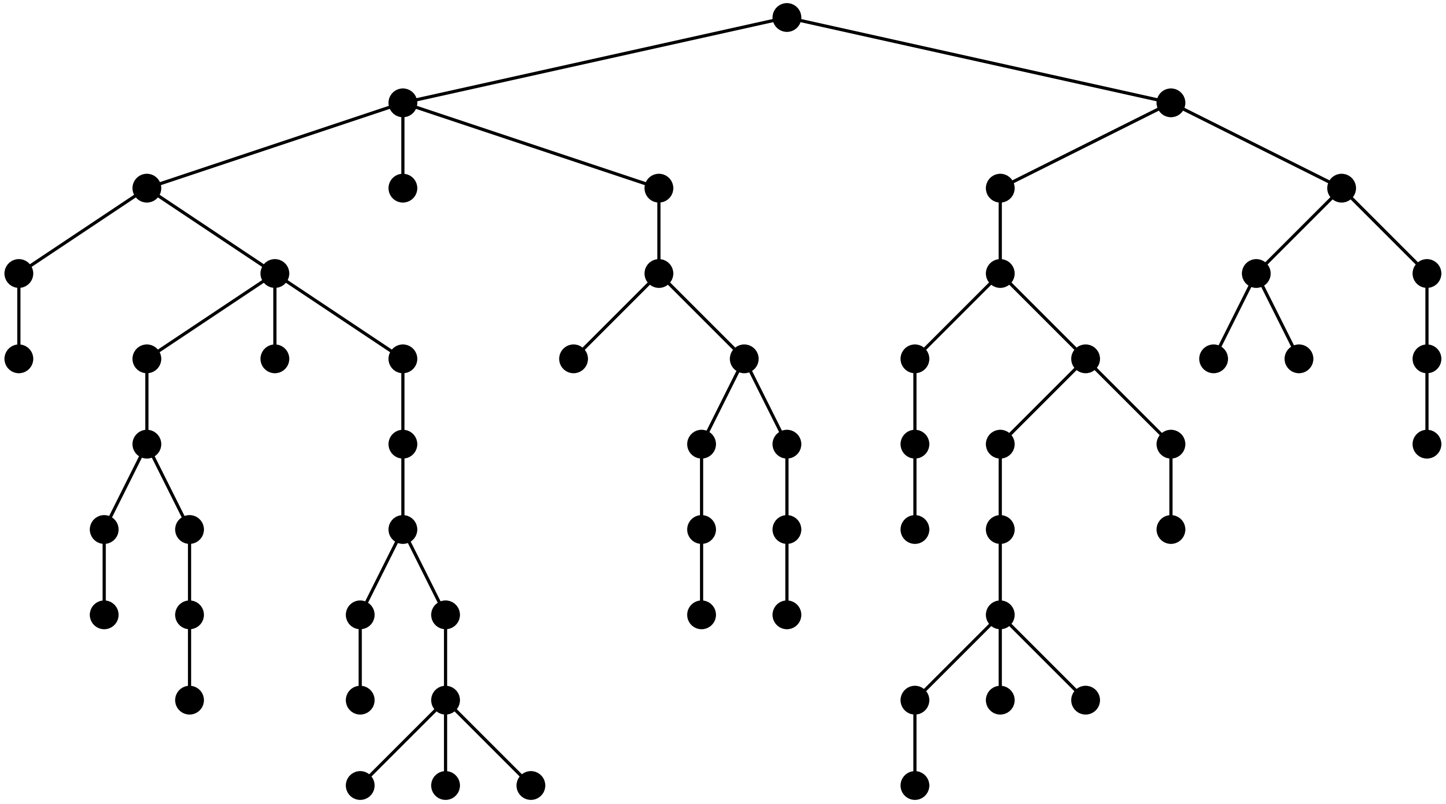
Can we get rid of this term?

Almost...

Optimal

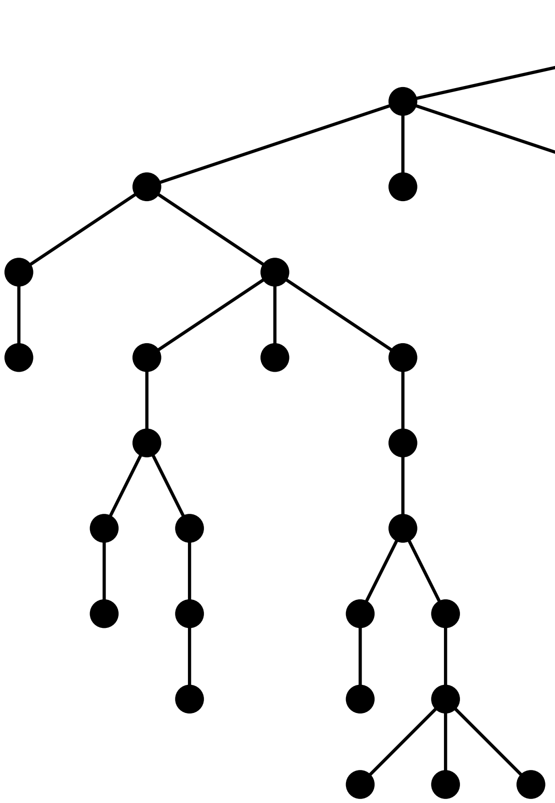
Indirection

We can use a similar technique to the one we encountered while designing level ancestor oracles



Indirection

We can use a similar technique to the one we encountered while designing level ancestor oracles

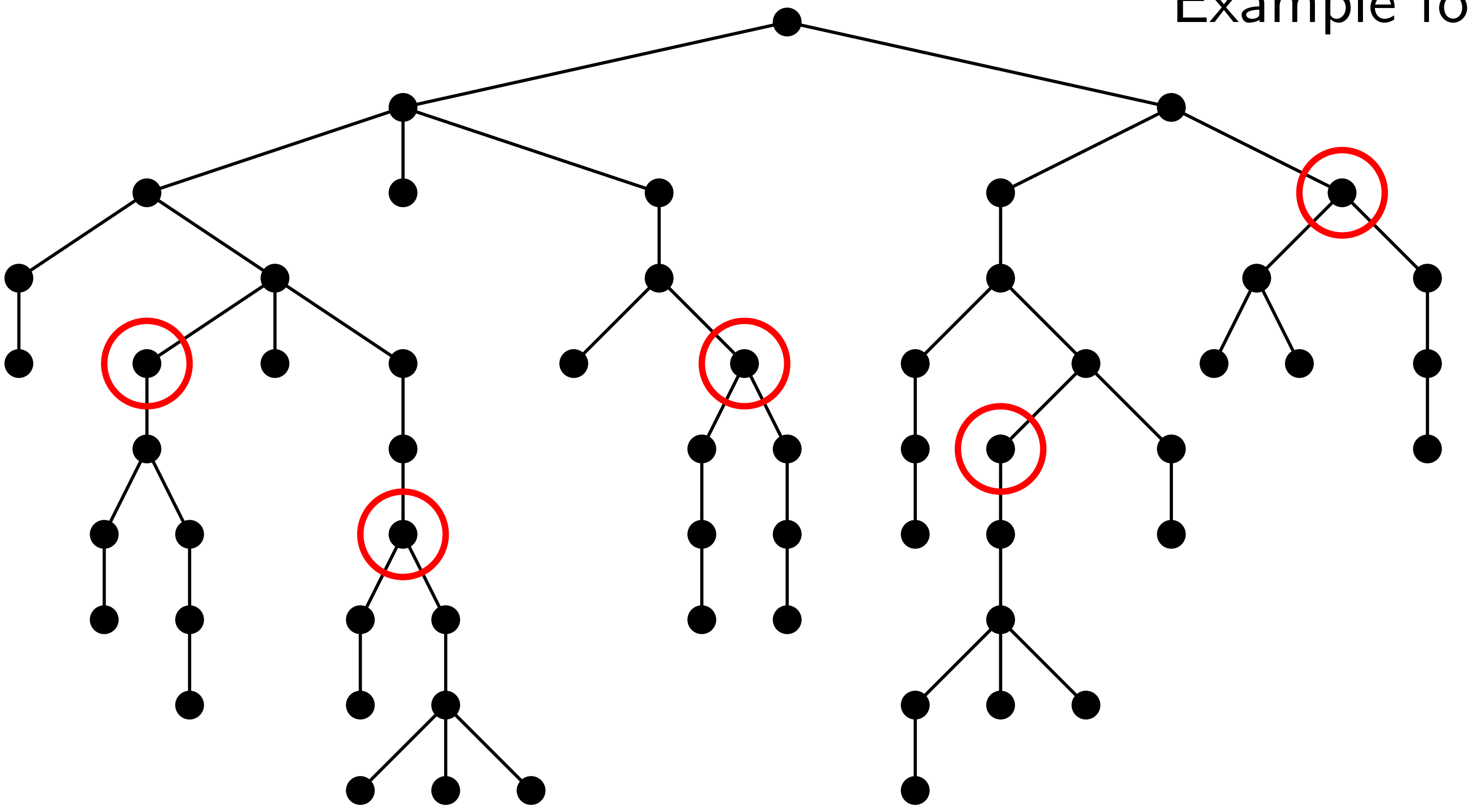


Find the set M of all maximally deep vertices with at least $|\Sigma|$ descendants

Indirection

We can use a similar technique to the one we encountered while designing level ancestor oracles

Example for $|\Sigma| = 7$



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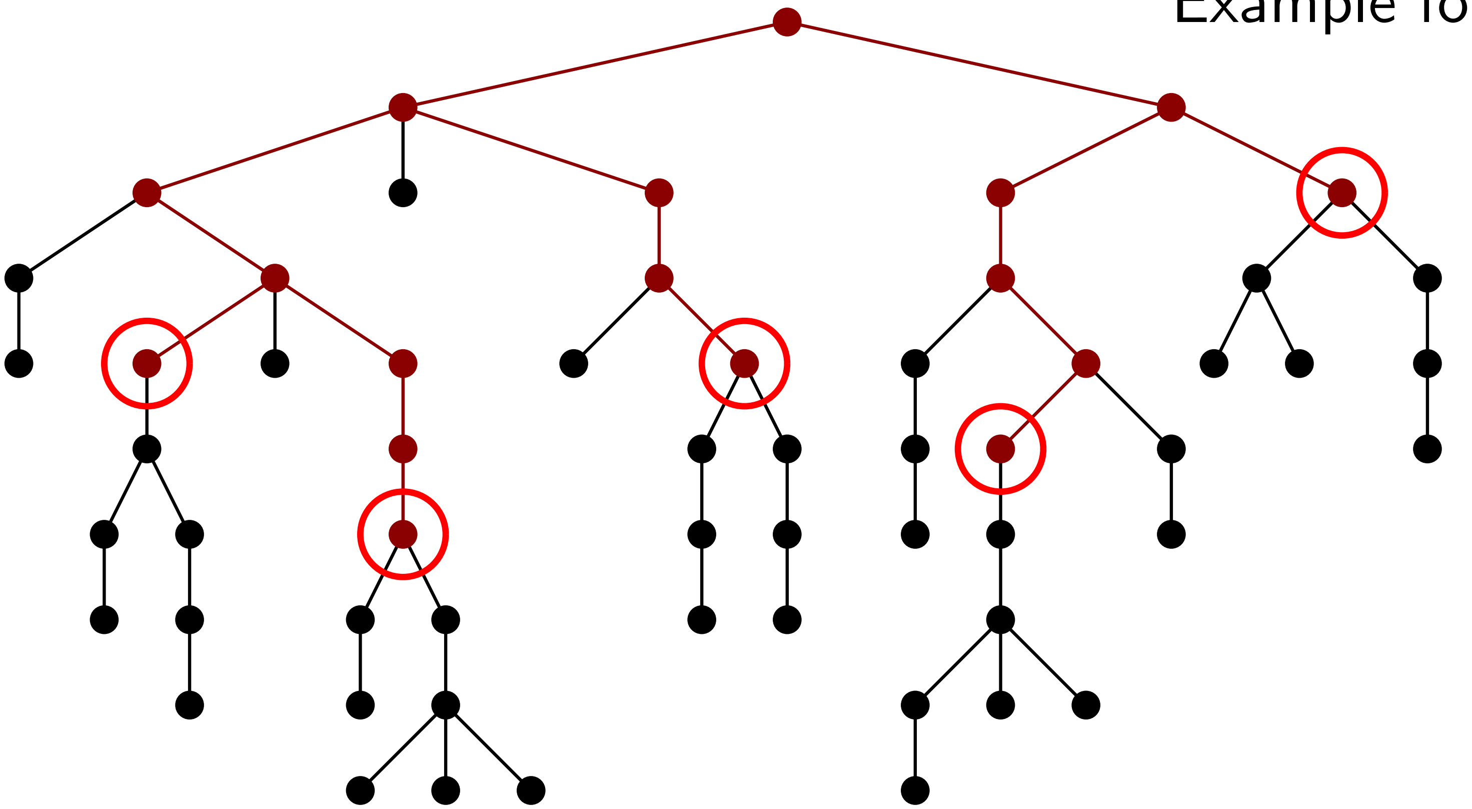

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Split the trie into a tree T' containing all the ancestors of the vertices in M and several bottom-trees in $T \setminus T'$.

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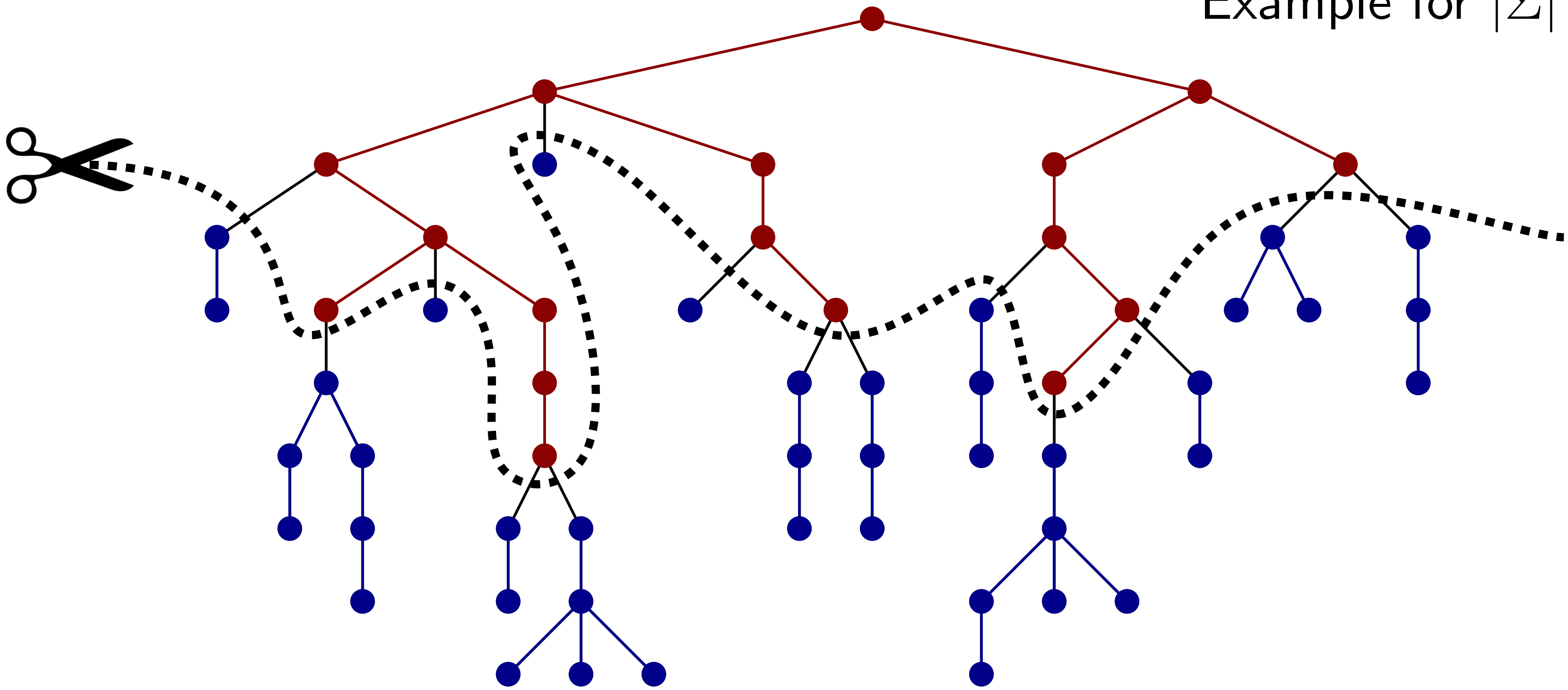

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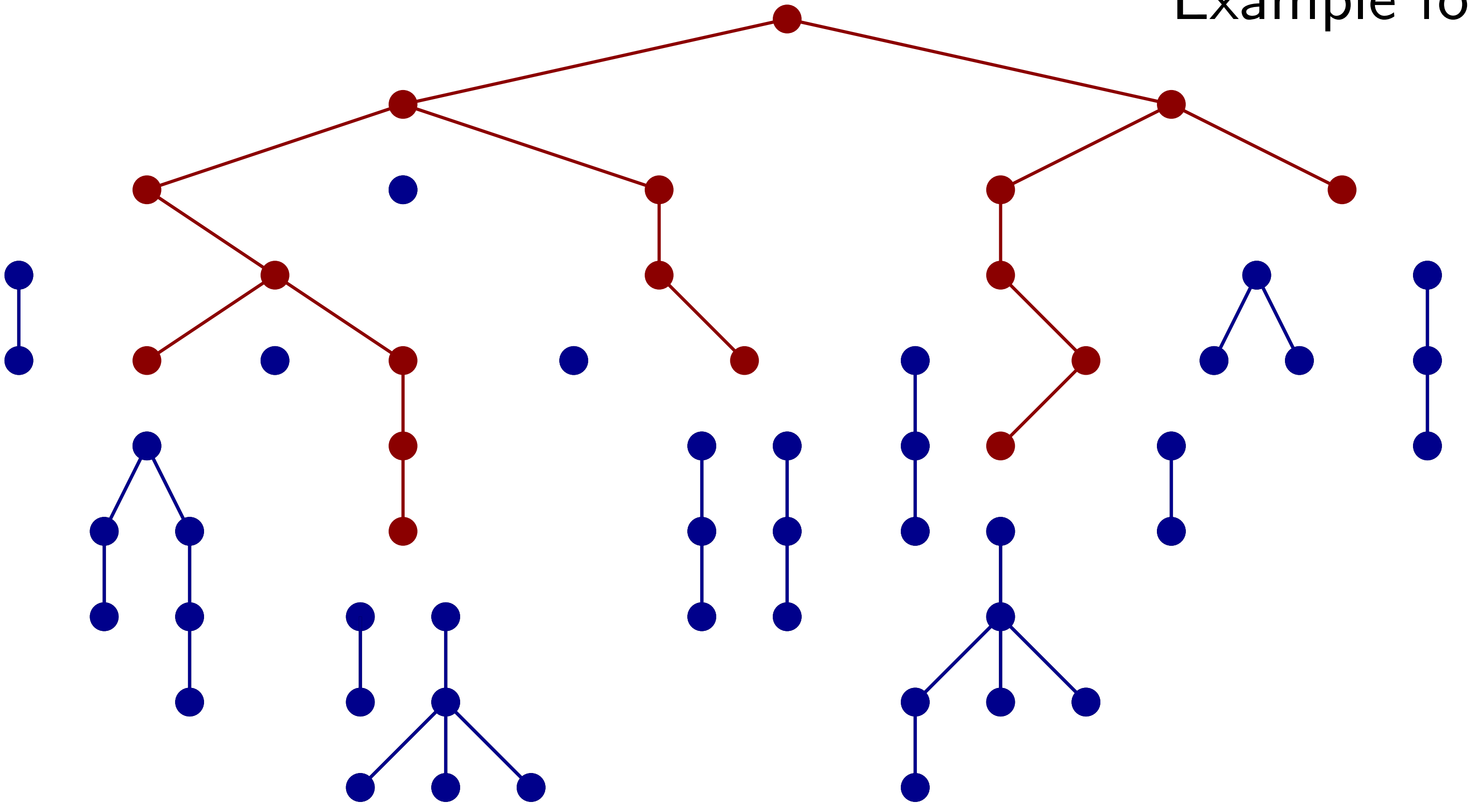

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Storing the top tree:

The number of leaves of T' is at most $\frac{n}{|\Sigma|}$

Fact: A tree with ℓ leaves has at most $\ell - 1$ branching nodes (i.e., nodes with at least 2 children)

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Storing the top tree:

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Fact: A tree with ℓ leaves has at most $\ell - 1$ branching nodes (i.e., nodes with at least 2 children)

- Store leaves using dense arrays

Space

$O(|\Sigma| \cdot \frac{n}{|\Sigma|}) = O(n)$

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Time to find the next node $O(1)$

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- Store each bottom tree using a weight-balanced BST

Total space of all bottom trees: $O(n)$

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Storing the bottom trees:

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Total space of all bottom trees: $O(n)$

- Each bottom tree has at most $|\Sigma|$ leaves

Time to navigate a bottom tree: $O(|P| + \log|\Sigma|)$

Representing Tries: Recap

	Space	**Query Time**
Array (dense)	$O(\lvert\Sigma\rvert \cdot n)$	$O(\lvert P\rvert)$
Array (sparse) / BST	$O(n)$	$O(\lvert P\rvert \log \lvert\Sigma\rvert)$
Weight-balanced BST	$O(n)$	$O(\lvert P\rvert + \log k)$
Indirection	$O(n)$	$O(\lvert P\rvert + \log \lvert\Sigma\rvert)$

Can be made dynamic with a time complexity of $O(|T| + \log|\Sigma|)$ per insertion/deletion of T

Application: String Sorting

Sort a collection of k strings $T_1, T_2, \ldots, T_k$ over Σ

$L = \max_{i=1,\ldots,k} |T_i|$

Obs: A string comparison requires time $O(L)$.
Naive sorting algorithm take time $O(Lk \log k)$ or $O(Lk)$

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- Create an empty trie
- For $i = 1, \ldots, k$:
    - Insert T_i into the trie
- An in-order visit of the trie returns the strings in lexicographic order

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$$\left.\begin{array}{l} \\ \\ \end{array}\right\} O\left(\sum_{i=1}^{k} (|T_i| + \log|\Sigma|)\right)$$

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$\left.\right\} O\left(n + k \log |\Sigma|\right)$

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- An in-order visit of the trie returns the strings in lexicographic order $O(n)$

Overall time: $O\left(n + k \log |\Sigma|\right)$

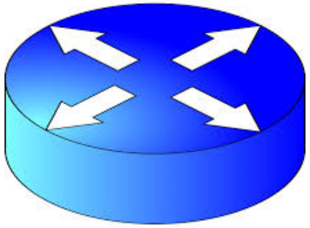
Application: Packet Routing

Among all the destinations that match, a packet gets routed to the one with the most specific rule

Packet

Src: 192.168.42.10

Dst: 101.167.200.15



Routing Table

Destination	Interface
169.0.0.0/11	eth1
169.48.0.0/12	ppp0
169.128.0.0/10	eth1
169.160.0.0/11	eth0
96.0.0.0/3	tun1
96.0.0.0/5	tun0
100.0.0.0/8	eth0
127.0.0.0/8	lo
default	wlan0

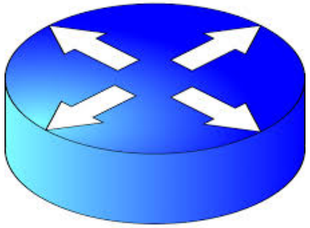
Application: Packet Routing

Among all the destinations that match, a packet gets routed to the one with the most specific rule

Packet

Src: 192.168.42.10

Dst: 0110010110100111. . .



Routing Table

Destination	Interface
10101001000$	eth1
101010010011$	ppp0
1010100110$	eth1
10101001101$	eth0
011$	tun1
011000$	tun0
01100100$	eth0
01111111$	lo
$	wlan0

Application: Packet Routing

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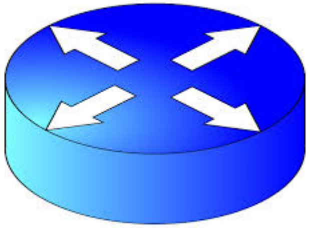
Packet

Src:	192.168.42.10
Dst:	011001011010011 1. . .



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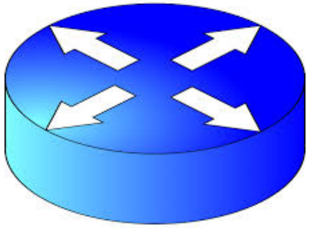
Src: 192.168.42.10

Dst: $\underbrace{0110010110100111\ldots}_{P}$



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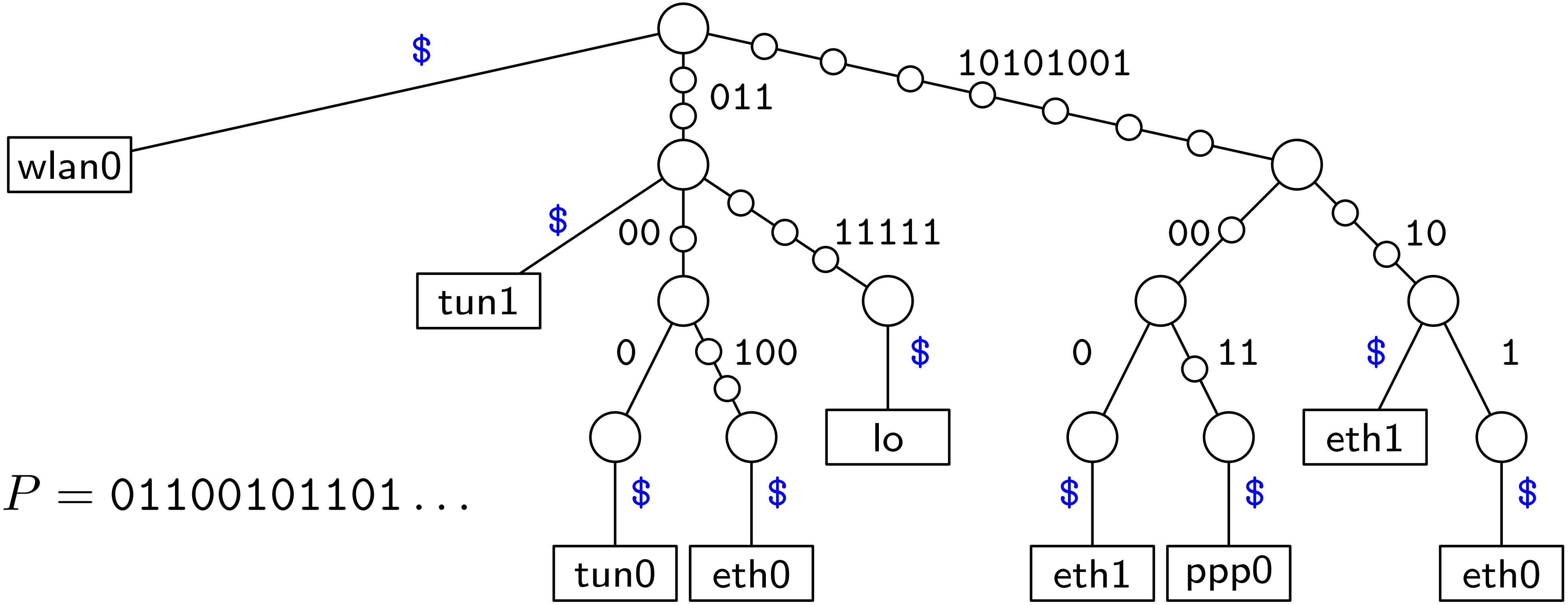
011000$	tun0
01100100$	eth0
01111111$	lo
$	wlan0

Given a pattern P we want the longest string in our collection that appears as a prefix of P

Application: Packet Routing

Build a trie T with all the addresses in the routing table.



$P = 01100101101\ldots$

- Find the node v corresponding to the maximal prefix that matches P

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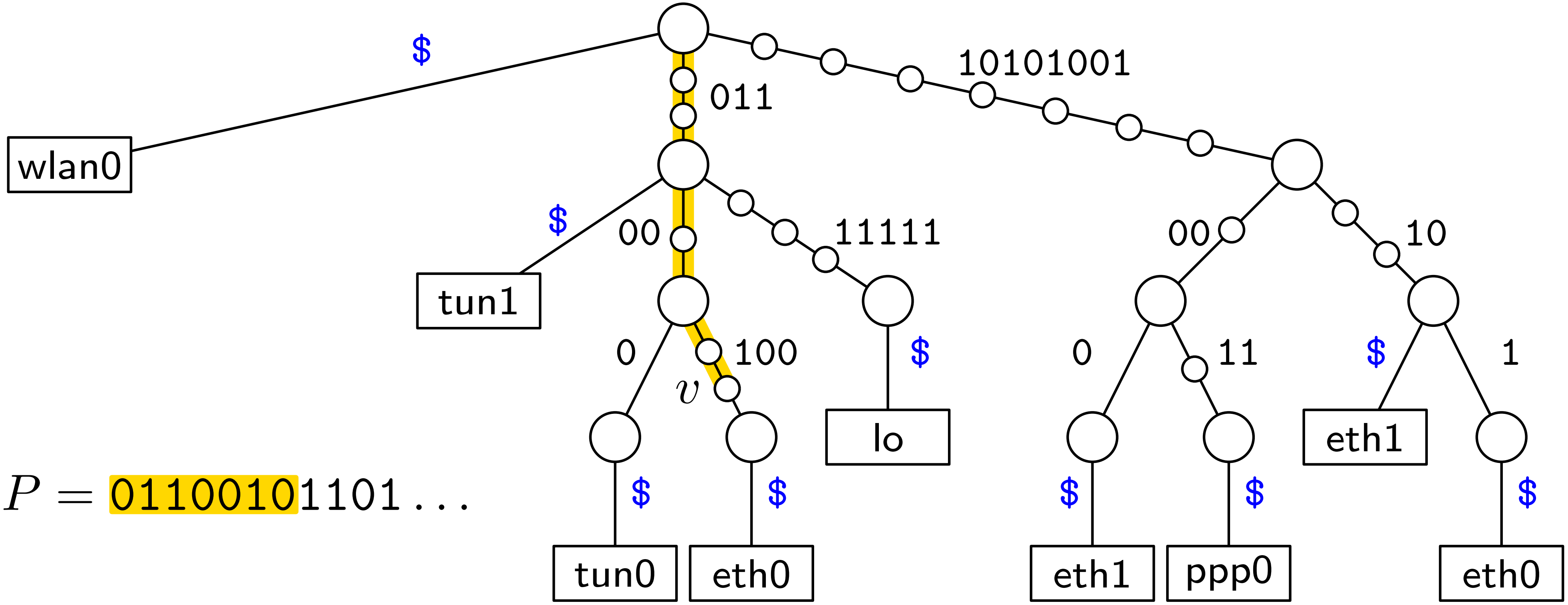

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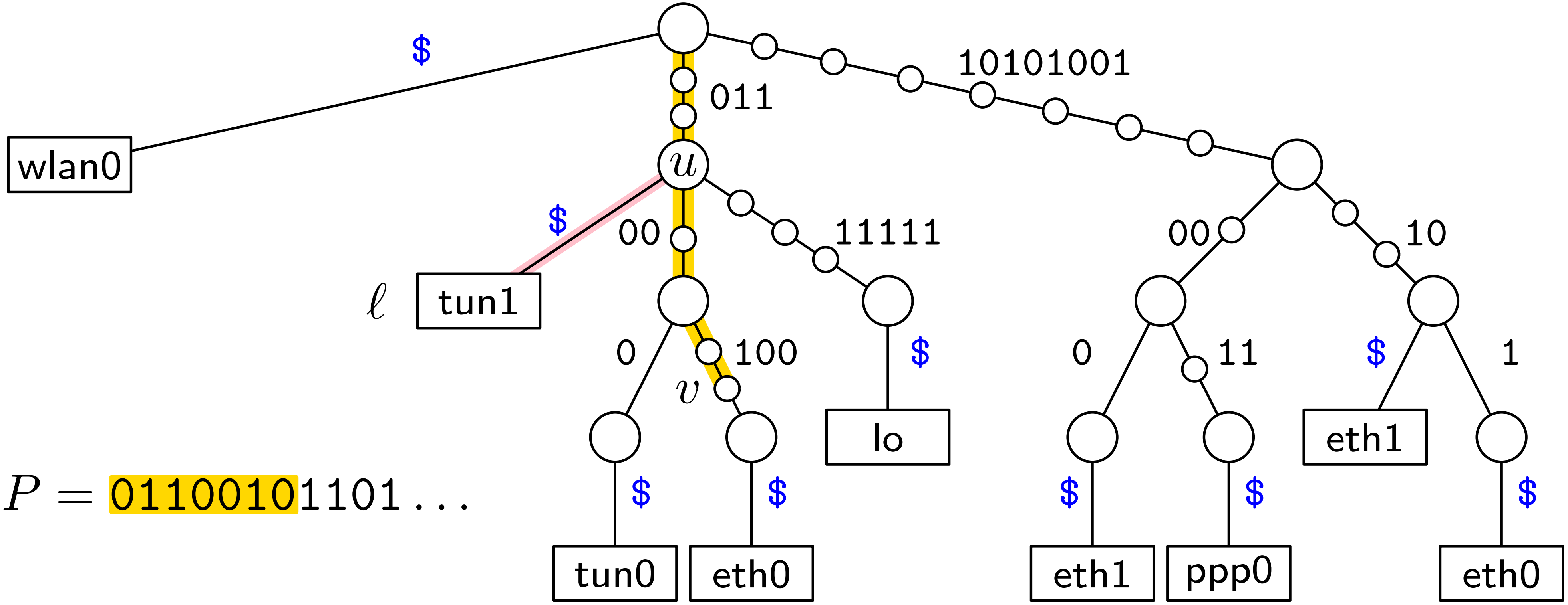

- Find the node v corresponding to the maximal prefix that matches P
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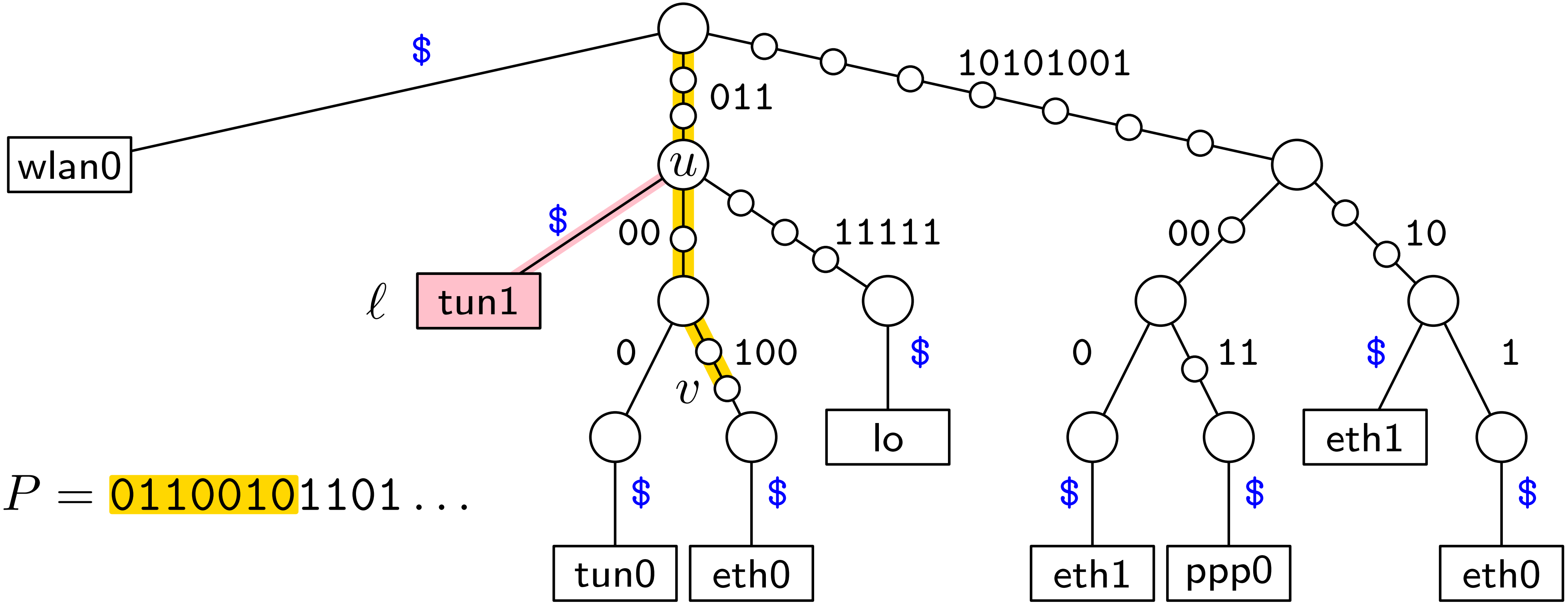

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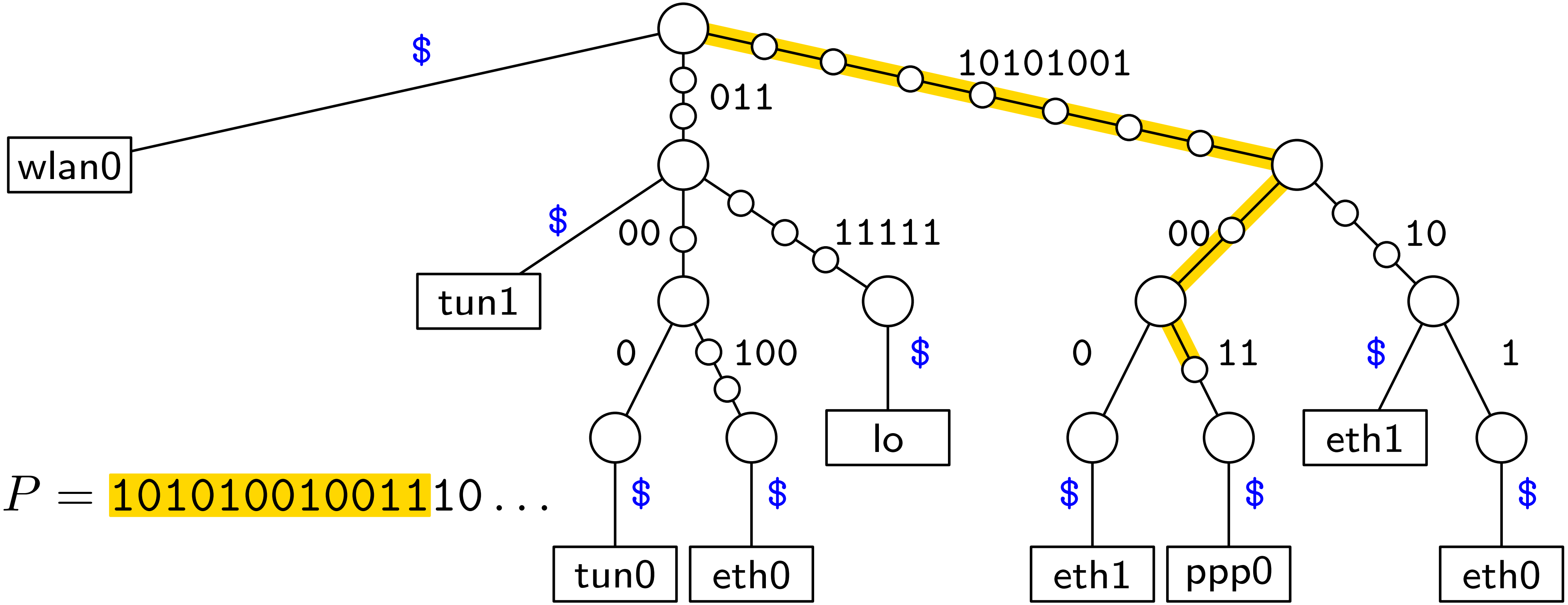

$P = 10101001001110\dots$

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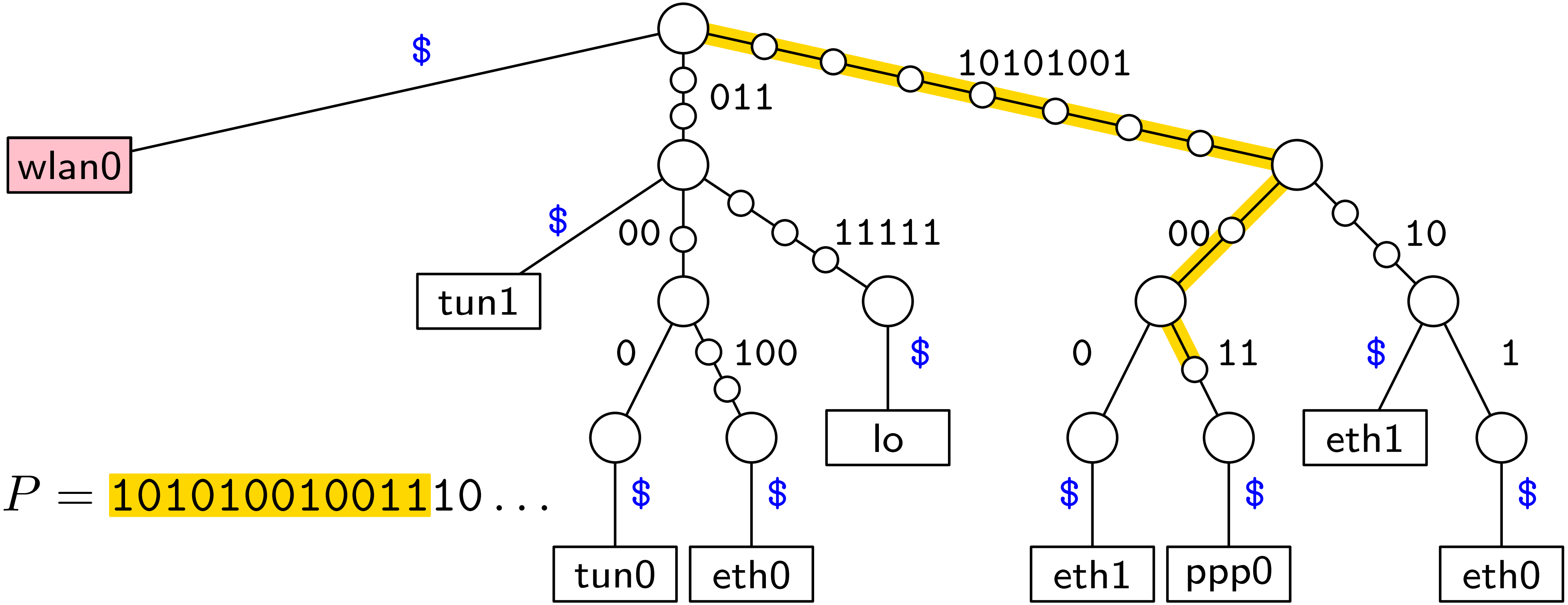

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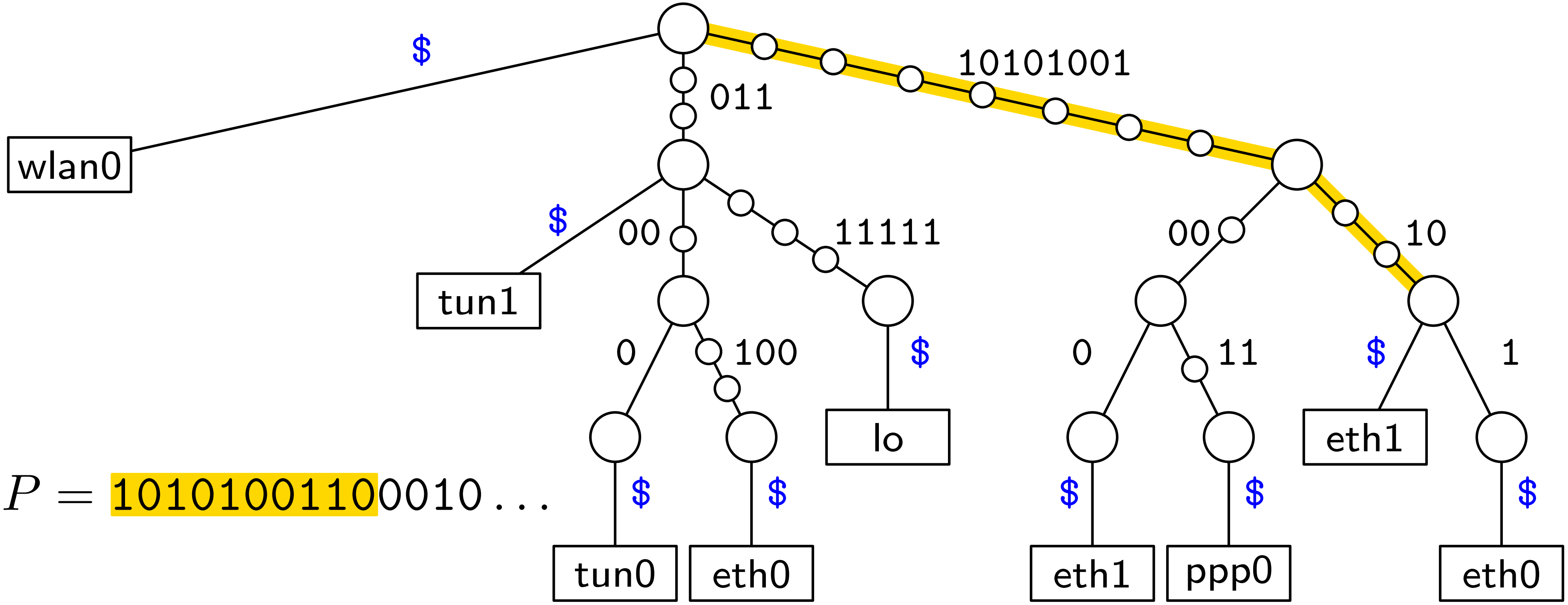

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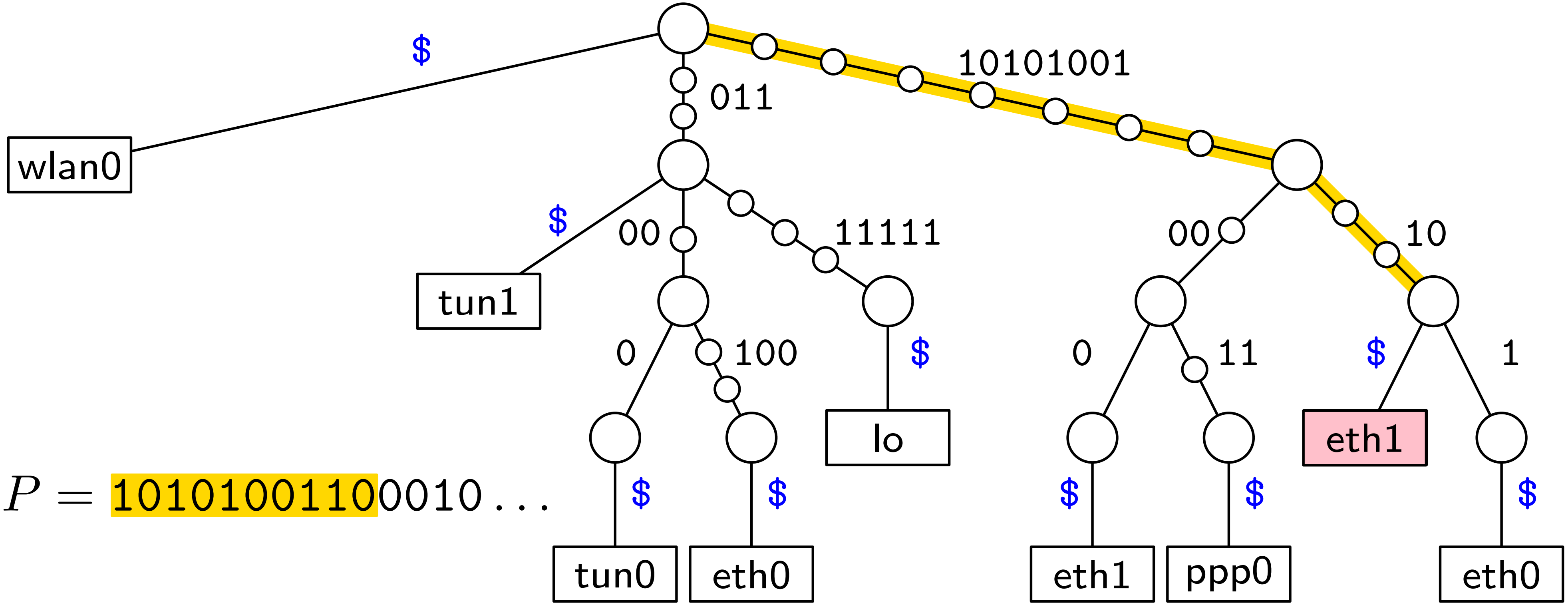

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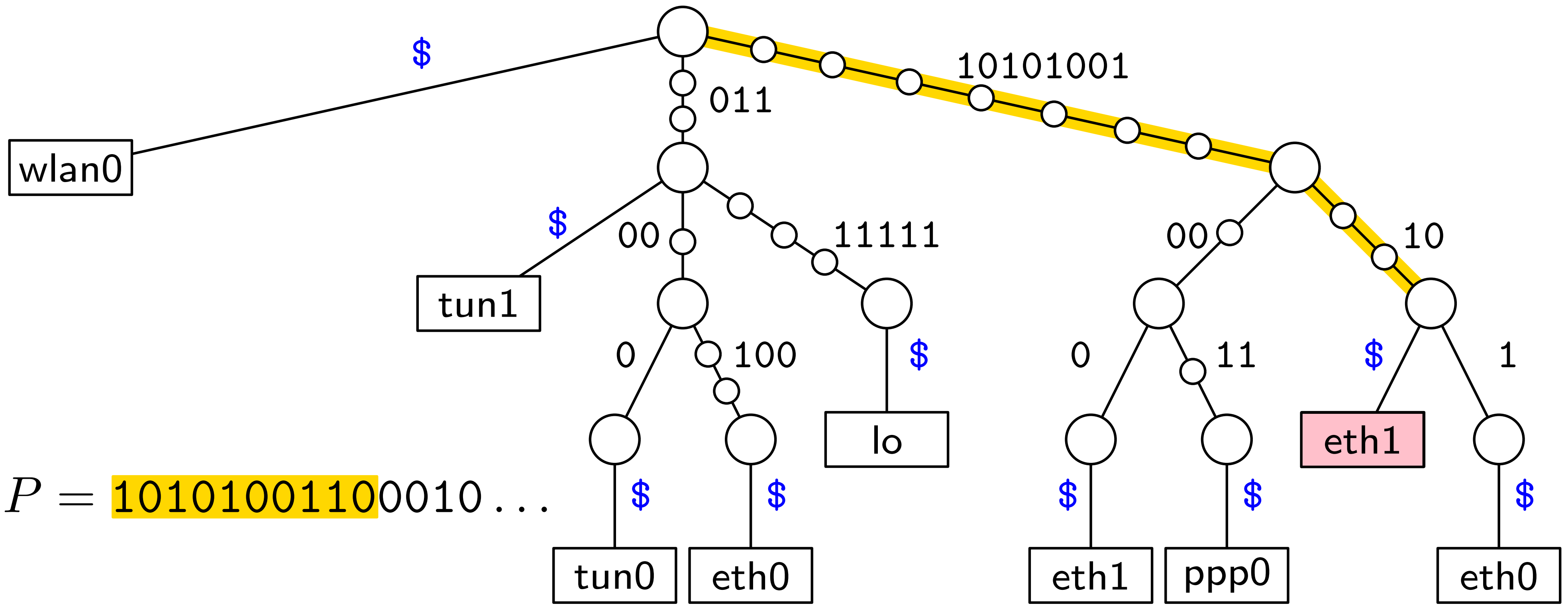

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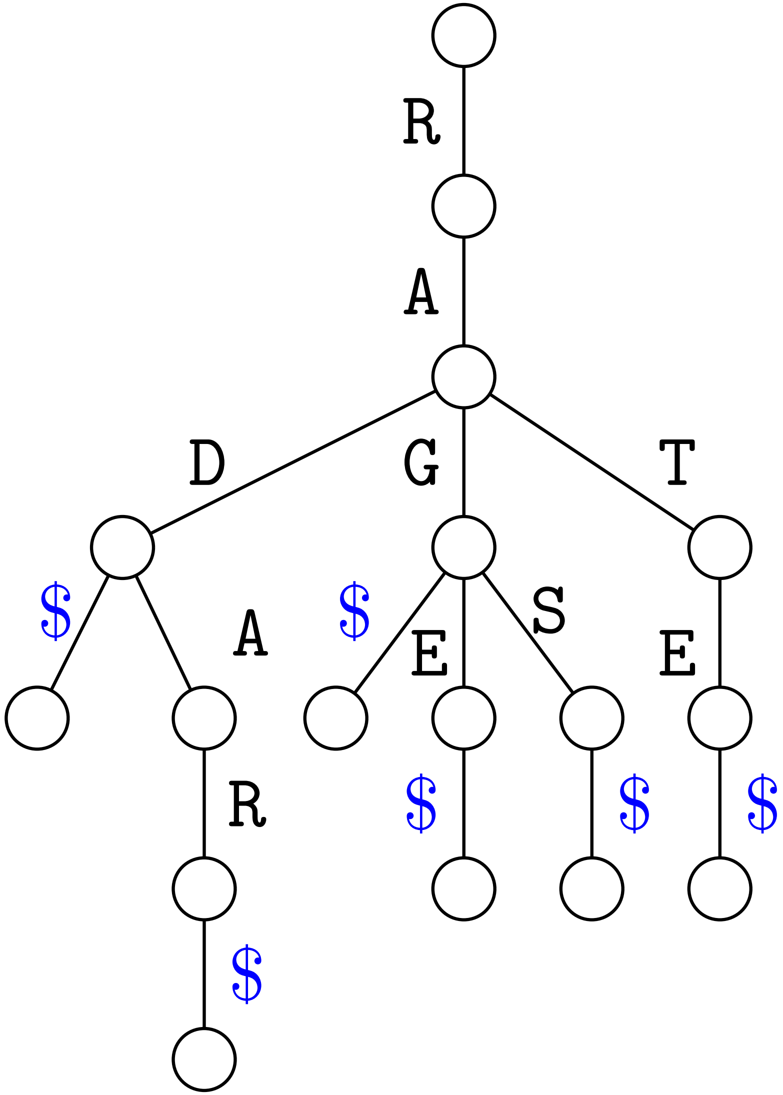
Time: O(address length)

Compressed Tries (Radix Trees)

Contract non-branching paths to a single edge labelled with the corresponding substring



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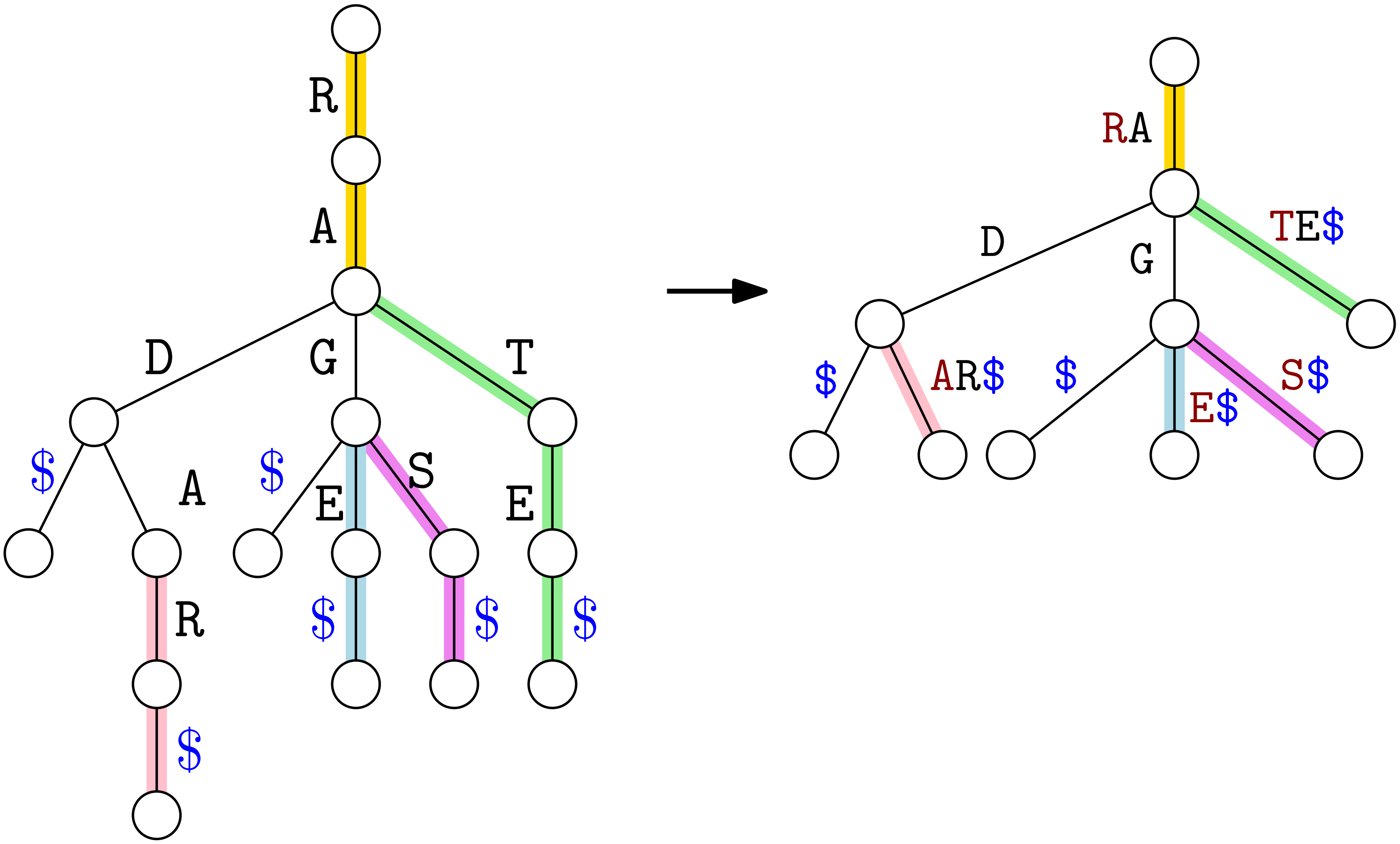

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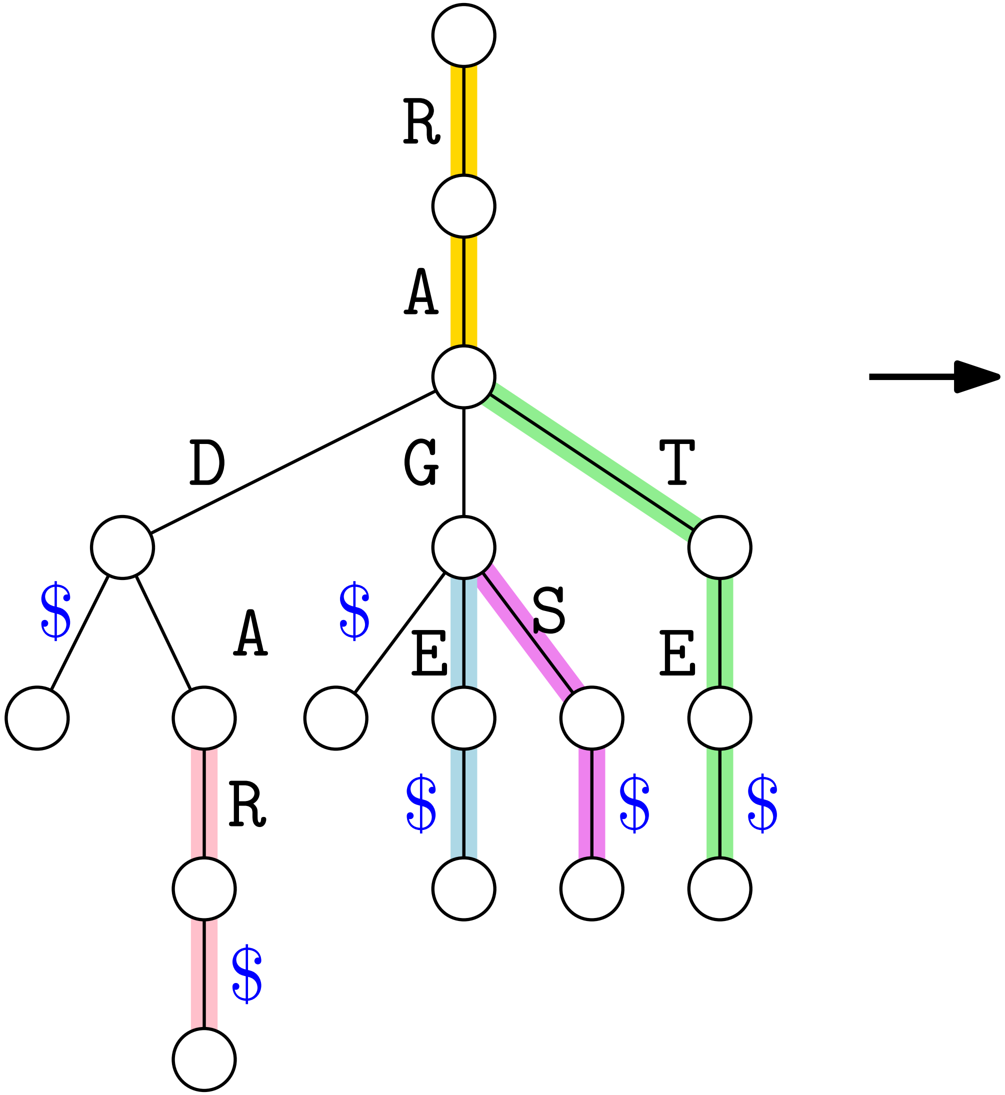

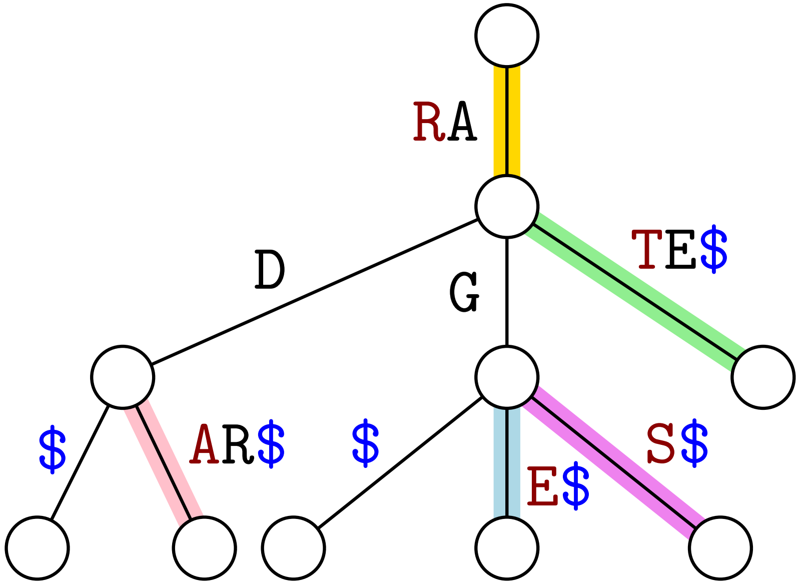

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Use the first character on each edge as the key

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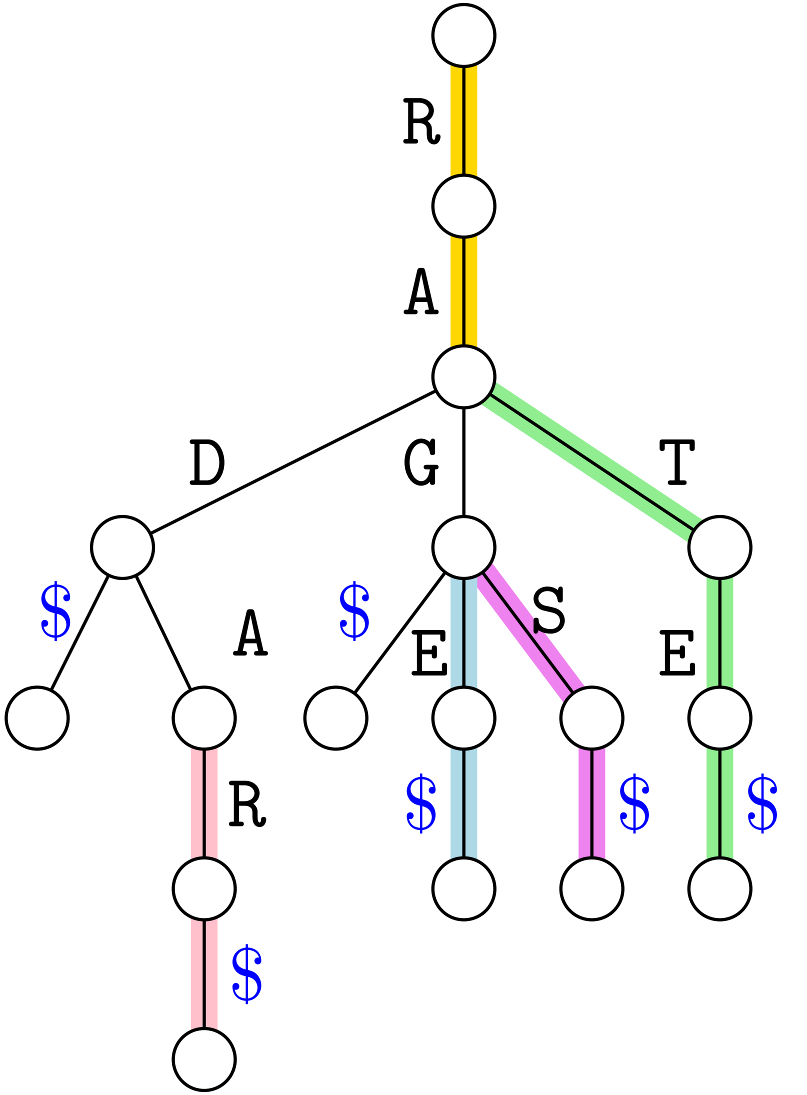
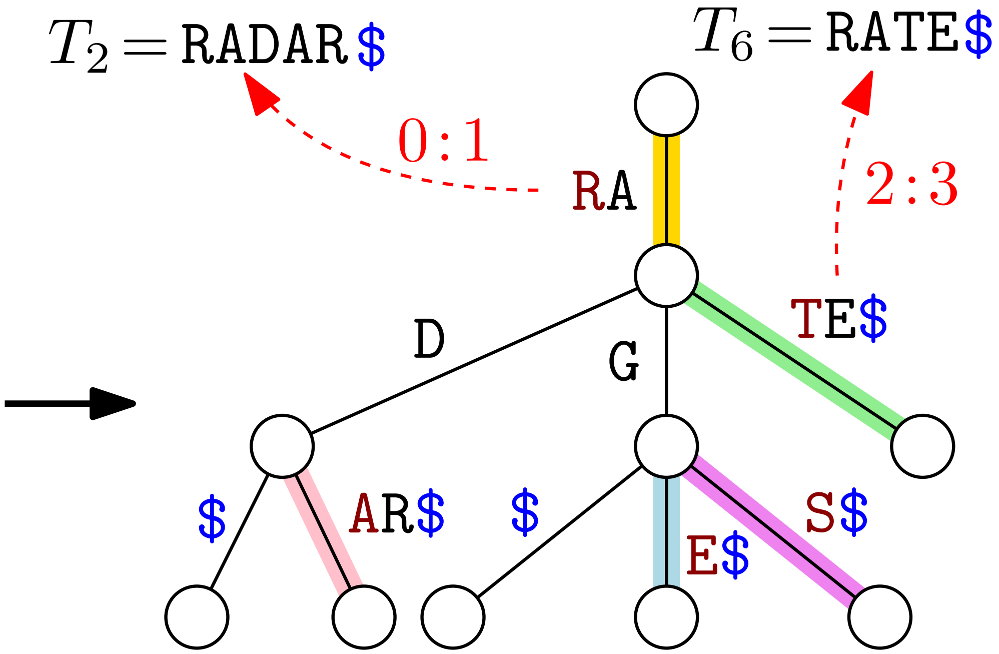




Previous constructions apply

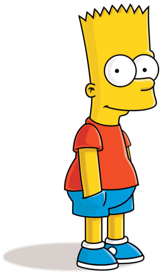
Use the first character on each edge as the key

Store edge labels as indices in the input strings

Suffix Trees

Back to String Matching

Problem: Given an alphabet Σ, a *text* $T \in \Sigma^*$ and a *pattern* $P \in \Sigma^*$, find some occurrence/all occurrences of P in T.



$$\Sigma = \{\texttt{A}, \texttt{B}, \ldots, \texttt{Z}, \texttt{a}, \texttt{b}, \ldots, \texttt{z}, \textvisiblespace\}$$

$$T = \texttt{B}\underline{\texttt{art}}\texttt{\textvisiblespace played\textvisiblespace d}\underline{\texttt{art}}\texttt{s\textvisiblespace at\textvisiblespace the\textvisiblespace p}\underline{\texttt{art}}\texttt{y}$$

$$P = \texttt{art}$$

Want: A data structure that can preprocesses T and answer string matching queries

Suffix Trees

The **suffix tree** of T is the compressed trie of all the suffixes of $T\$$

$\Sigma = \{\mathtt{A}, \mathtt{B}, \mathtt{N}, \mathtt{S}\}$ $\qquad T = \mathtt{BANANAS}\$$

Suffix Trees

The **suffix tree** of T is the compressed trie of all the suffixes of $T\$$

$$\Sigma = \{\mathtt{A}, \mathtt{B}, \mathtt{N}, \mathtt{S}\} \qquad T = \overset{\mathtt{01234567}}{\mathtt{BANANAS\$}}$$

7	$\mathtt{\$}$
6	$\mathtt{S\$}$
5	$\mathtt{AS\$}$
4	$\mathtt{NAS\$}$
3	$\mathtt{ANAS\$}$
2	$\mathtt{NANAS\$}$
1	$\mathtt{ANANAS\$}$
0	$\mathtt{BANANAS\$}$

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7 $

6 S$

5 AS$

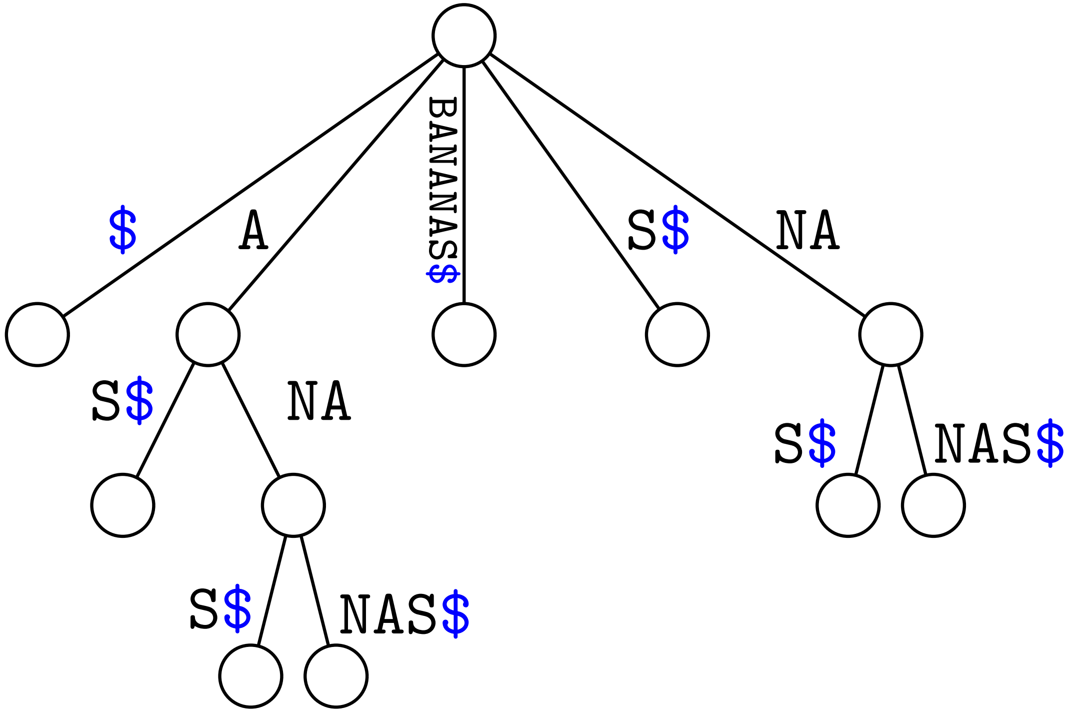
4 NAS$

3 ANAS$

2 NANAS$

1 ANANAS$

0 BANANAS$



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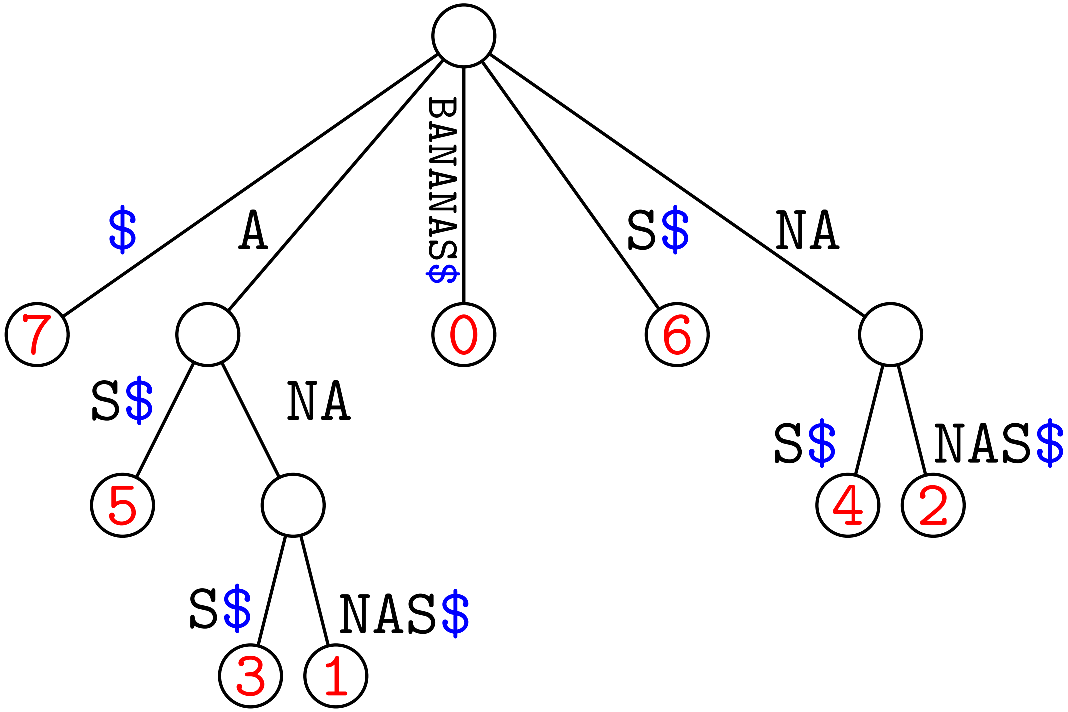

Label edges with indices into T

Label leaves with the index of the start of the corresponding suffix

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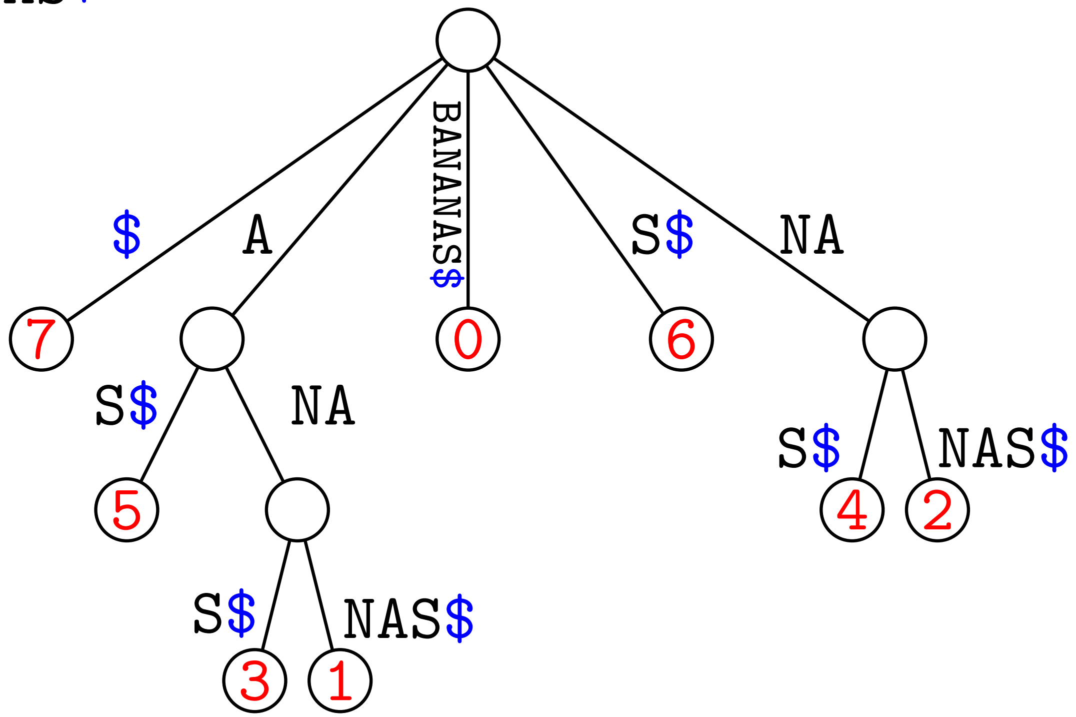
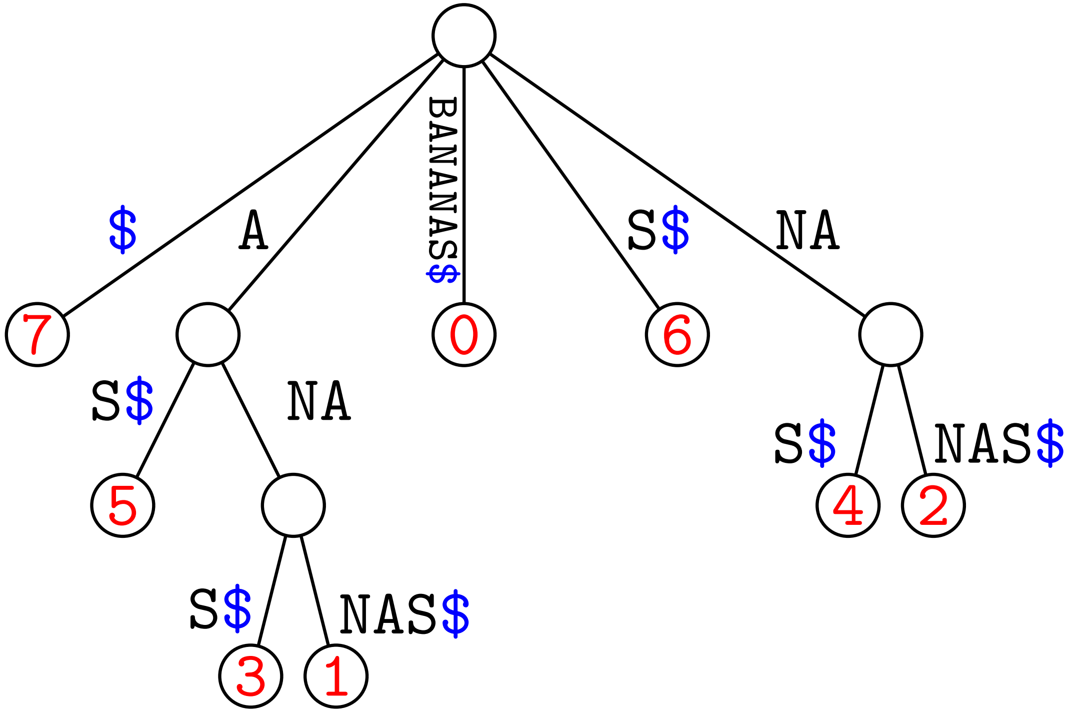


Label edges with indices into T

Label leaves with the index of the start of the corresponding suffix

Space: $\quad O(\#\text{ nodes}) = O(\#\text{ leaves}) = O(|T|)$

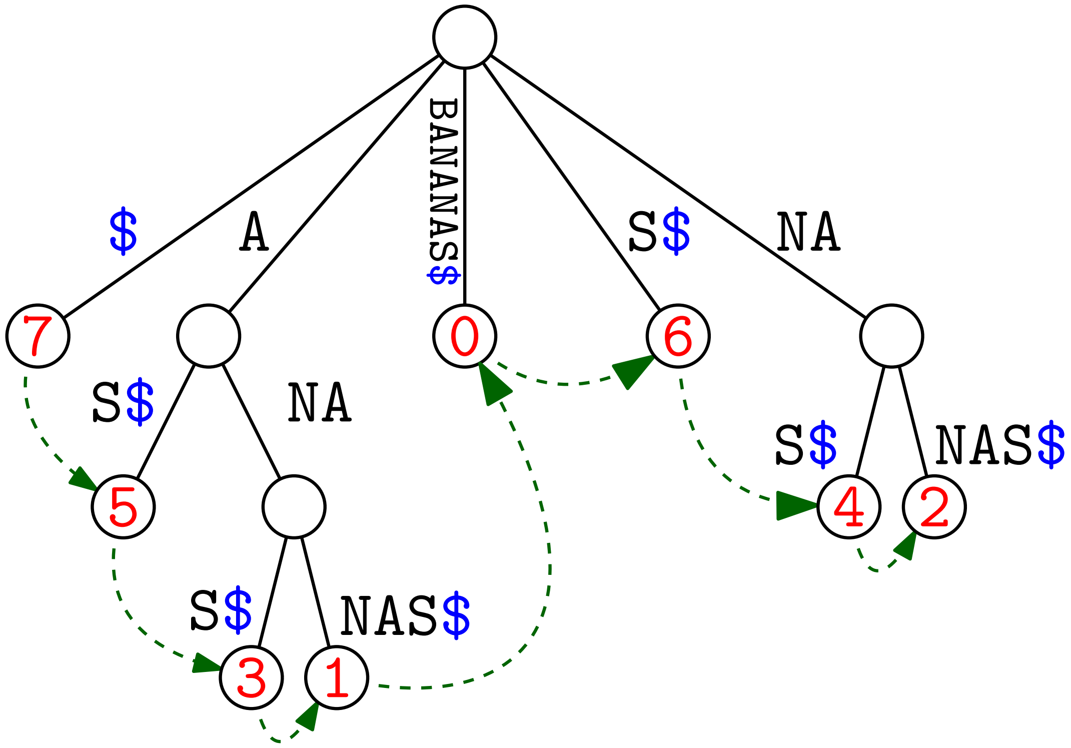
Applications: String Matching



Searching for a pattern P returns a compact representation of **all** occurrences of P in T

- Find the node v corresponding to P
- The occurrences of P are all and only the leaves in the subtree of v

Applications: String Matching



Searching for a pattern P returns a compact representation of **all** occurrences of P in T

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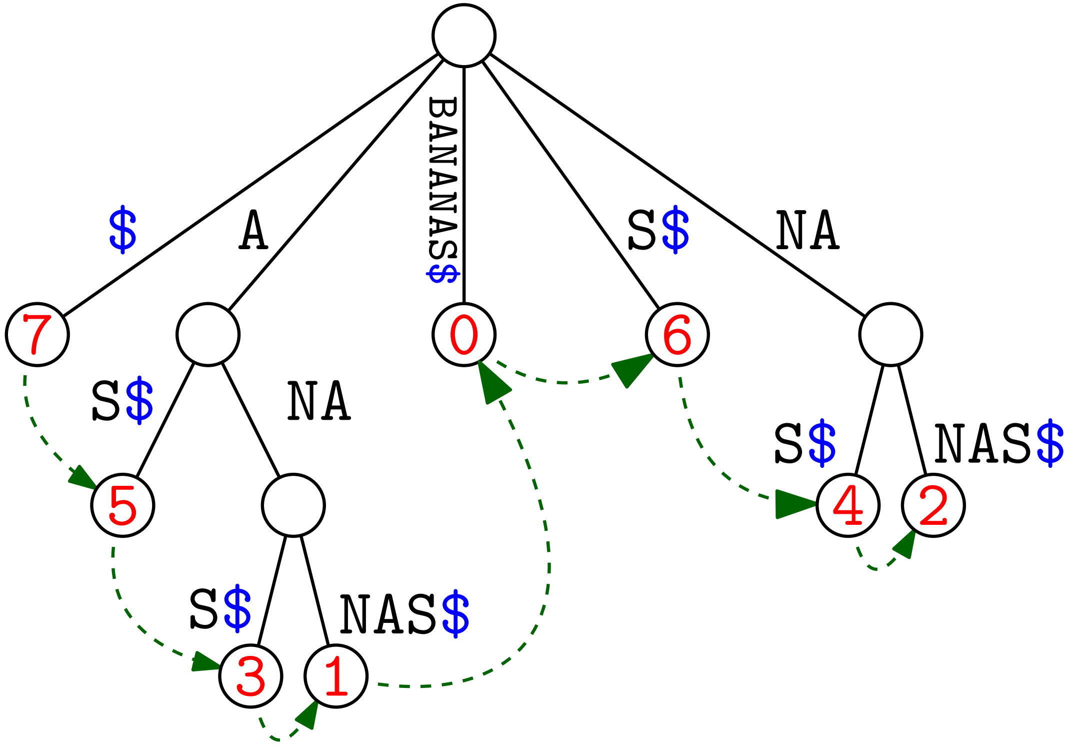
- The occurrences of P are all and only the leaves in the subtree of v
- Arrange leaves in a linked list to find the next match in $O(1)$ time

Applications: String Matching



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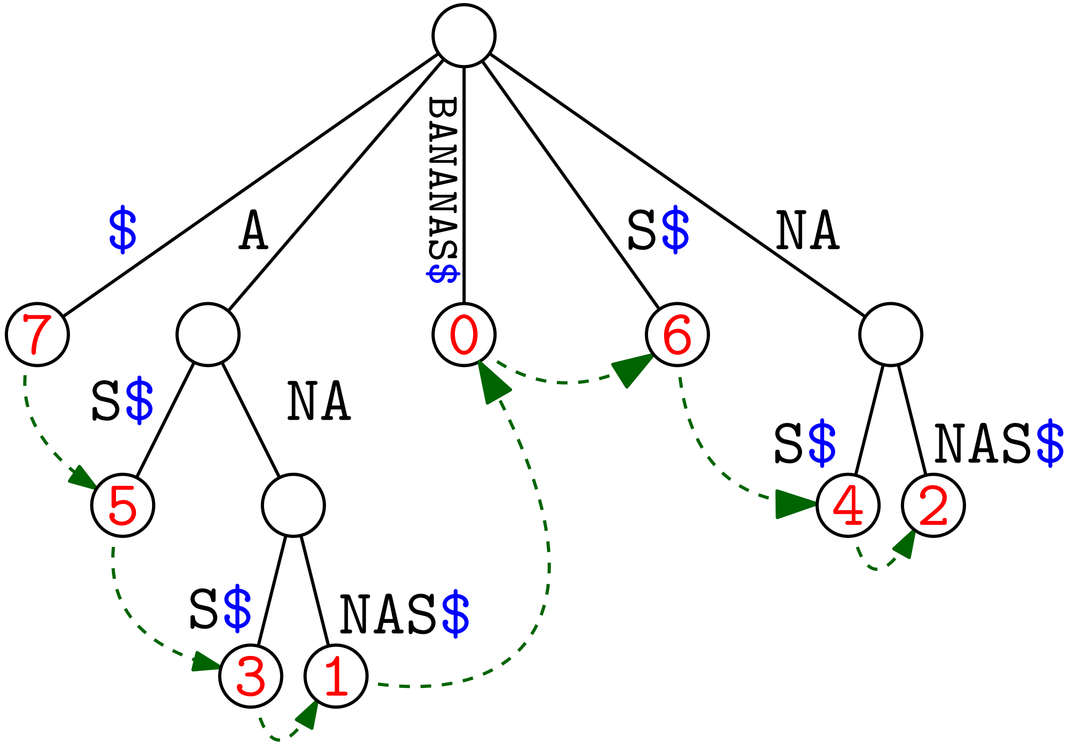

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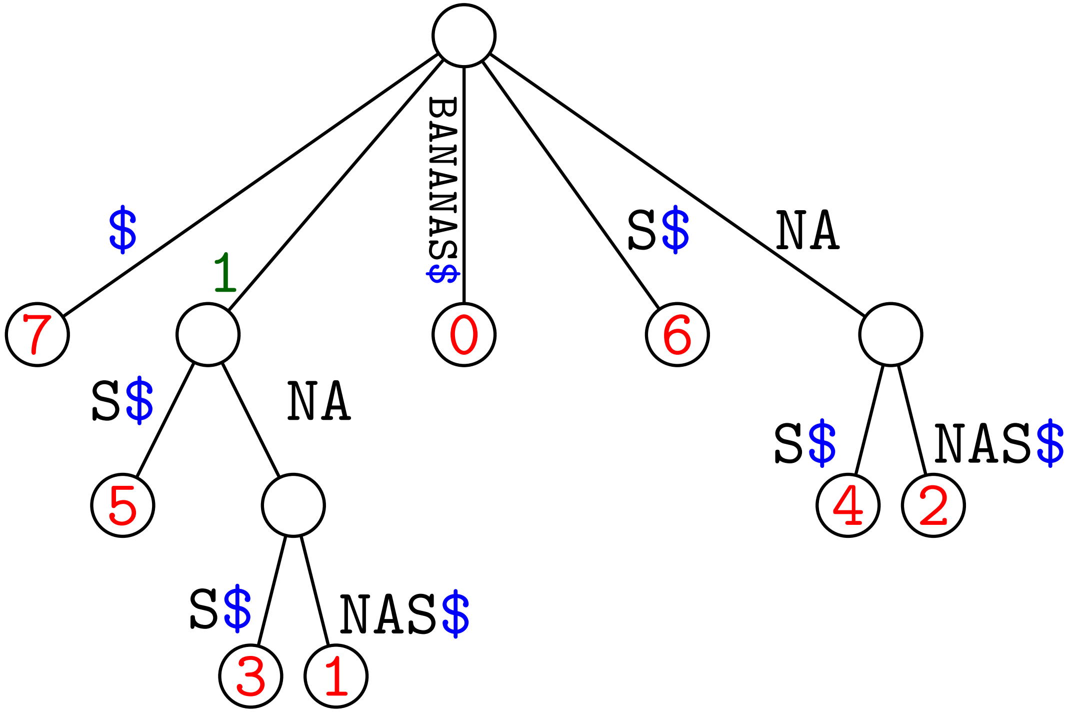
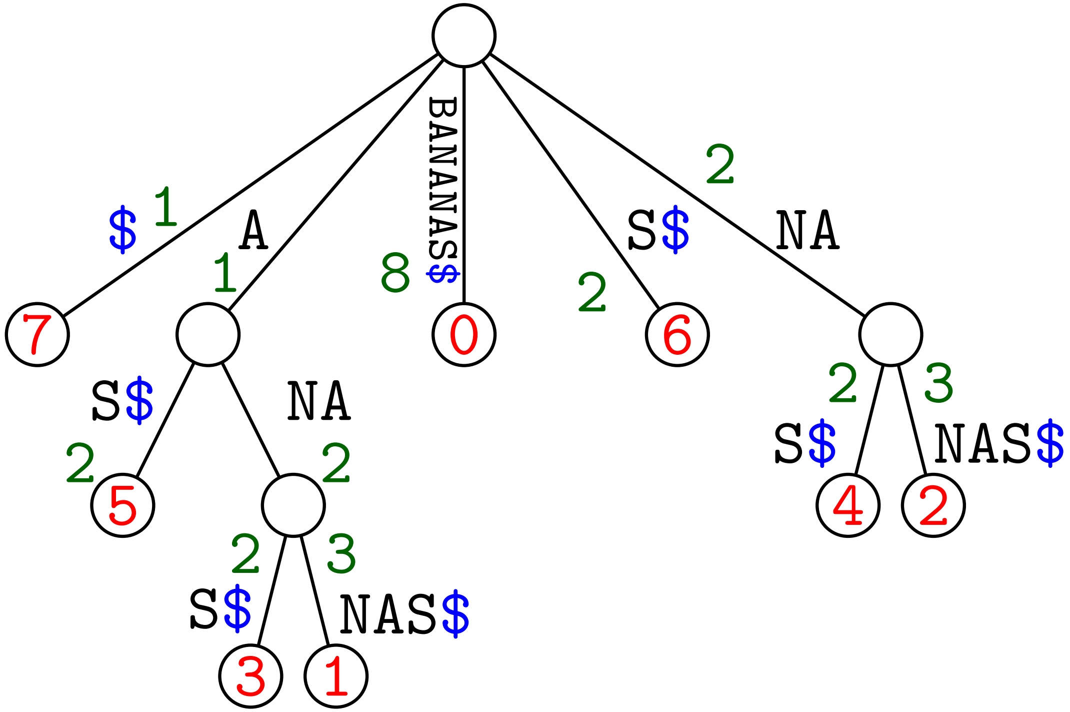
Number of matches in time $O(|P| + \log|\Sigma|)$ (store # leaves in the subtree)

Applications: Longest Repeated Substring



Find the longest string that appears at least twice in T as a substring:

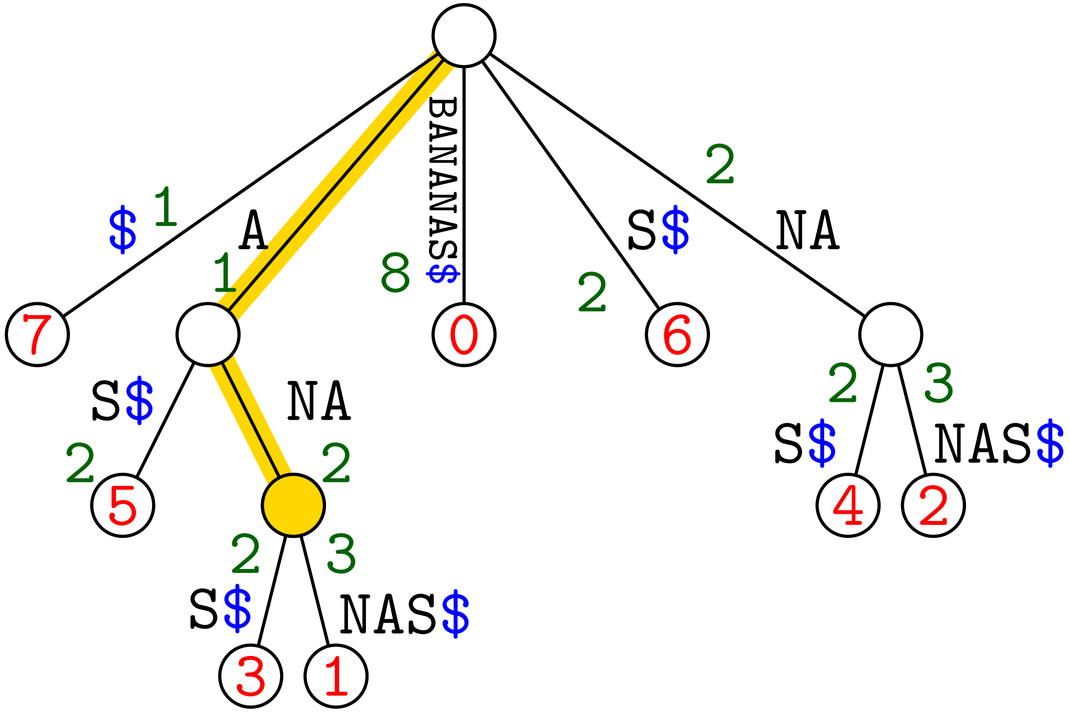
Applications: Longest Repeated Substring



Find the longest string that appears at least twice in T as a substring:

- Assign a length to each edge equal to the number of symbols in its label

Applications: Longest Repeated Substring



Find the longest string that appears at least twice in T as a substring:

- Assign a length to each edge equal to the number of symbols in its label
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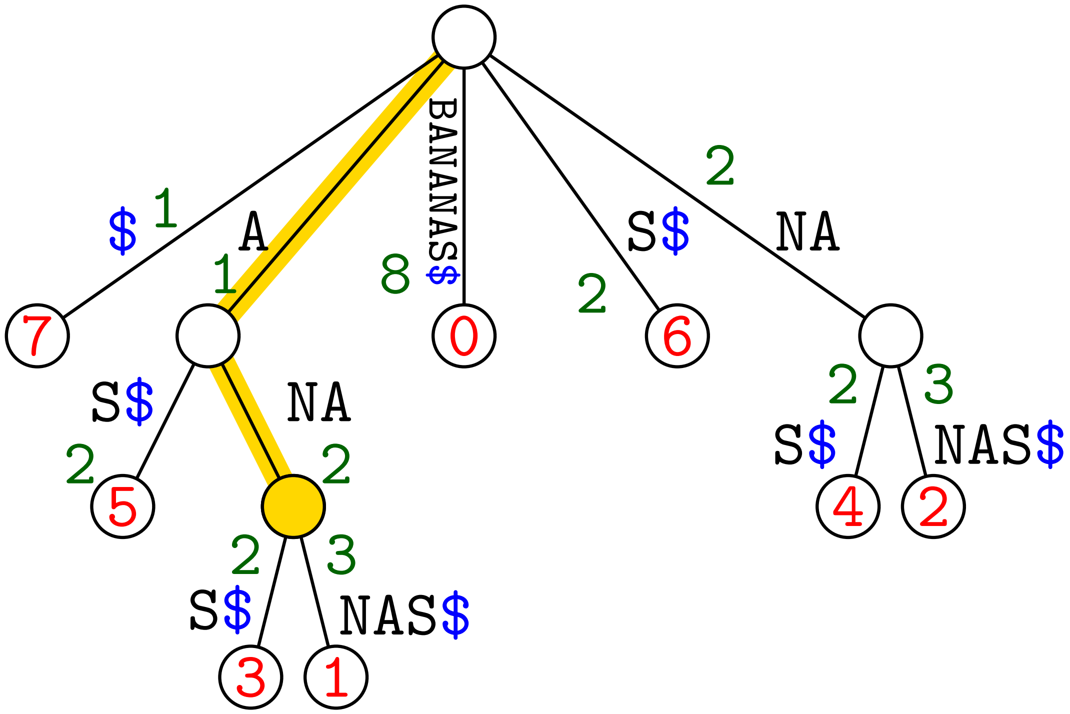

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- Assign a length to each edge equal to the number of symbols in its label
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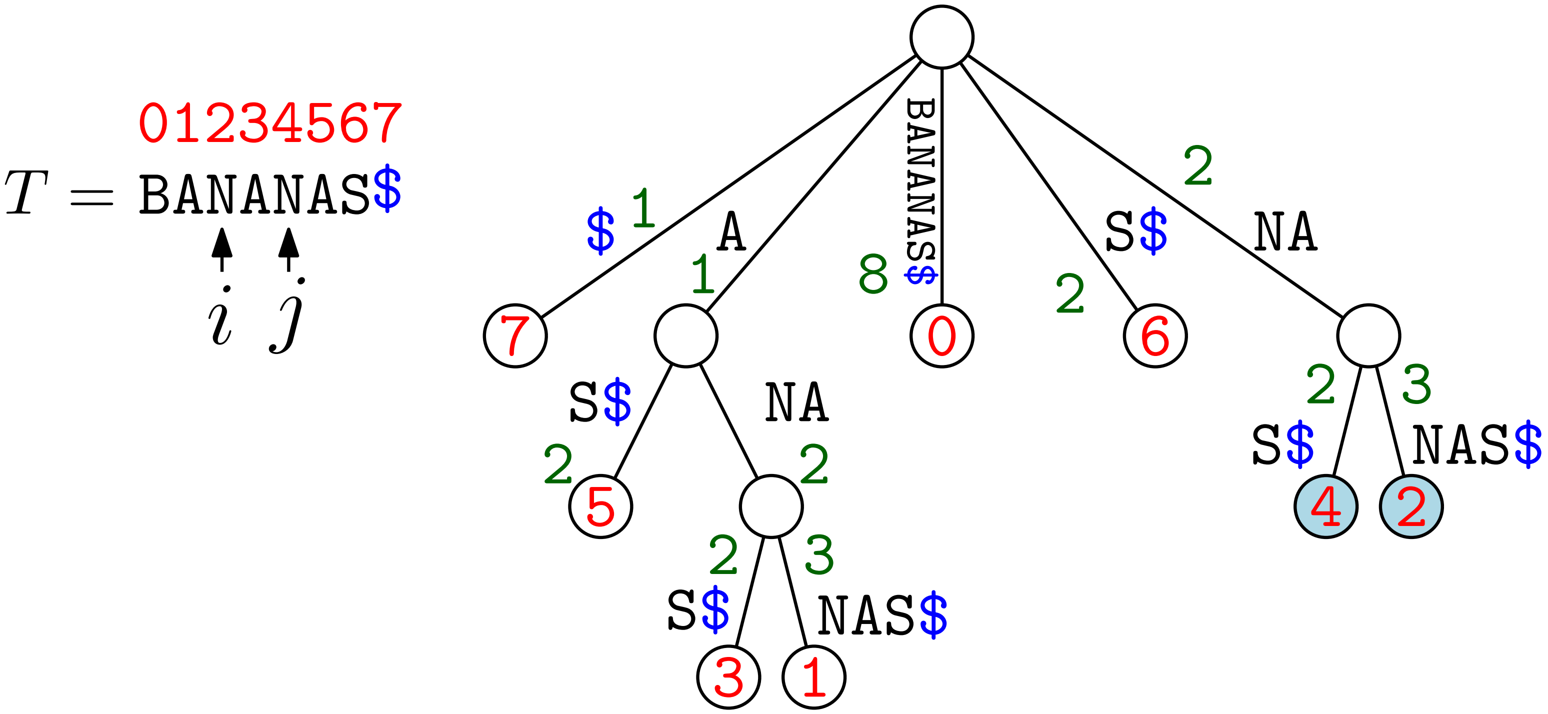
Time: $O(|T|)$

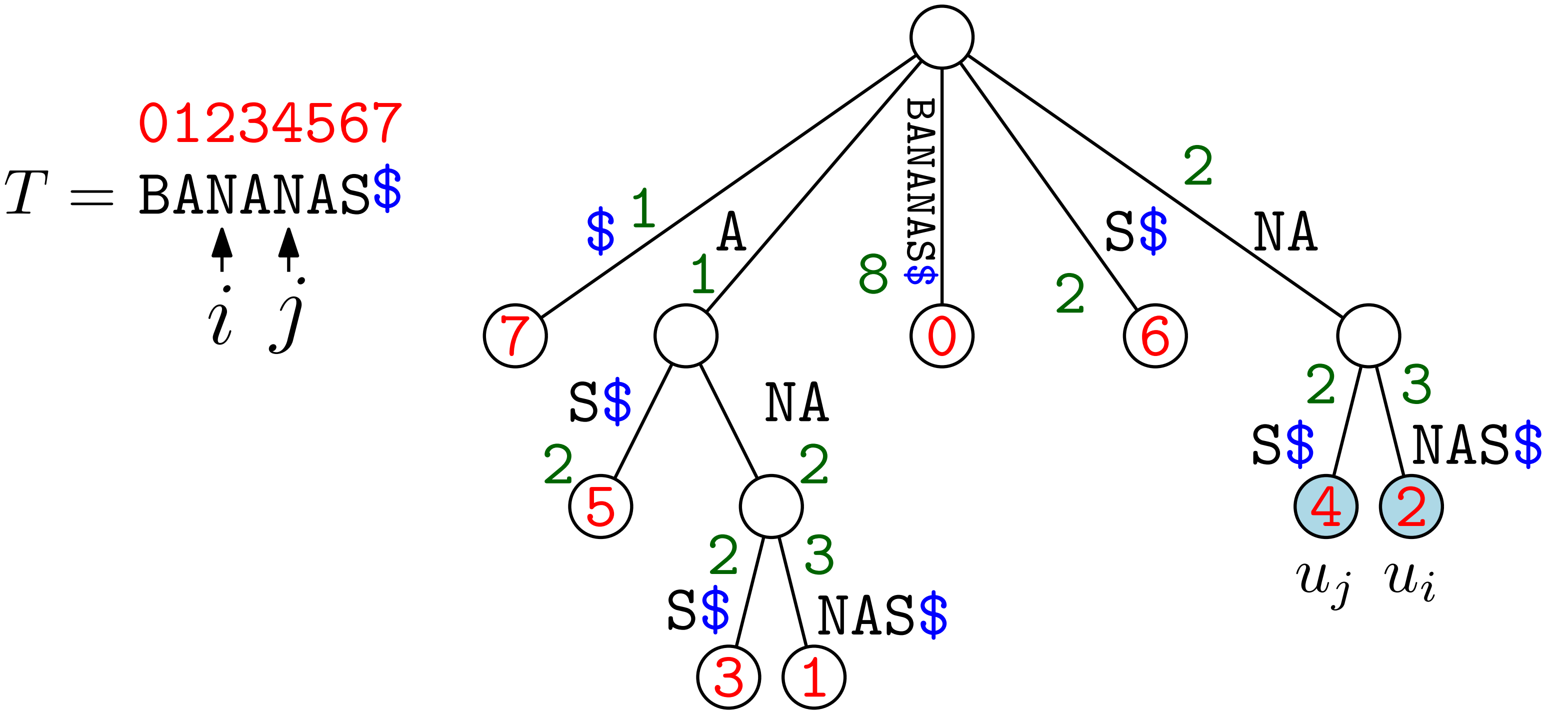
Applications: Longest Common Prefix



Given indicies i and j, find the longest common prefix of $T[i:]$ and $T[j:]$

- Look at the leaves u_i, u_j corresponding to $T[i:]$ and $T[j:]$

Applications: Longest Common Prefix



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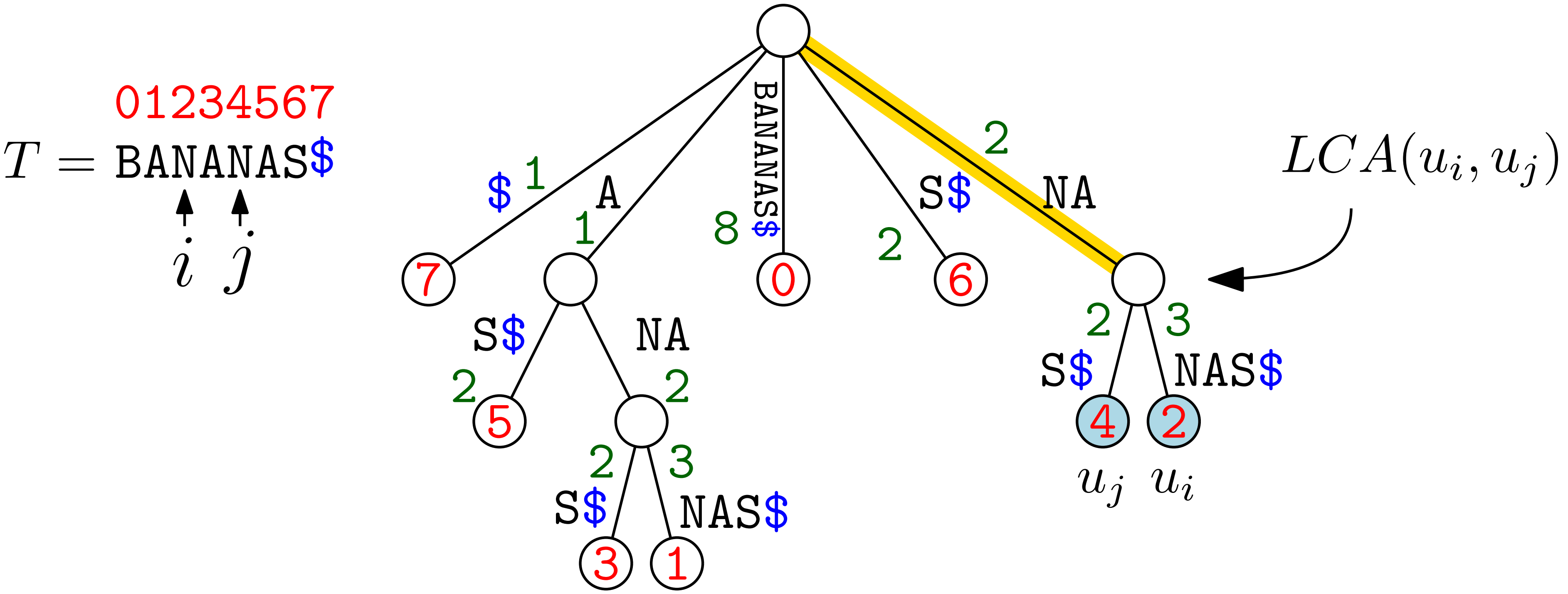
Applications: Longest Common Prefix



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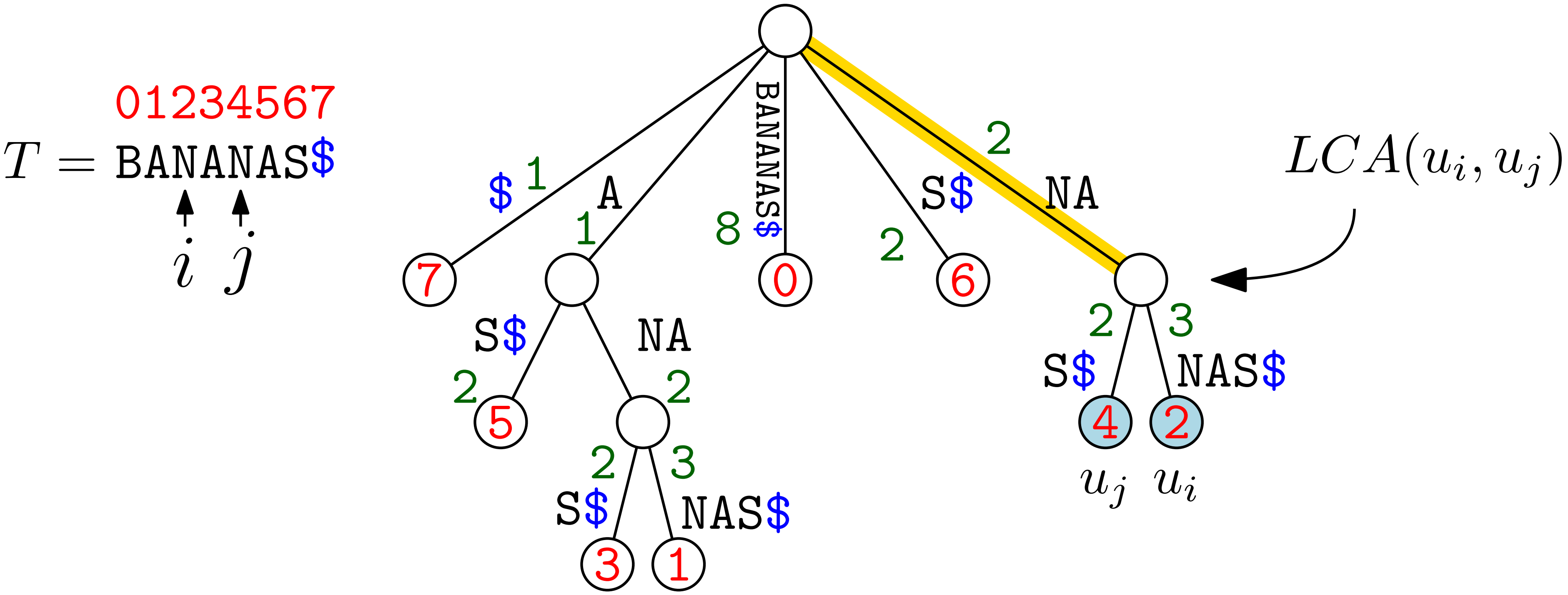

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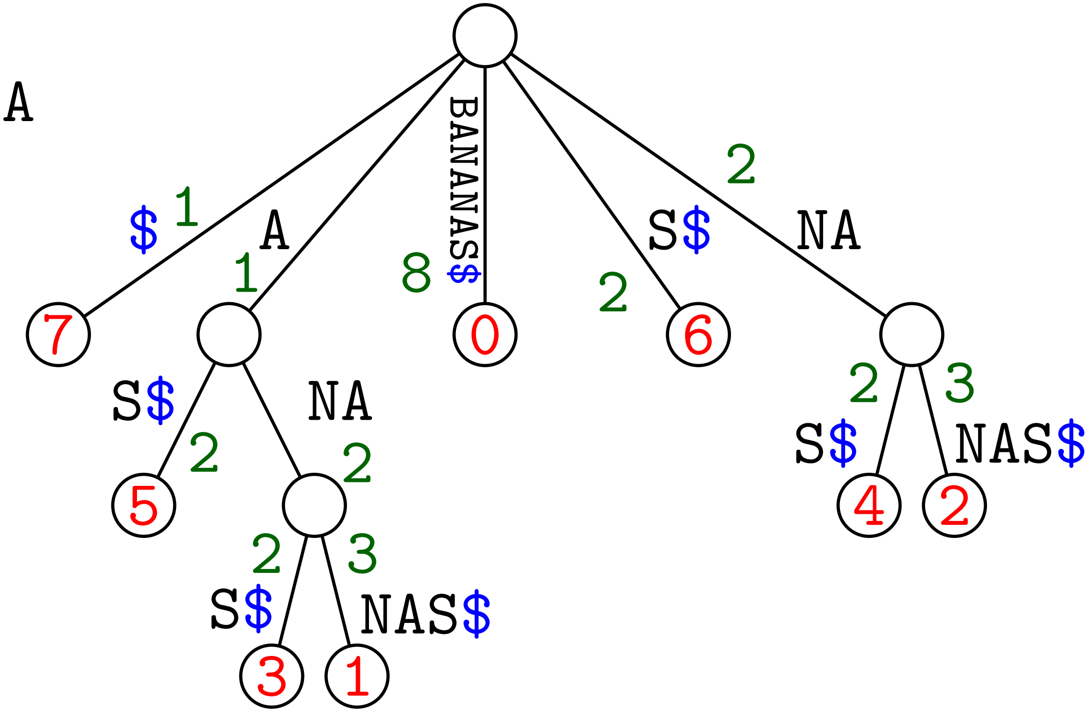
We already know how to answer LCA queries in constant time!

Applications: Fiding Additional Matches

$P = T[2:3] =$ NA



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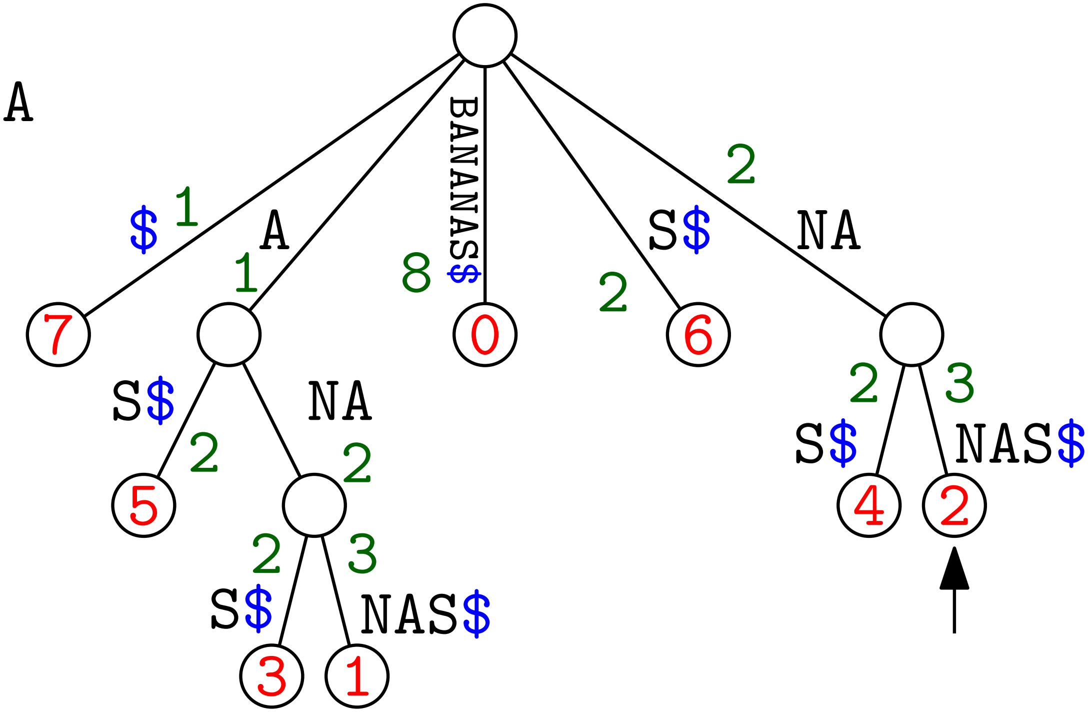

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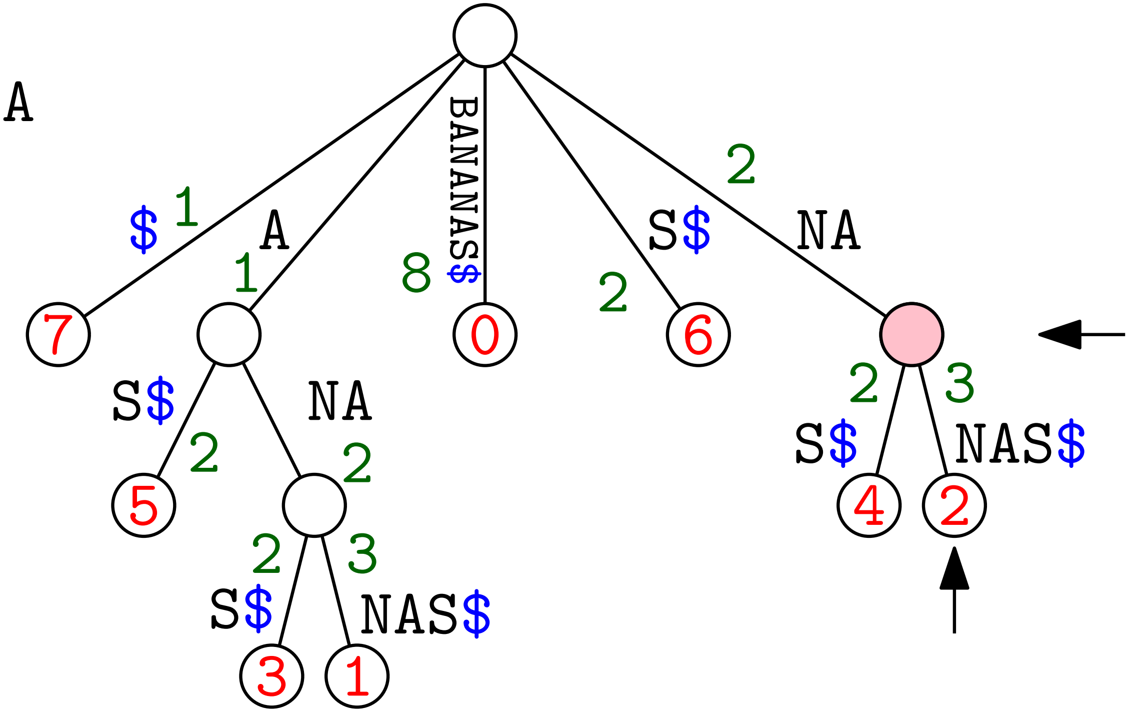

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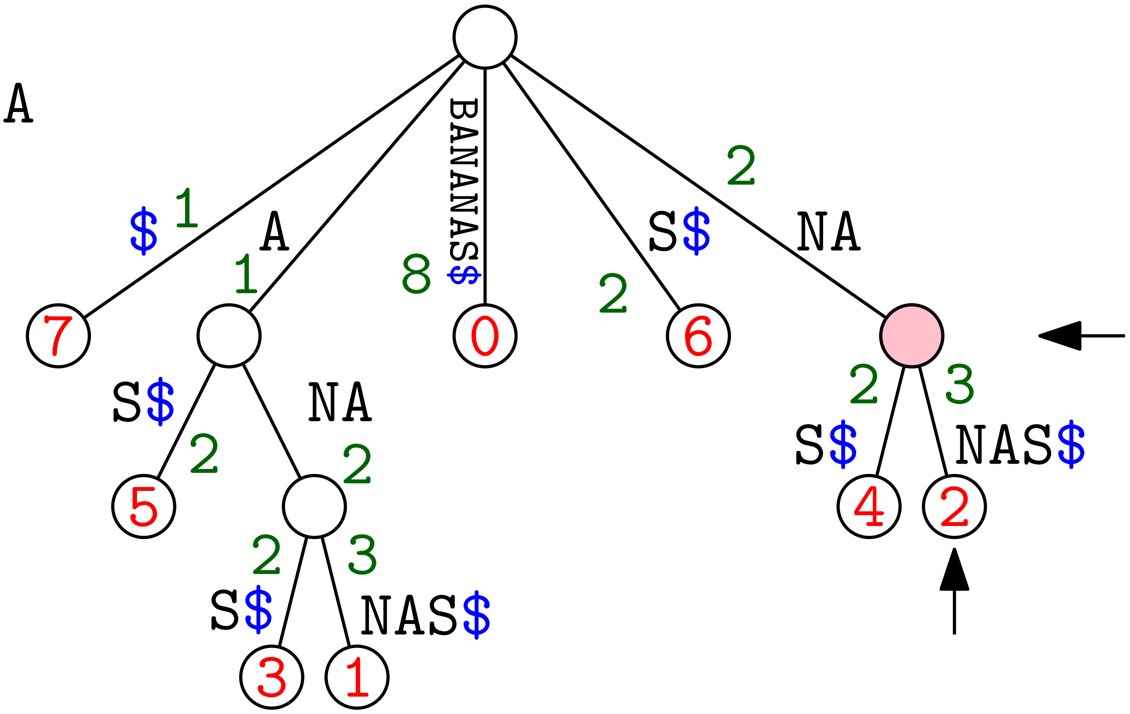

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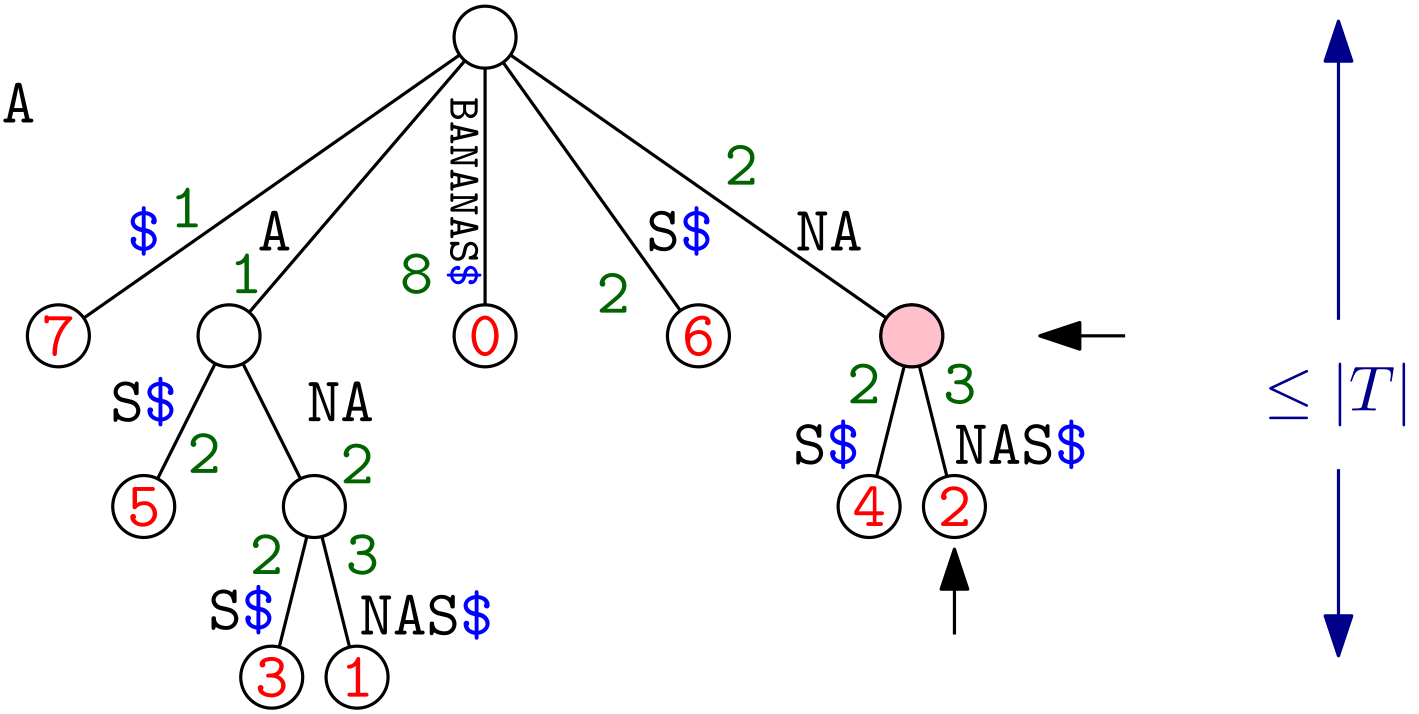

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We can answer weighted LA queries in $O(\log \log |T|)$ time!

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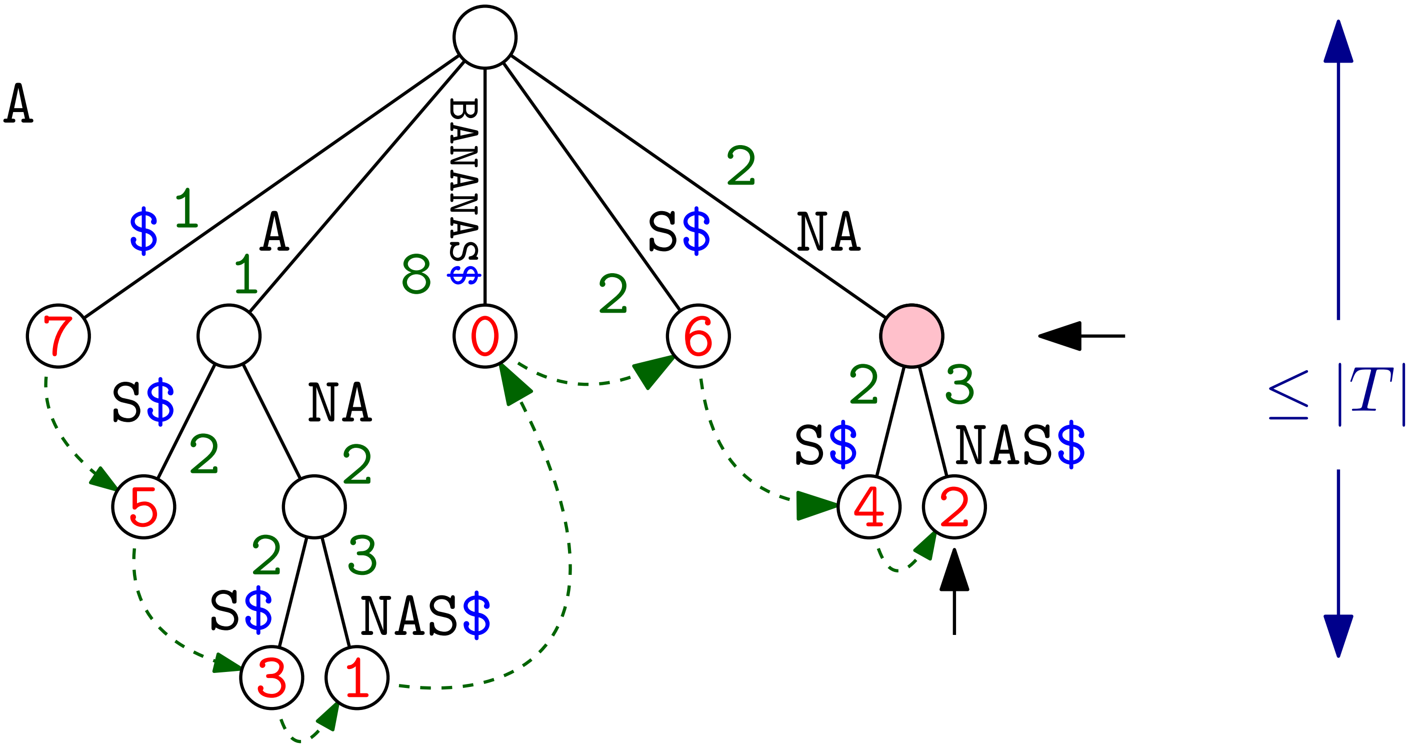

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- We want to quickly find the node that corresponds to P
- Start from the leaf corresponding to $T[i:]$
- Walk **up** the tree for "$|T| - j$" characters
- This is a **weighted** level ancestor query!
- Link leaves to find the other occurrences in $O(1)$ additional time each

We can answer weighted LA queries in $O(\log\log|T|)$ time!

Applications: Document Retrieval

Preprocess collection of documents $T_1, T_2, \ldots, T_k$ to quickly find all documents that contain a pattern P

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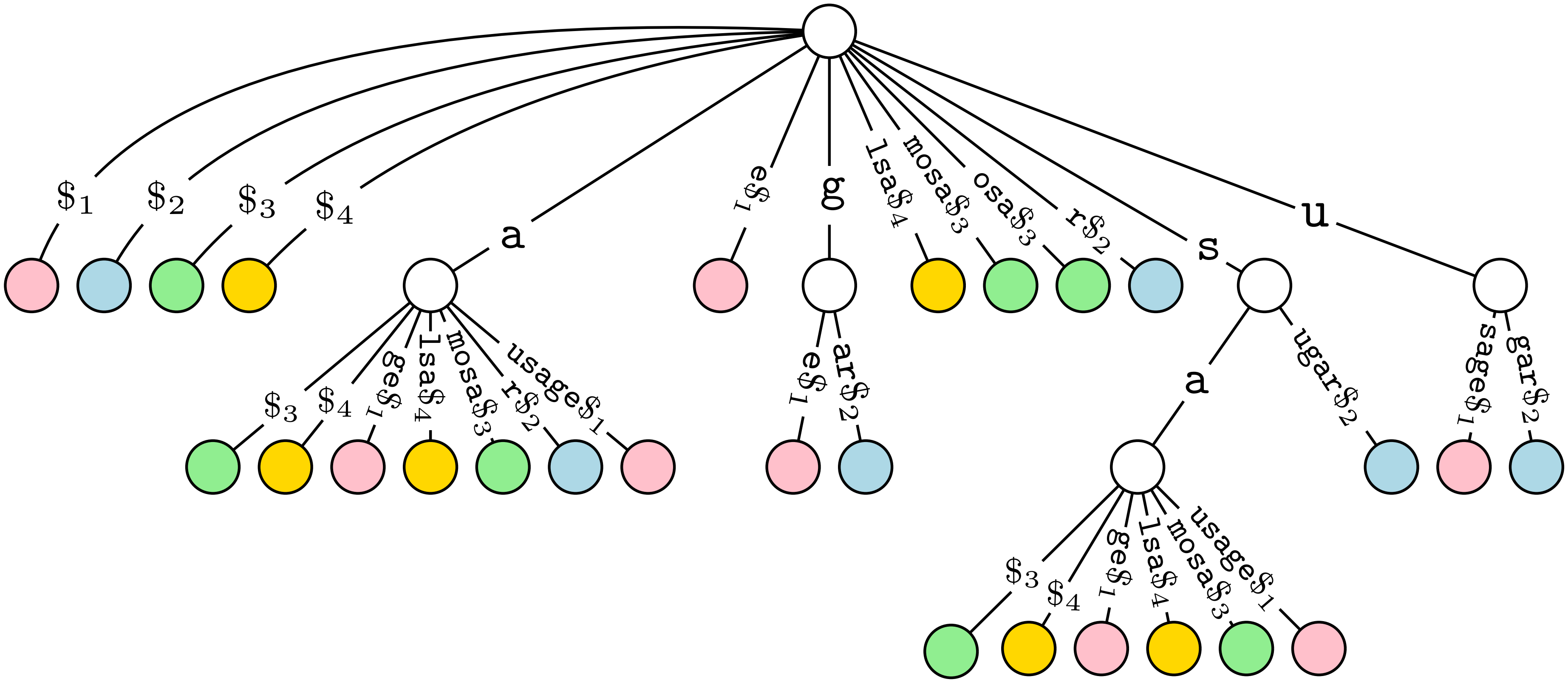
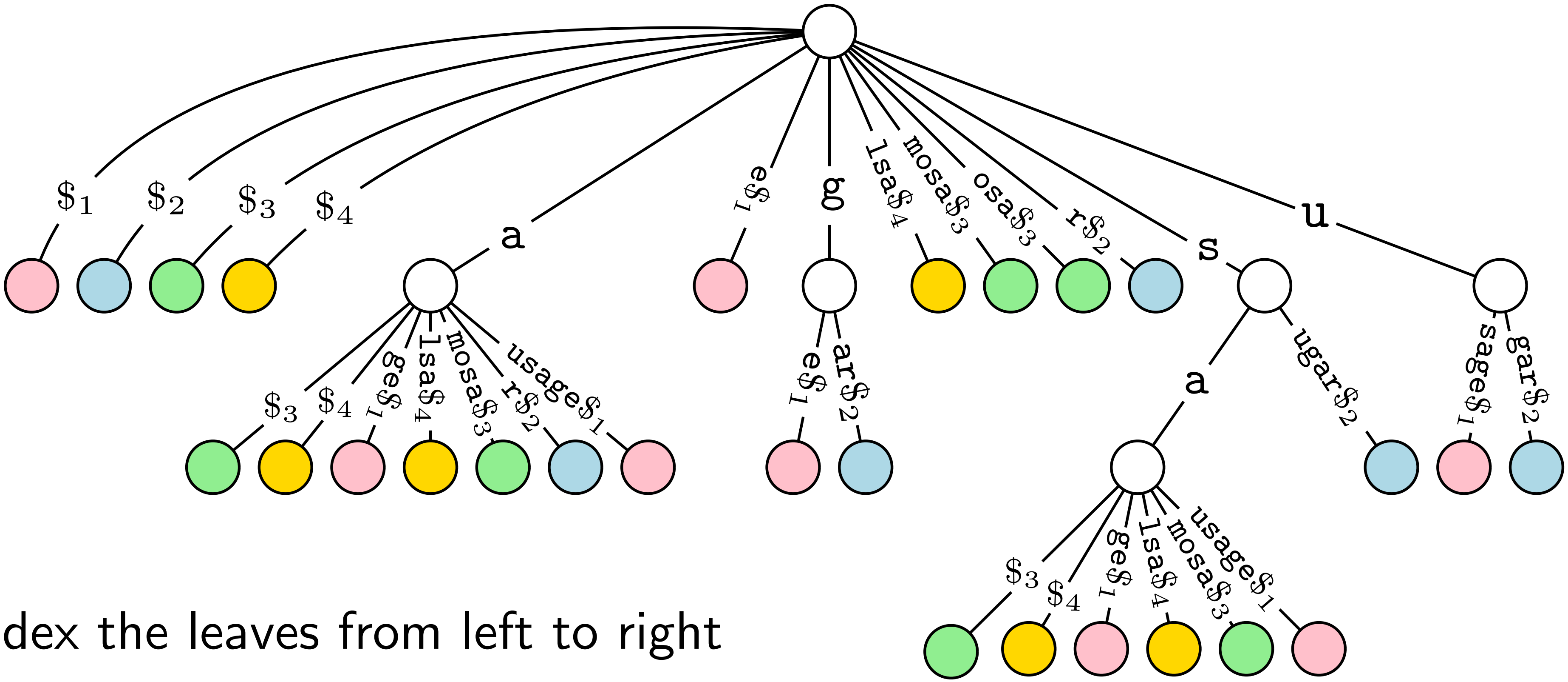
Use the end symbol $\$_i$ for document T_i and build a suffix-tree with the suffxes of all the strings $T_i\$_i$

sausage$\$_1$ sugar$\$_2$ samosa$\$_3$ salsa$\$_4$

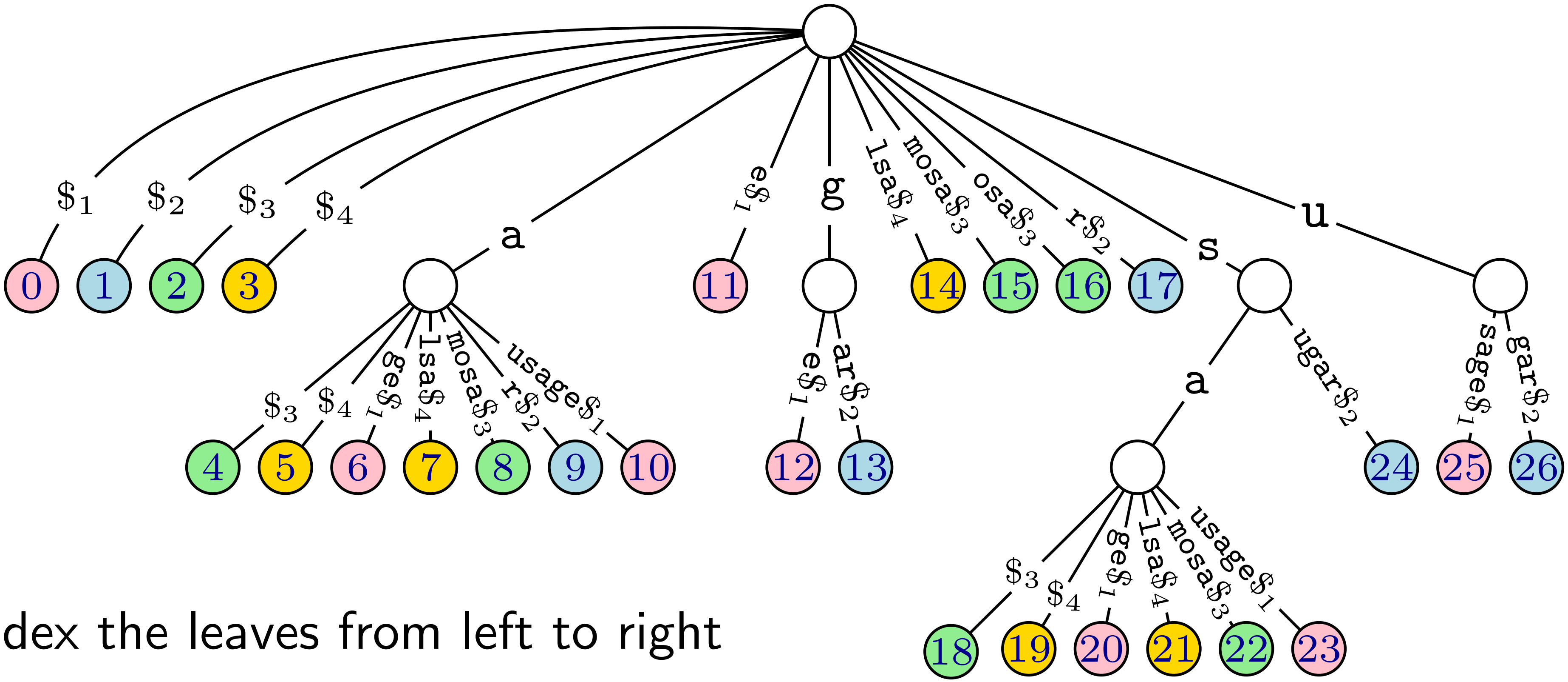


Applications: Document Retrieval



Index the leaves from left to right

Applications: Document Retrieval



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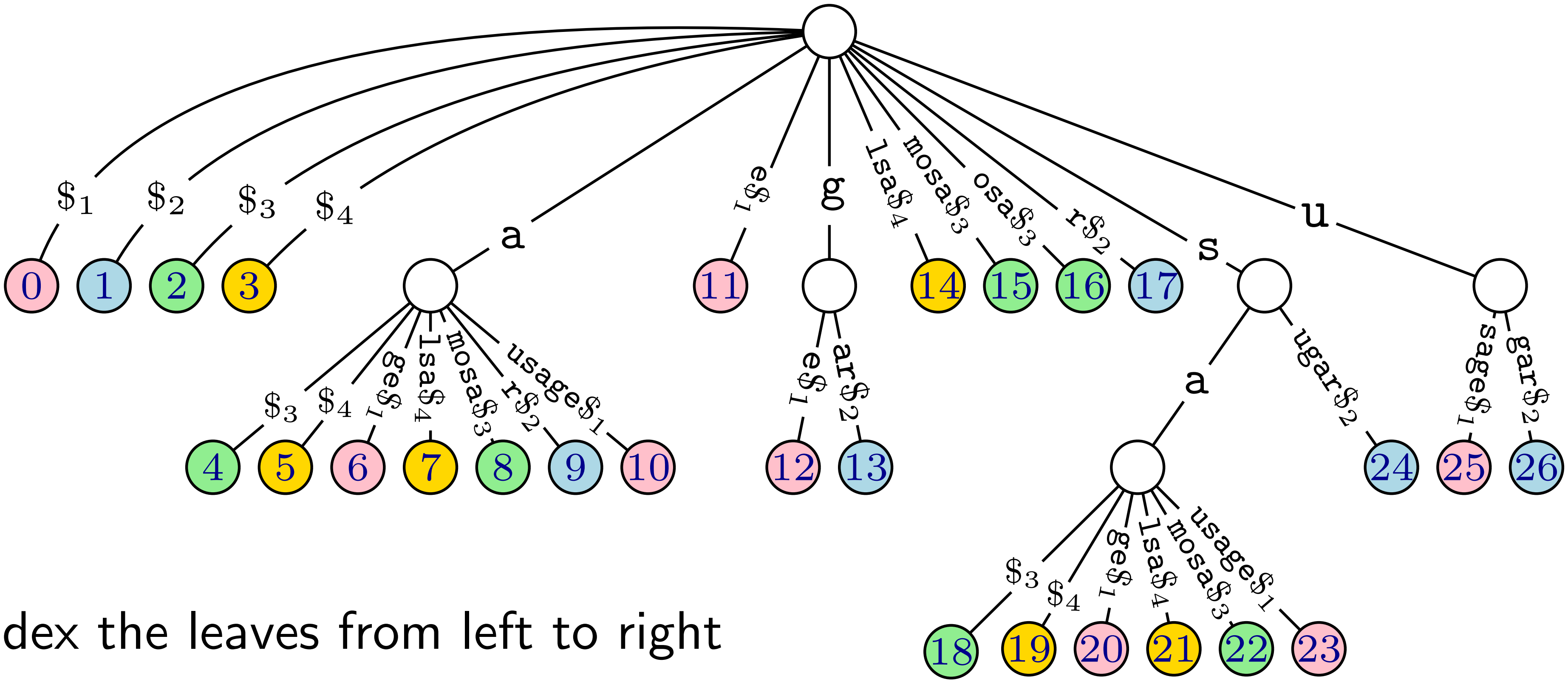
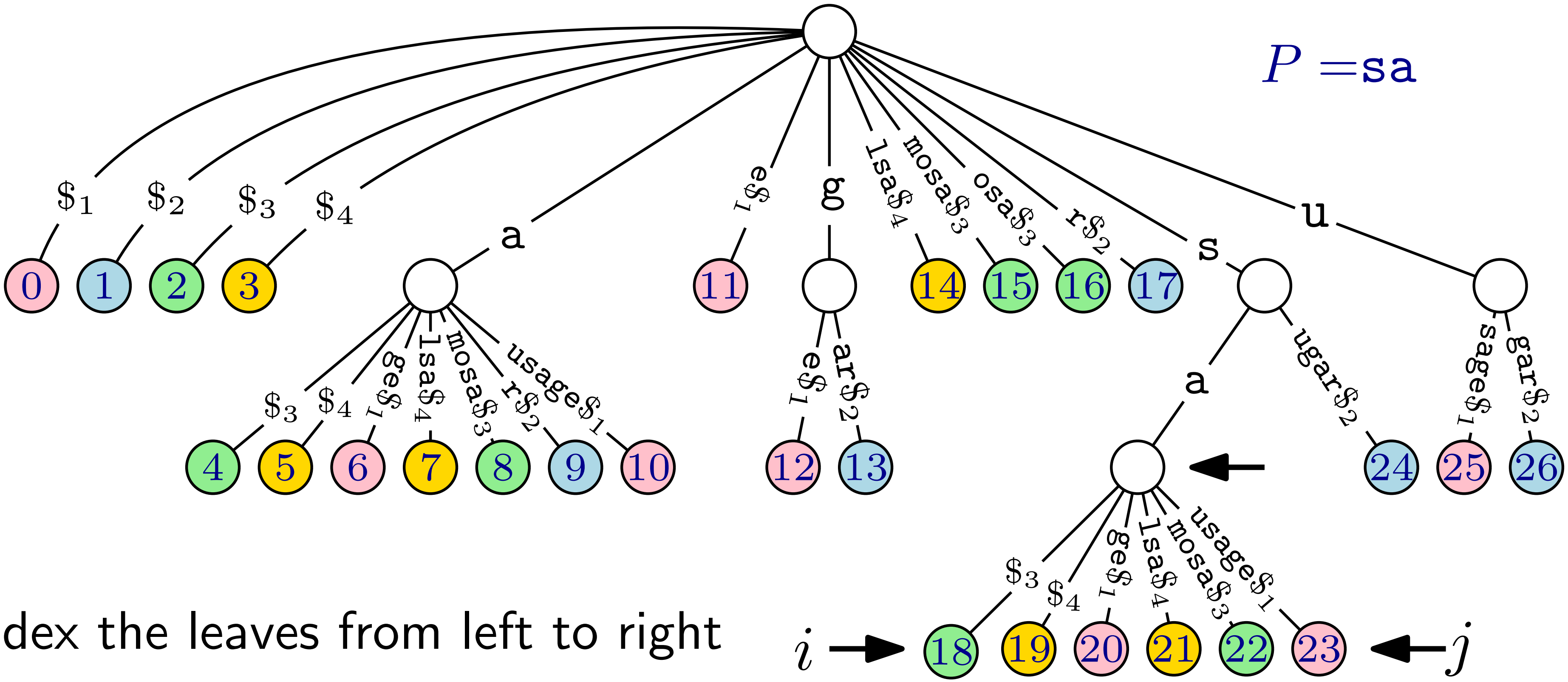


Index the leaves from left to right

Store an array A where $A[i]$ poinst to the document of leaf i

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
🟥	🟦	🟩	🟨	🟩	🟨	🟥	🟨	🟩	🟦	🟥	🟥	🟥	🟦	🟨	🟩	🟩	🟦	🟩	🟨	🟥	🟨	🟩	🟥	🟦	🟥	🟦

Applications: Document Retrieval



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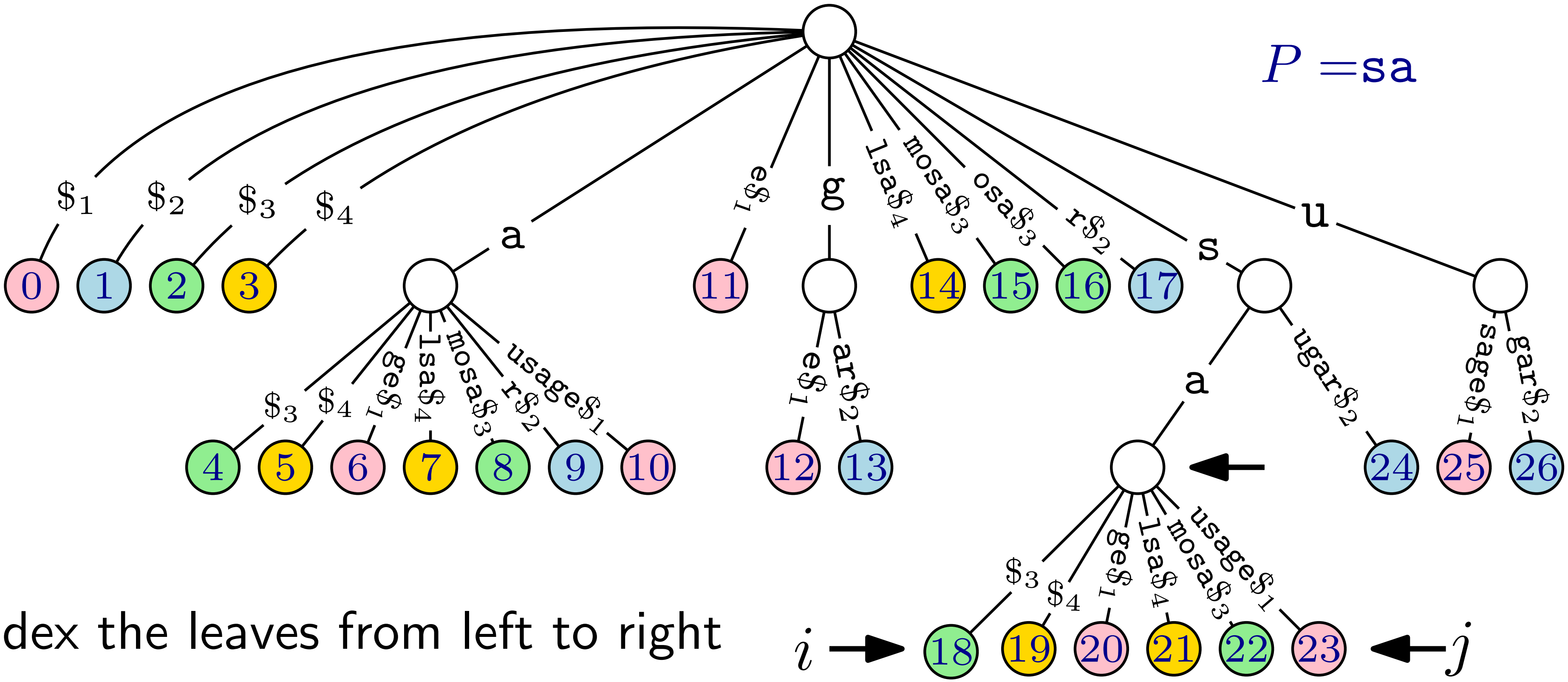
Searching for a pattern P returns the interval $A[i:j]$ containing all and only the leaves corresponding to the matches of P

Applications: Document Retrieval



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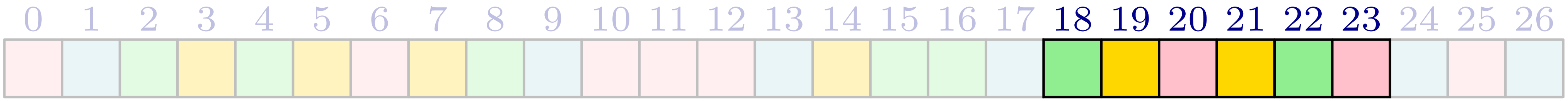

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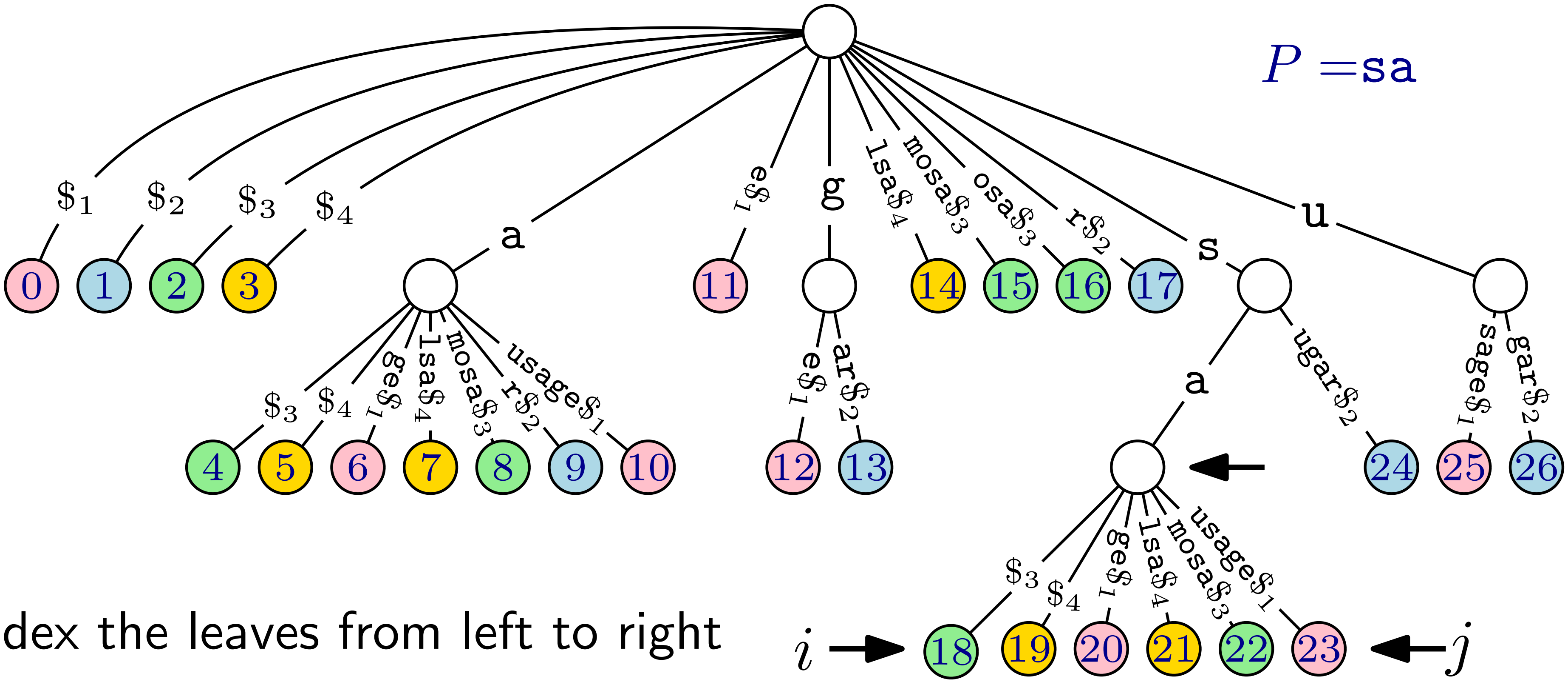
Find all distinct documents (colors) in $A[i:j]$

Applications: Document Retrieval



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🟥	🟦	🟩	🟨	🟩	🟨	🟥	🟨	🟩	🟦	🟥	🟥	🟥	🟦	🟨	🟩	🟩	🟦	🟩	🟨	🟥	🟨	🟩	🟥	🟦	🟥	🟦

Searching for a pattern P retu

only the leaves corresponding t

Find all distinct documents (c

Time:

$O(|P| + \log|\Sigma| + \#$ retrieved documents$)$

via range minimum queries

Constructing Suffix Trees & Suffix Arrays

Suffix Arrays & Suffix Trees

$T =$ BANANAS

Sort all suffixes along with their start index

0	BANANAS$
1	ANANAS$
2	NANAS$
3	ANAS$
4	NAS$
5	AS$
6	S$
7	$

Suffix Arrays & Suffix Trees

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7	\$
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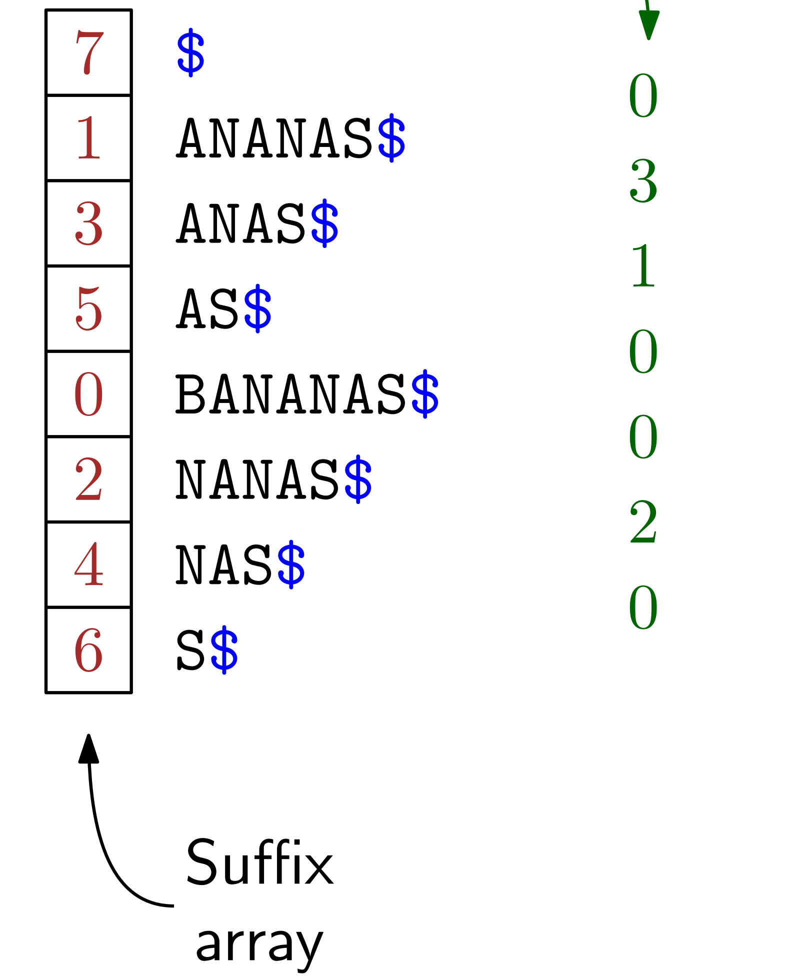
6	S$

Suffix array

Suffix Arrays & Suffix Trees

$T =$ `BANANAS`

Length of the longest common prefix between adjacent suffixes (w.r.t. the sorted order)



7	$	
		0
1	ANANAS$	
		3
3	ANAS$	
		1
5	AS$	
		0
0	BANANAS$	
		0
2	NANAS$	
		2
4	NAS$	
		0
6	S$	

Suffix array

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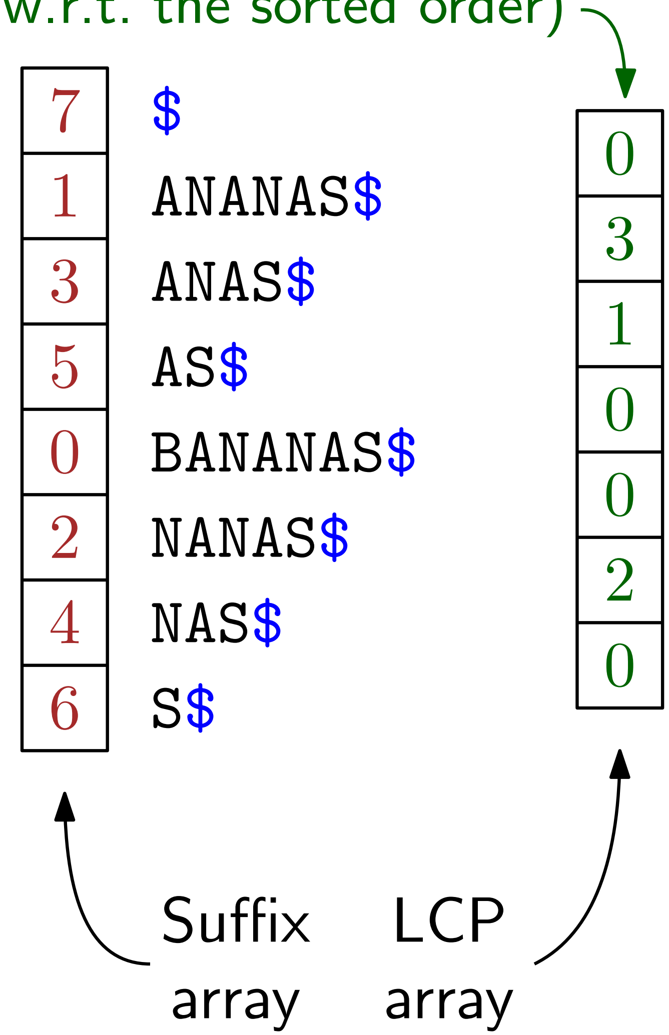

7	$
1	ANANAS$
3	ANAS$
5	AS$
0	BANANAS$
2	NANAS$
4	NAS$
6	S$

0
3
1
0
0
2
0

Suffix array

LCP array

Suffix Arrays & Suffix Trees

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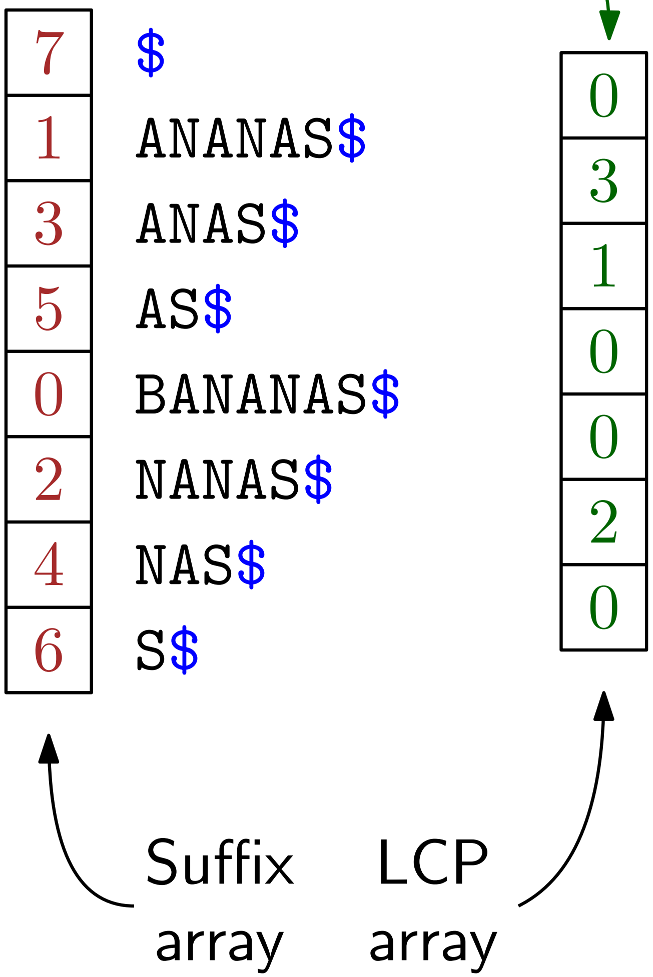

7	$	0
1	ANANAS$	3
3	ANAS$	1
5	AS$	0
0	BANANAS$	0
2	NANAS$	2
4	NAS$	0
6	S$	

Suffix array

LCP array

We can construct a suffix tree from the Suffix and LCP arrays

A construction similar to the one of cartesian trees yields the subtree of branching vertices

0	3	1	0	0	2	0

Suffix Arrays & Suffix Trees

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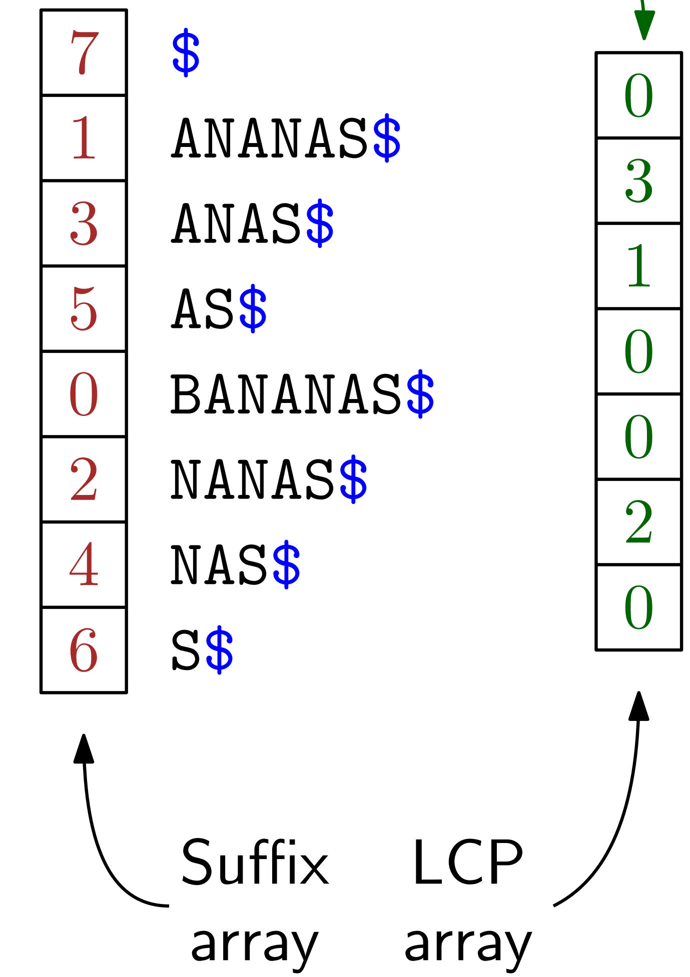

Suffix array	
7	$
1	ANANAS$
3	ANAS$
5	AS$
0	BANANAS$
2	NANAS$
4	NAS$
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LCP array
0
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1
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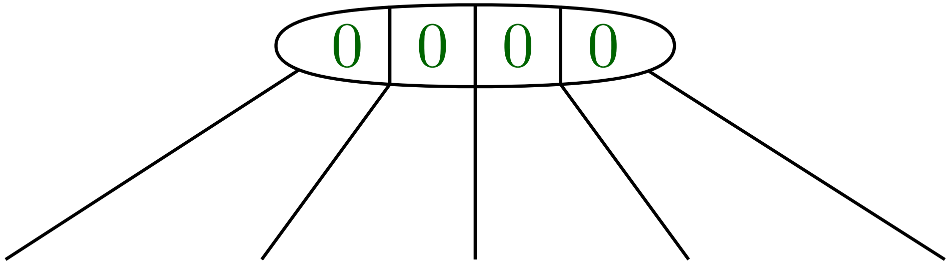

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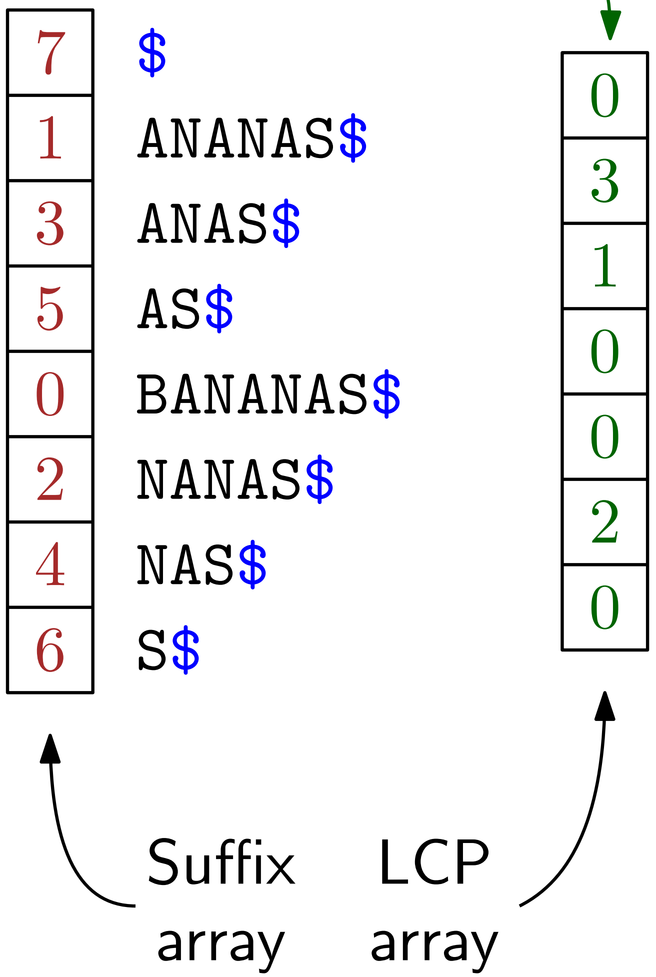

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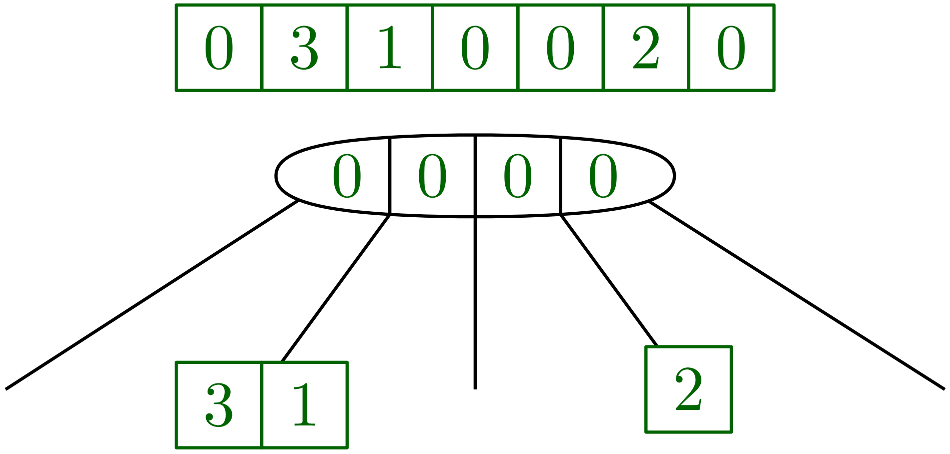

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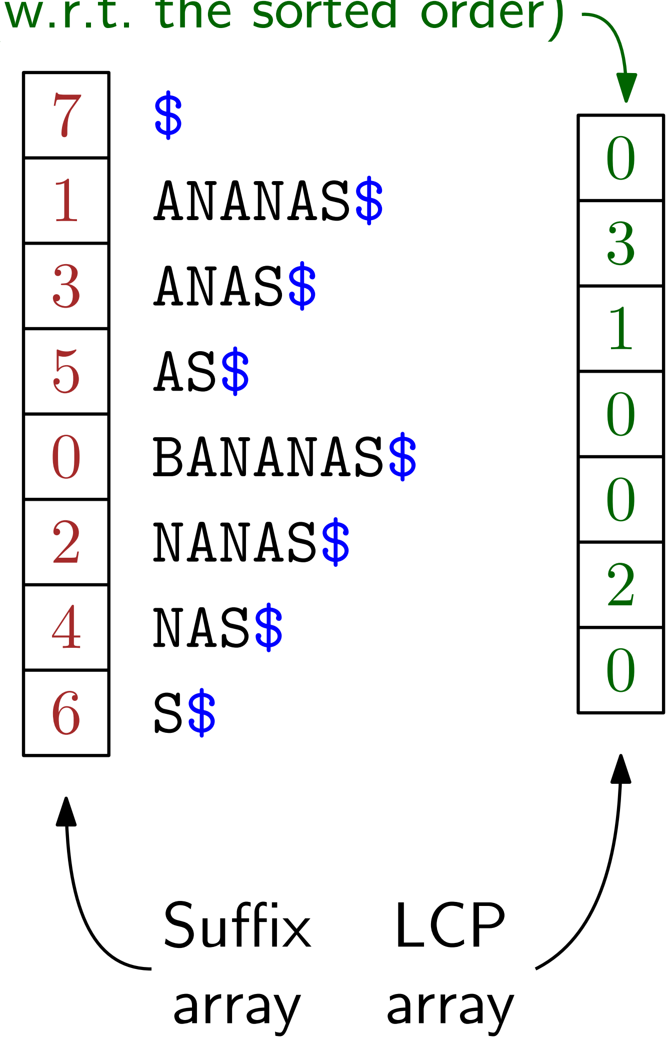

Suffix array		LCP array
7	$	0
1	ANANAS$	3
3	ANAS$	1
5	AS$	0
0	BANANAS$	0
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6	S$	

Suffix array

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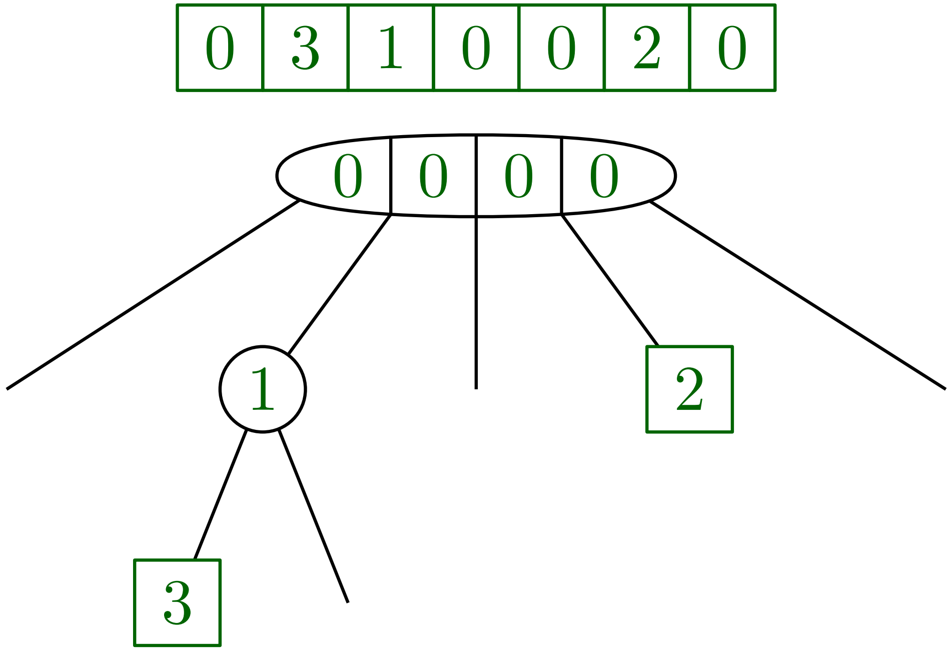

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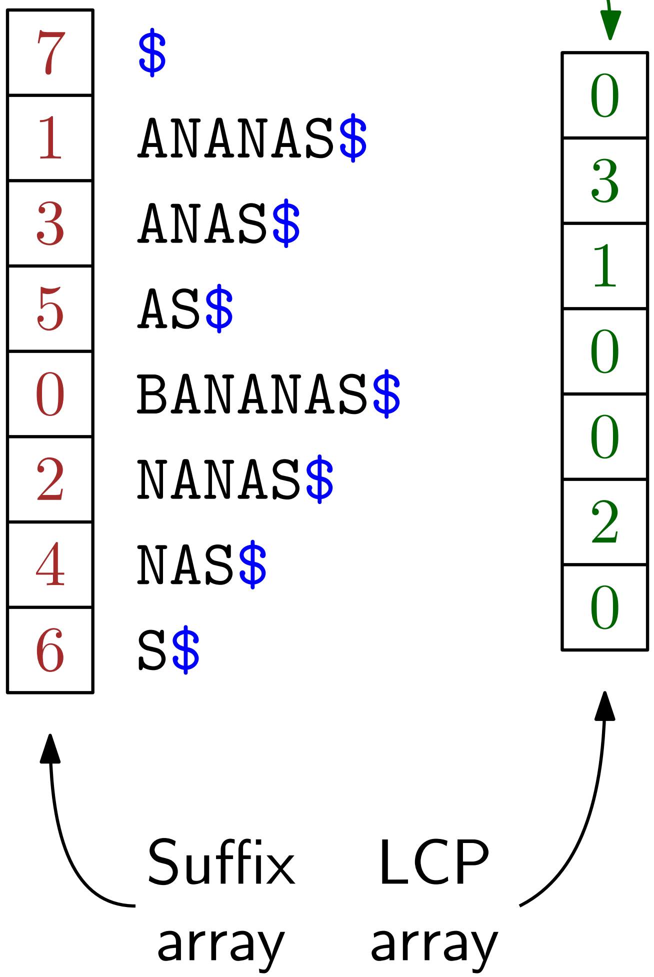

7	$
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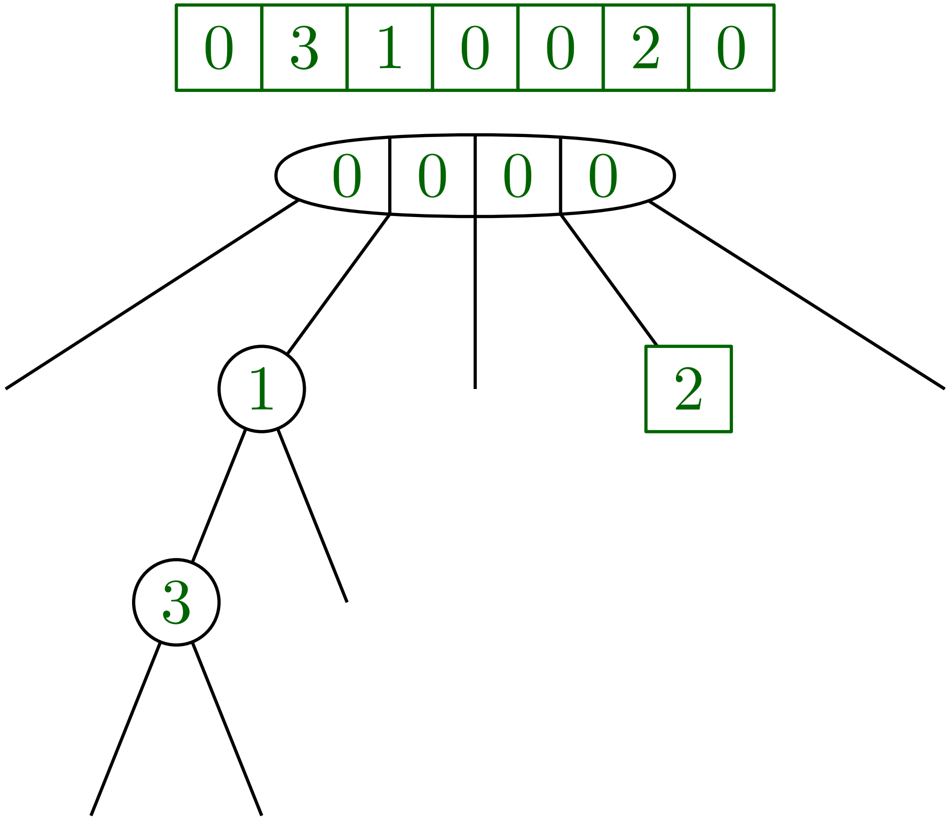

0	3	1	0	0	2	0

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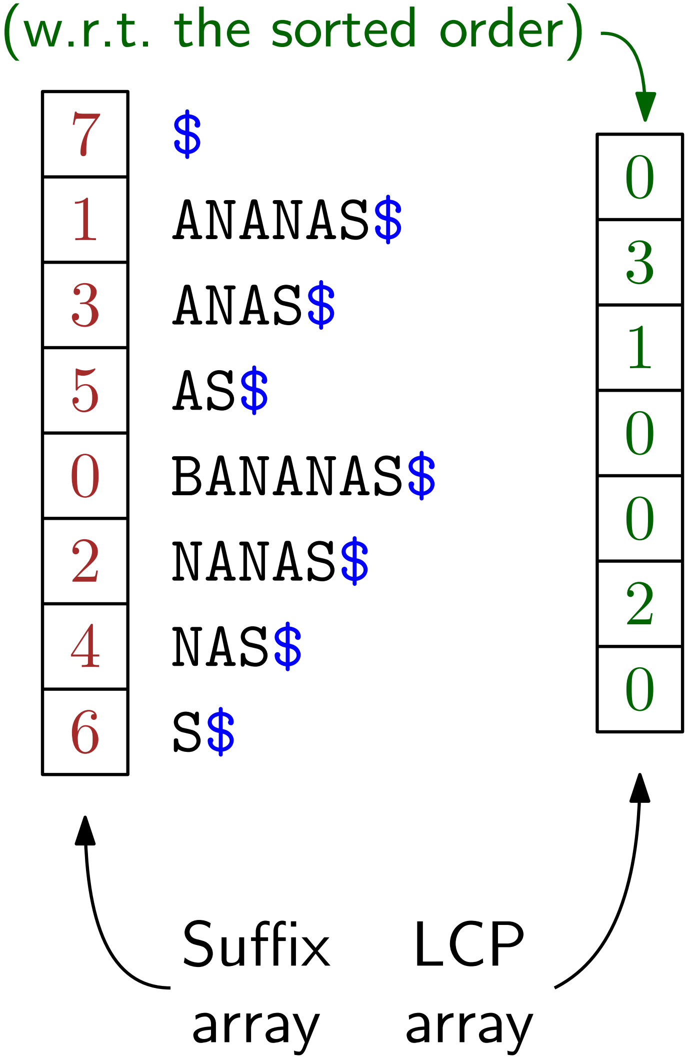

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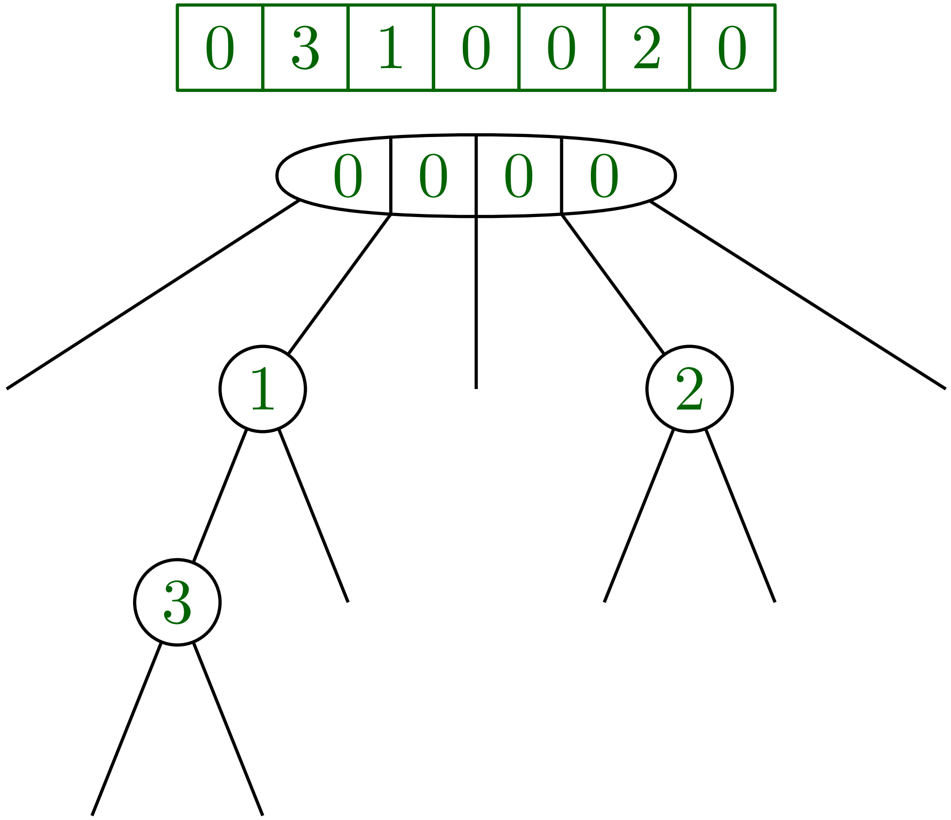

0	3	1	0	0	2	0

Suffix Arrays & Suffix Trees

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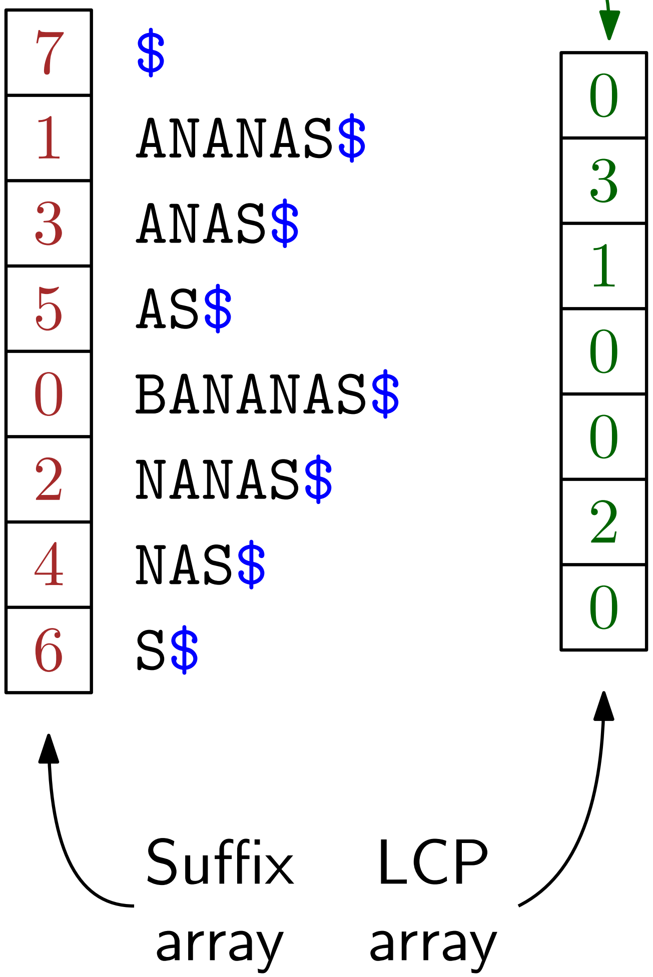
Length of the longest common prefix between adjacent suffixes (w.r.t. the sorted order)



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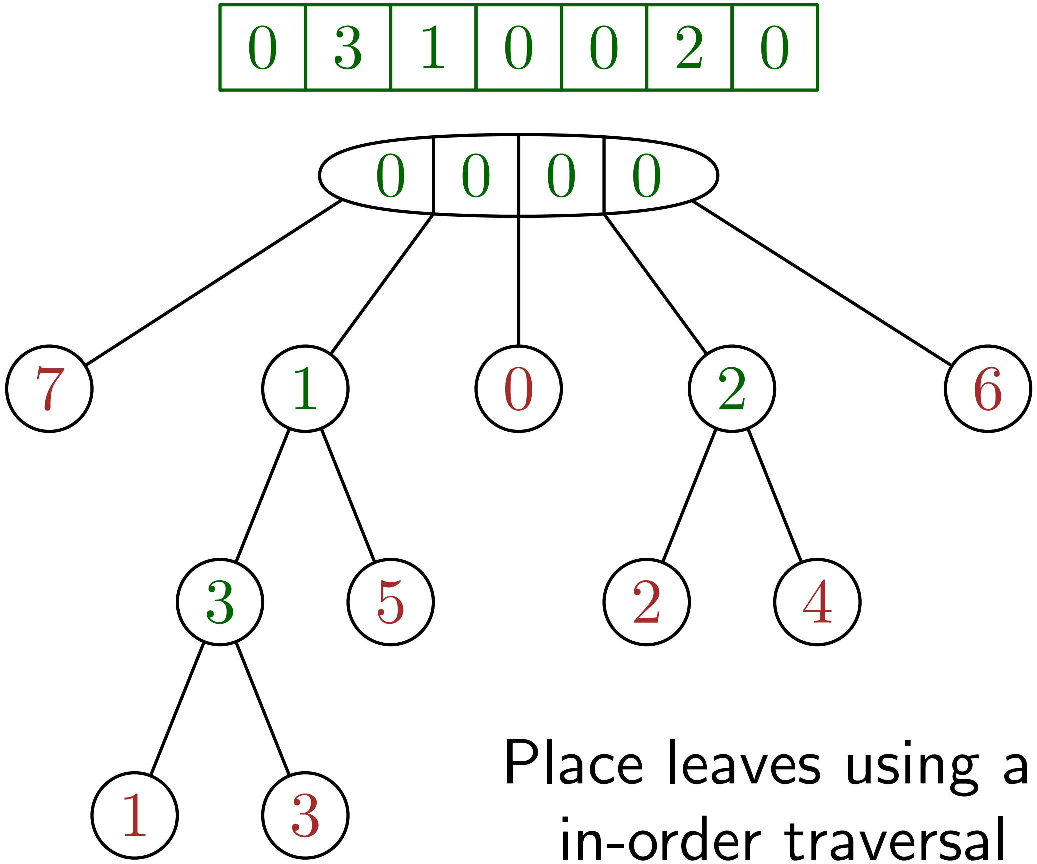
5	AS$	0
0	BANANAS$	0
2	NANAS$	2
4	NAS$	0
6	S$	

We can construct a suffix tree from the Suffix and LCP arrays

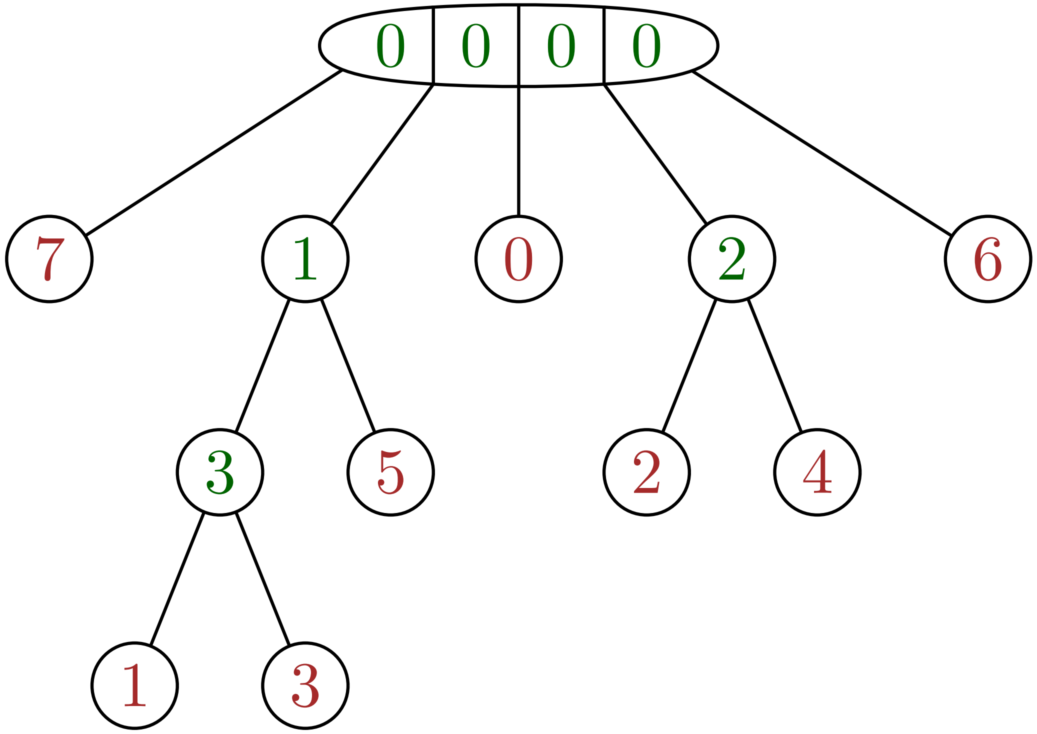
A construction similar to the one of cartesian trees yields the subtree of branching vertices



Place leaves using an in-order traversal

Suffix Arrays & Suffix Trees

Branching vertices are labelled with their *letter depth*, i.e., the number of letters of the prefix encoded in the path from the root to the vertex



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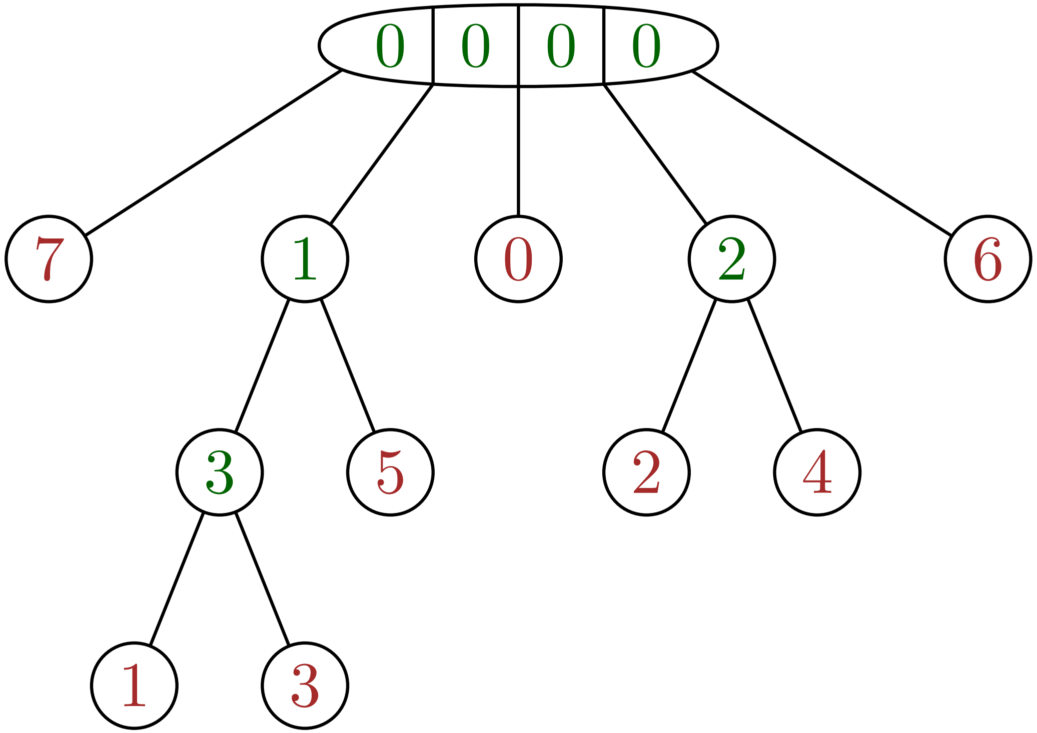
Edge labels are easy to reconstruct with a post-order visit



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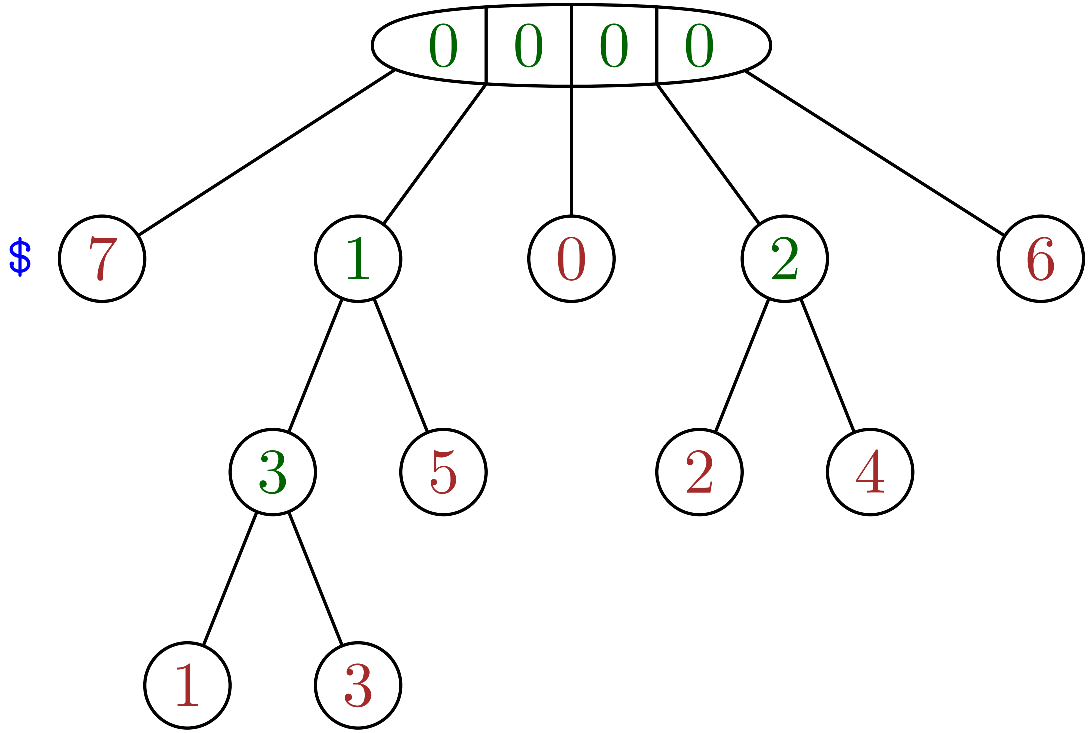

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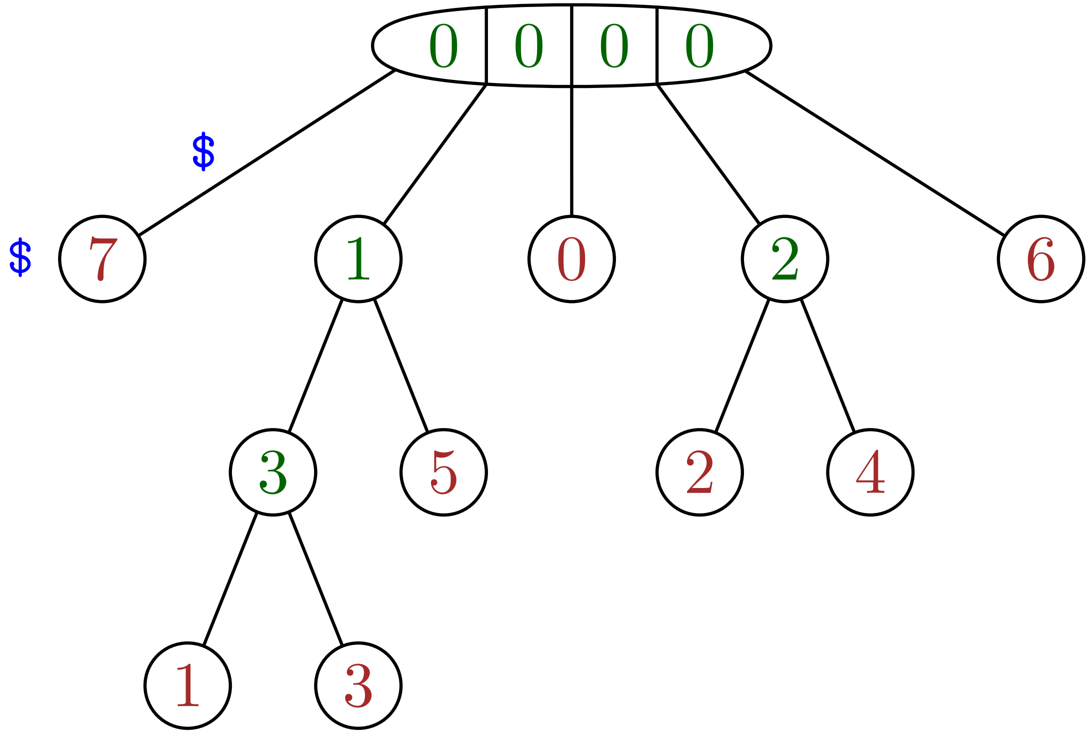

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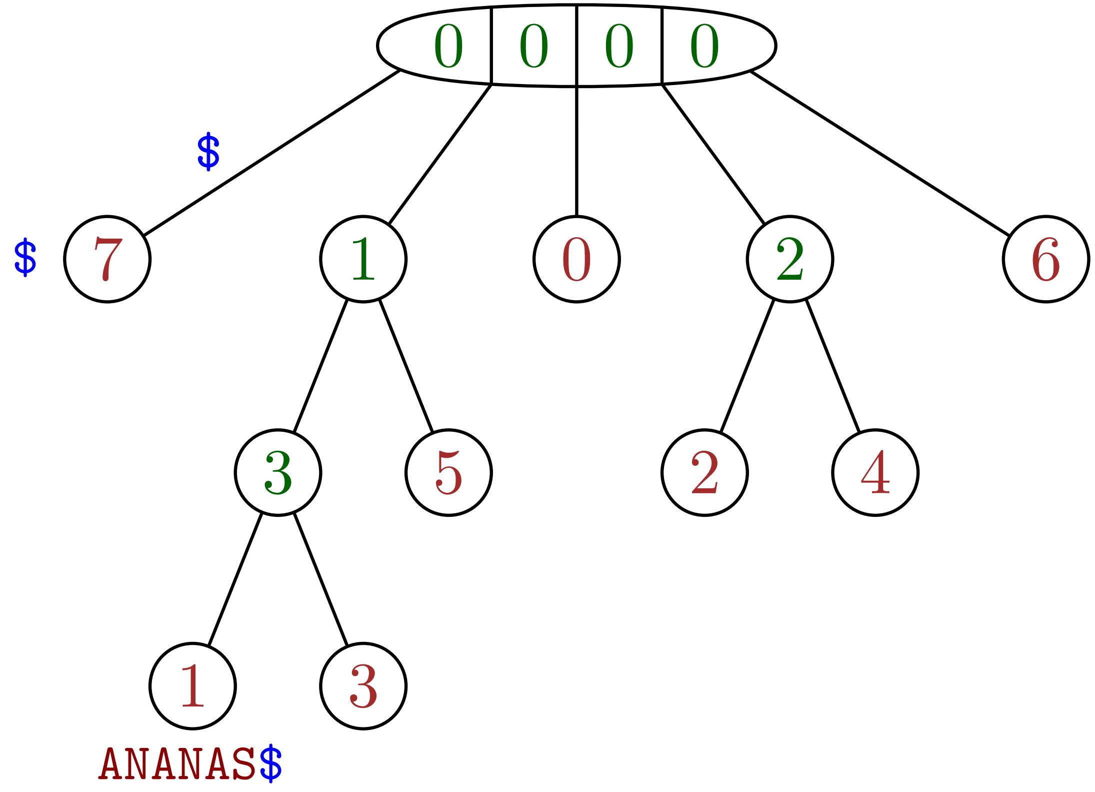

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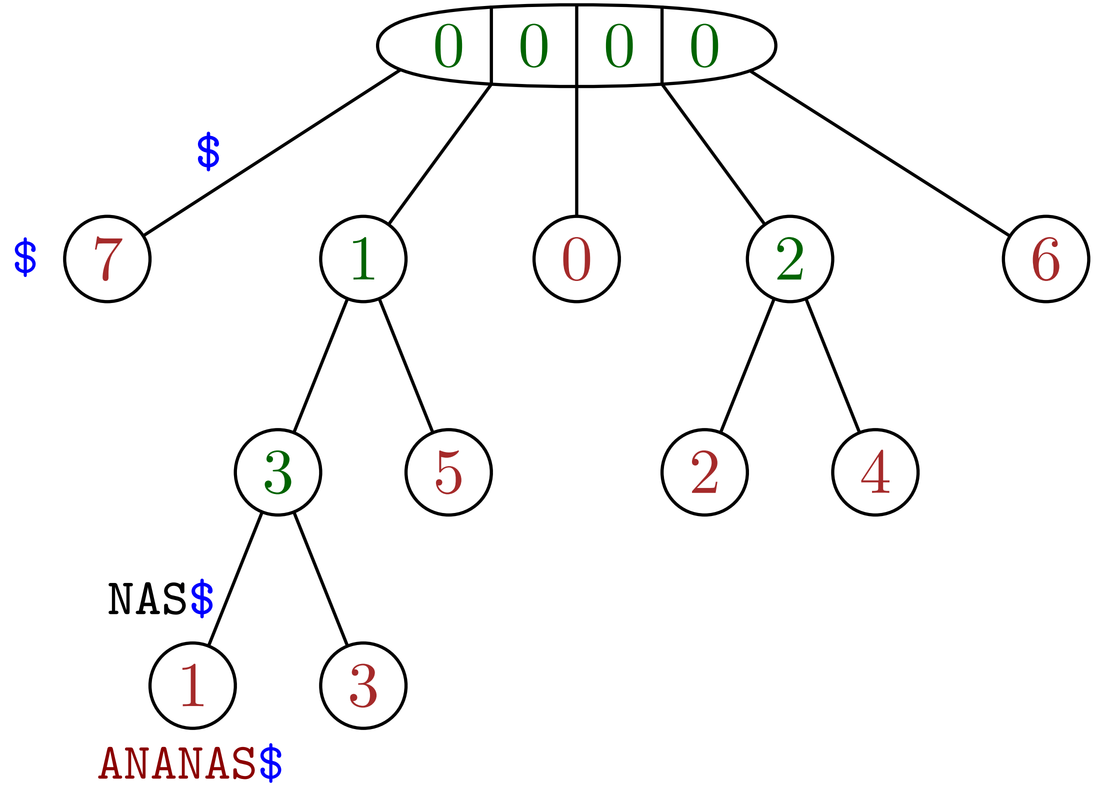

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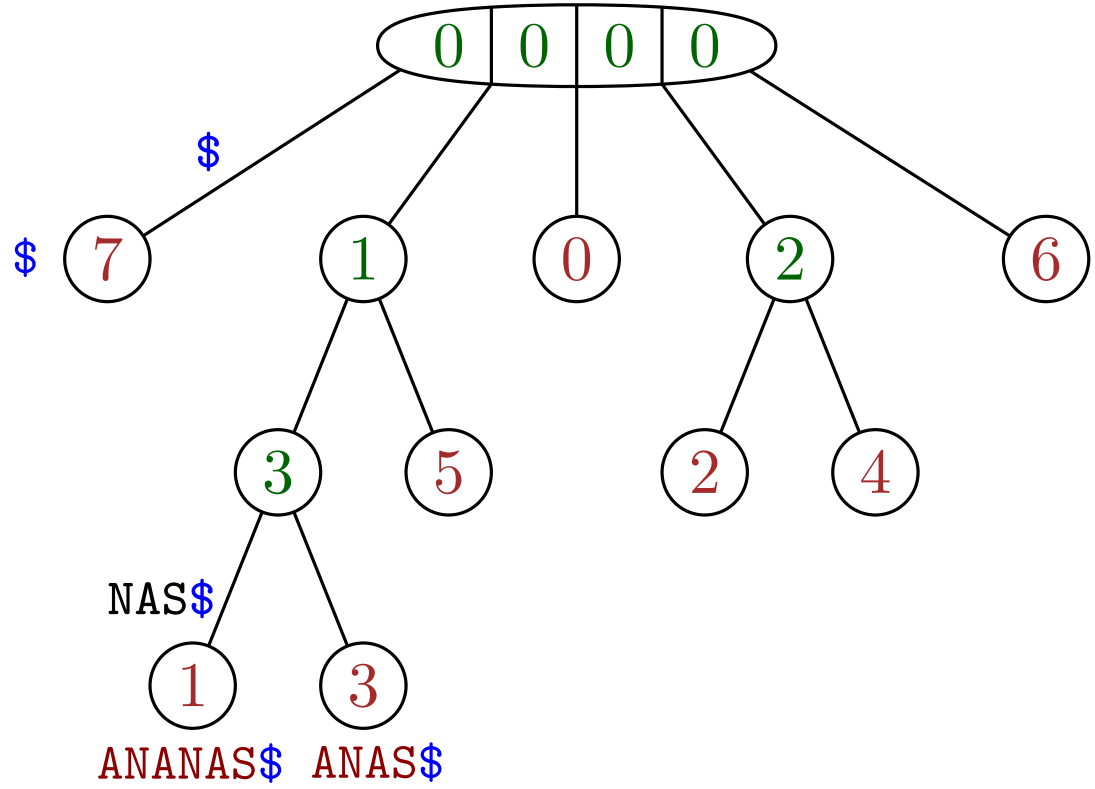

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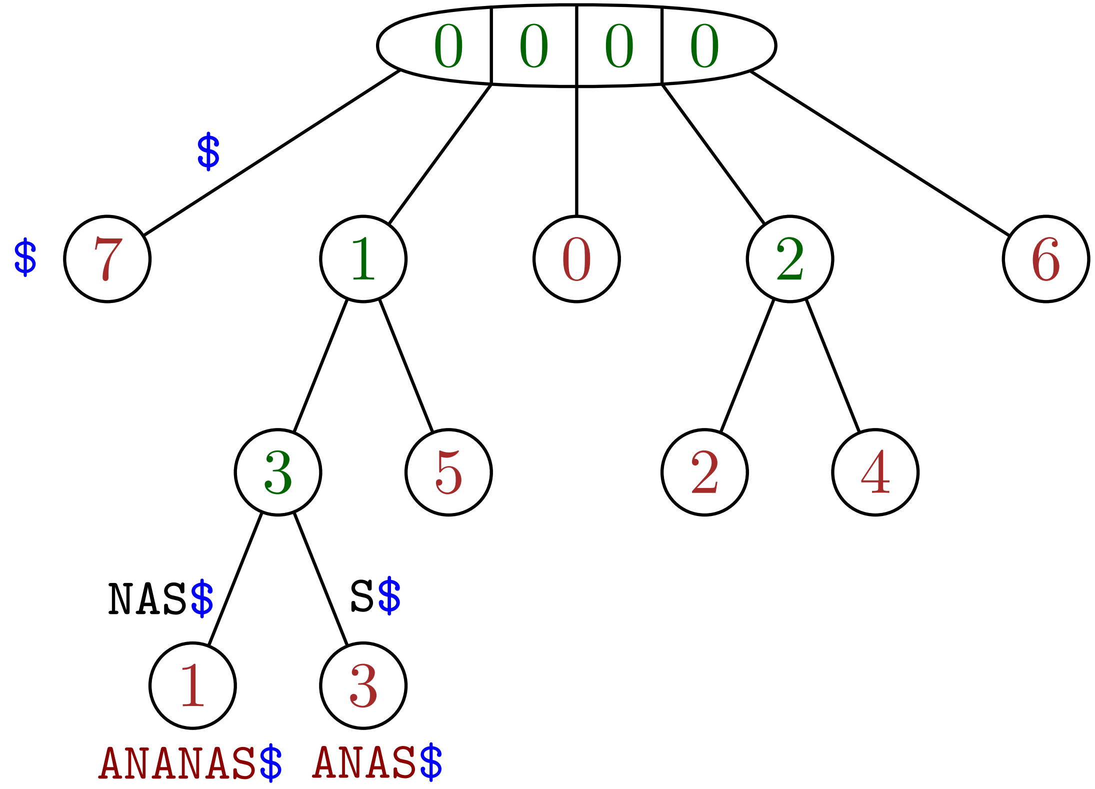

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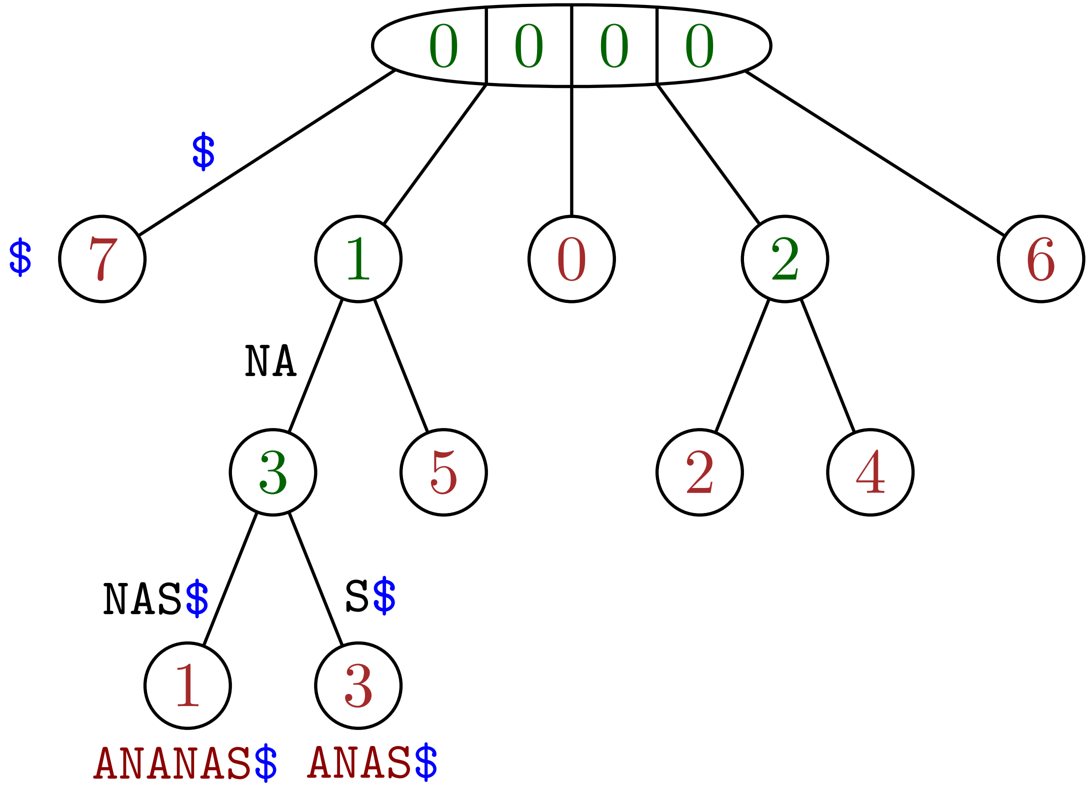

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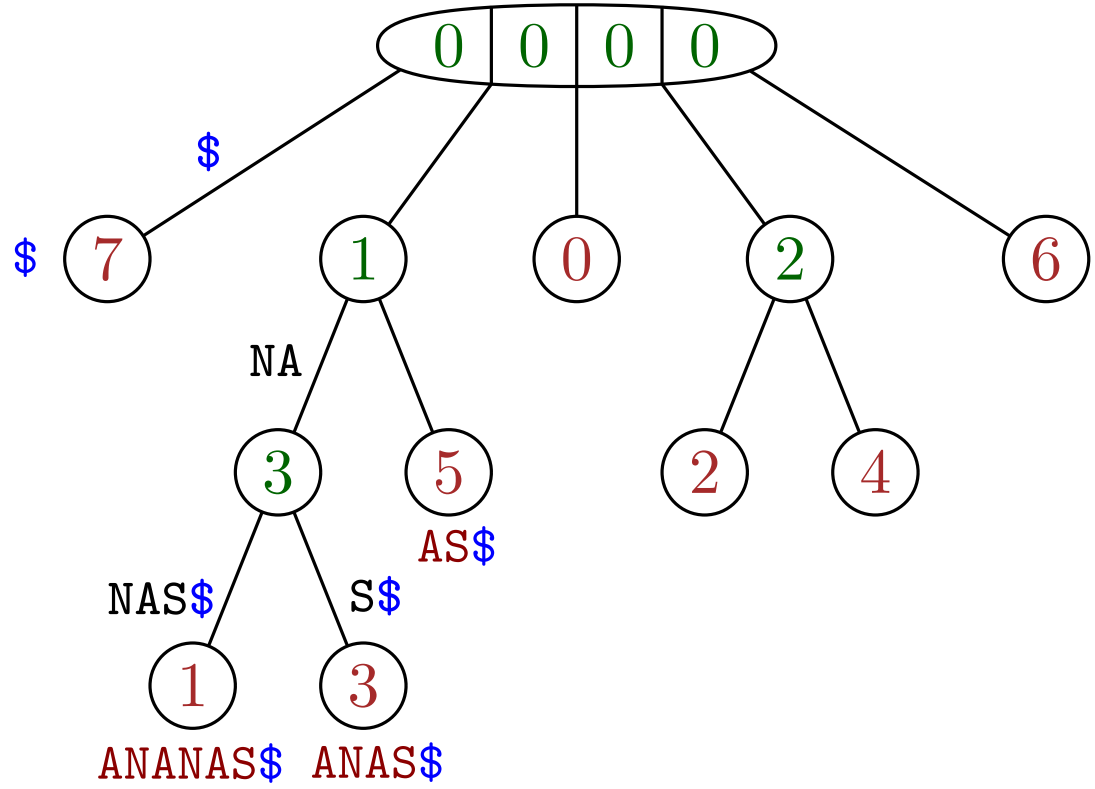

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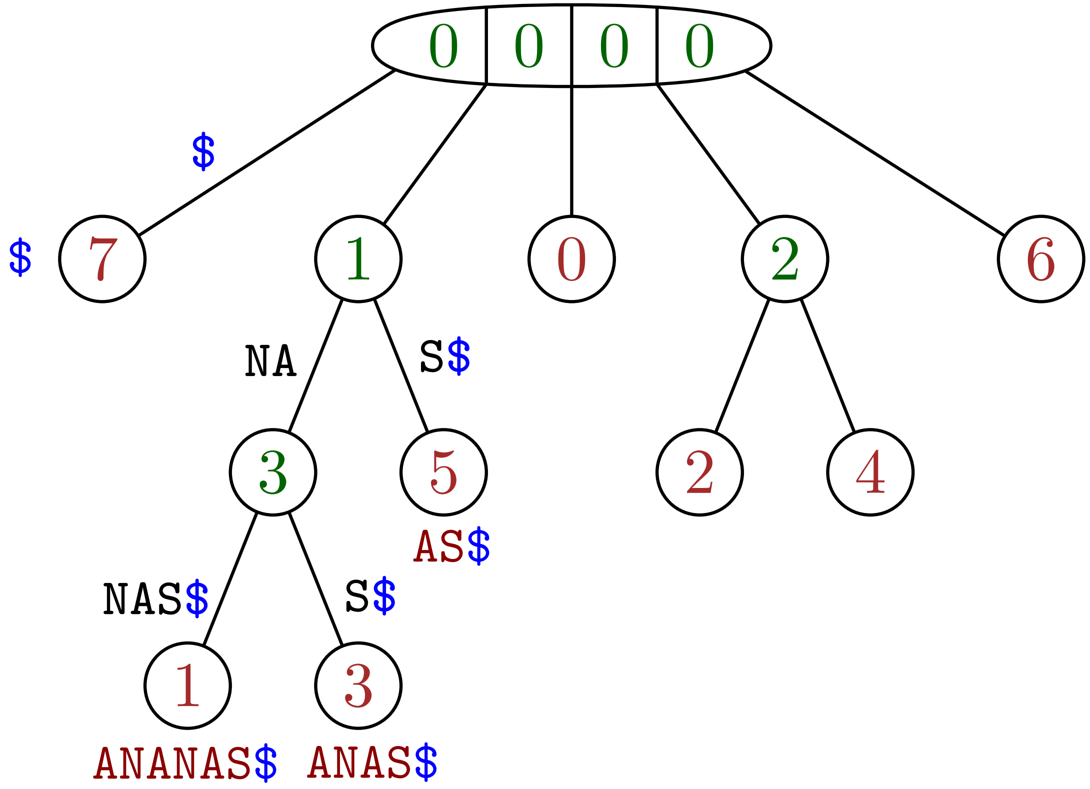

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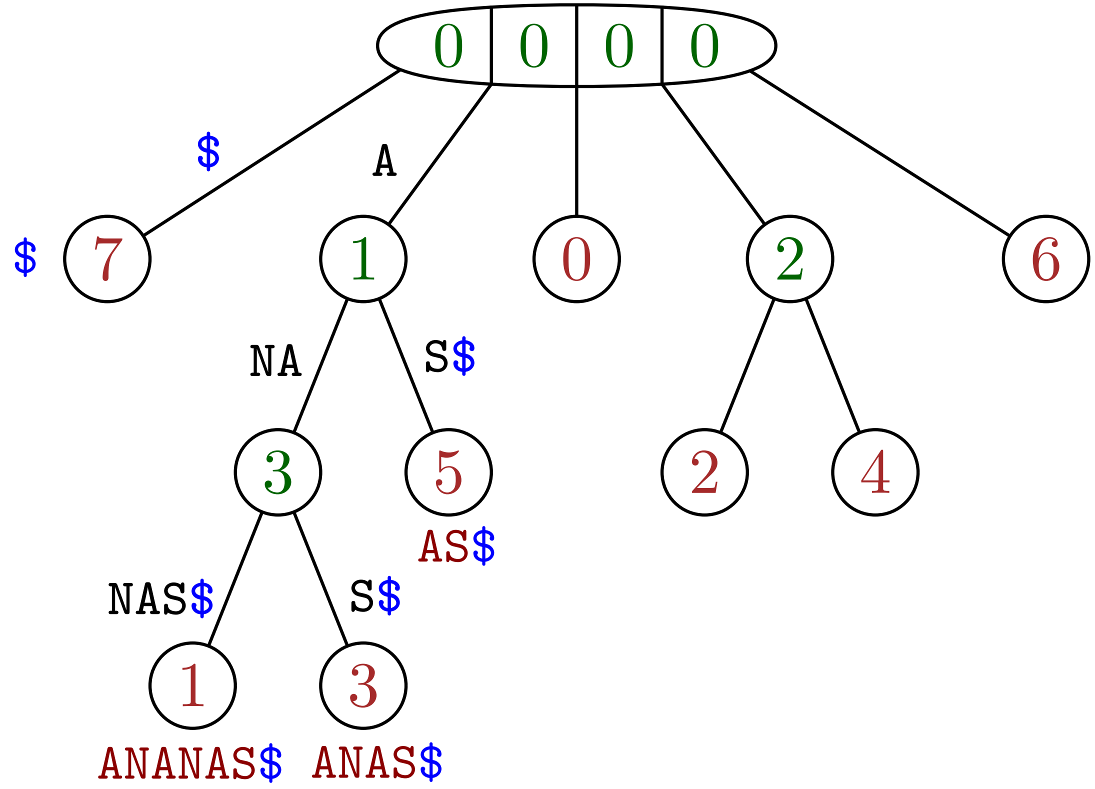

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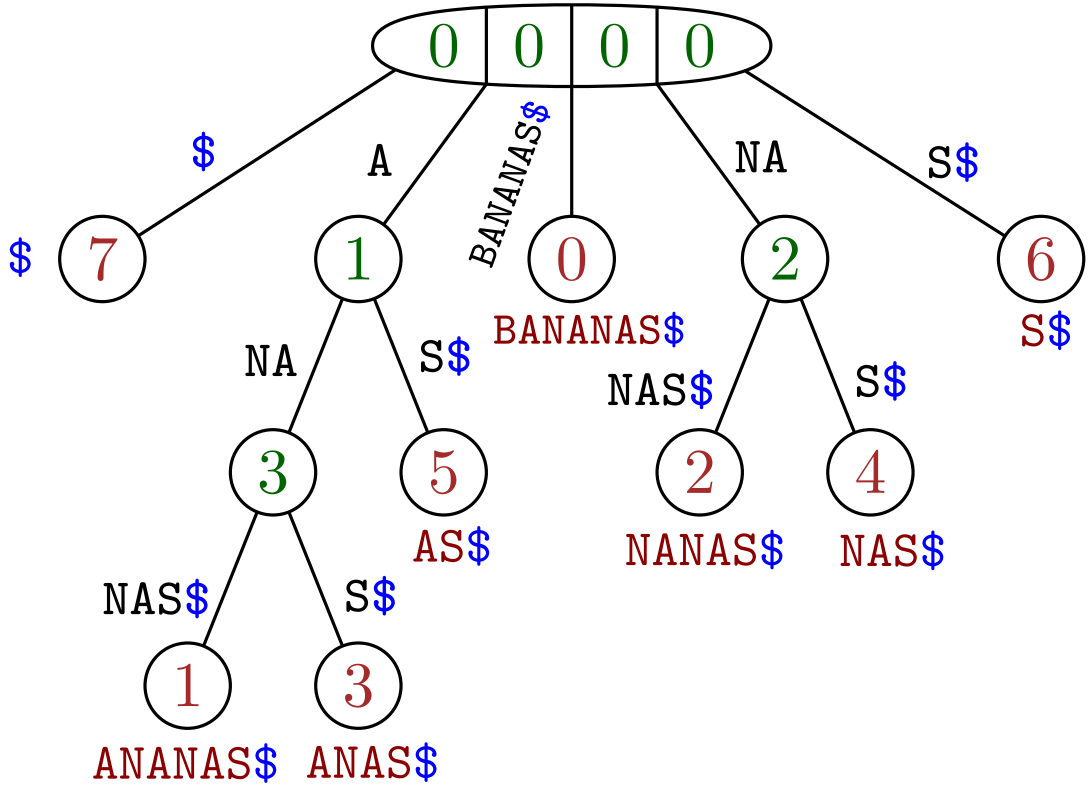

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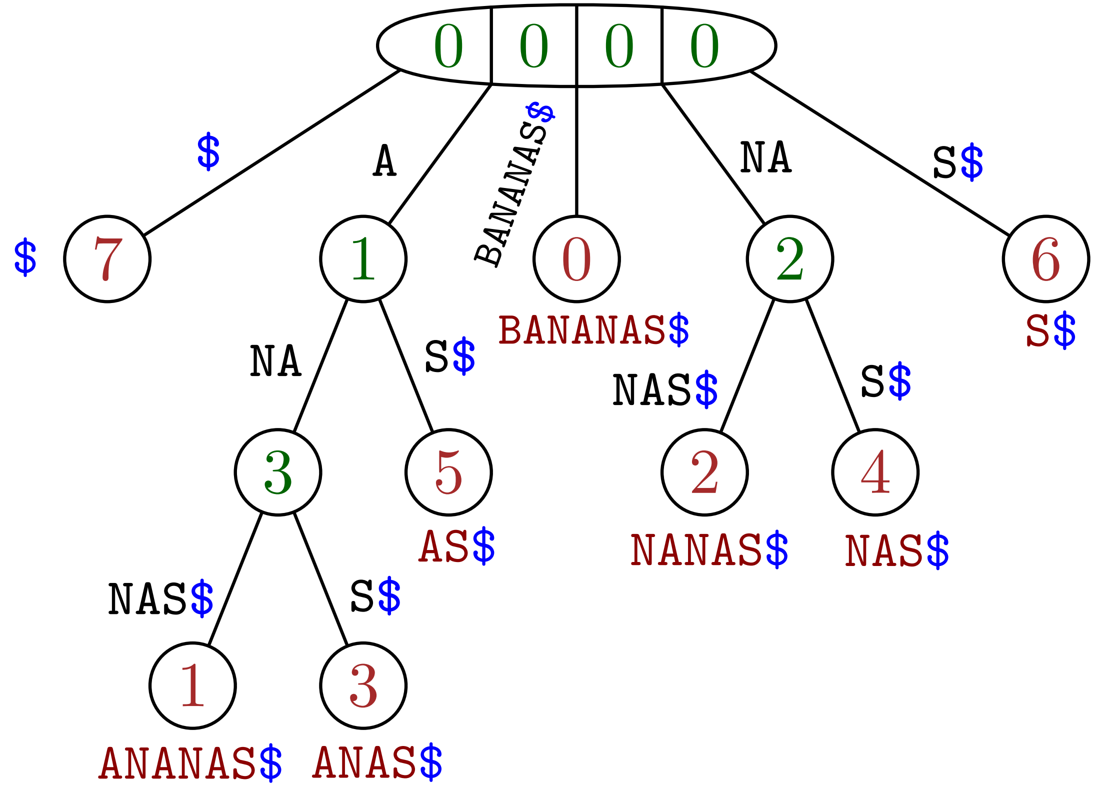

Construction time
(from Suffix + LCP Arrays):
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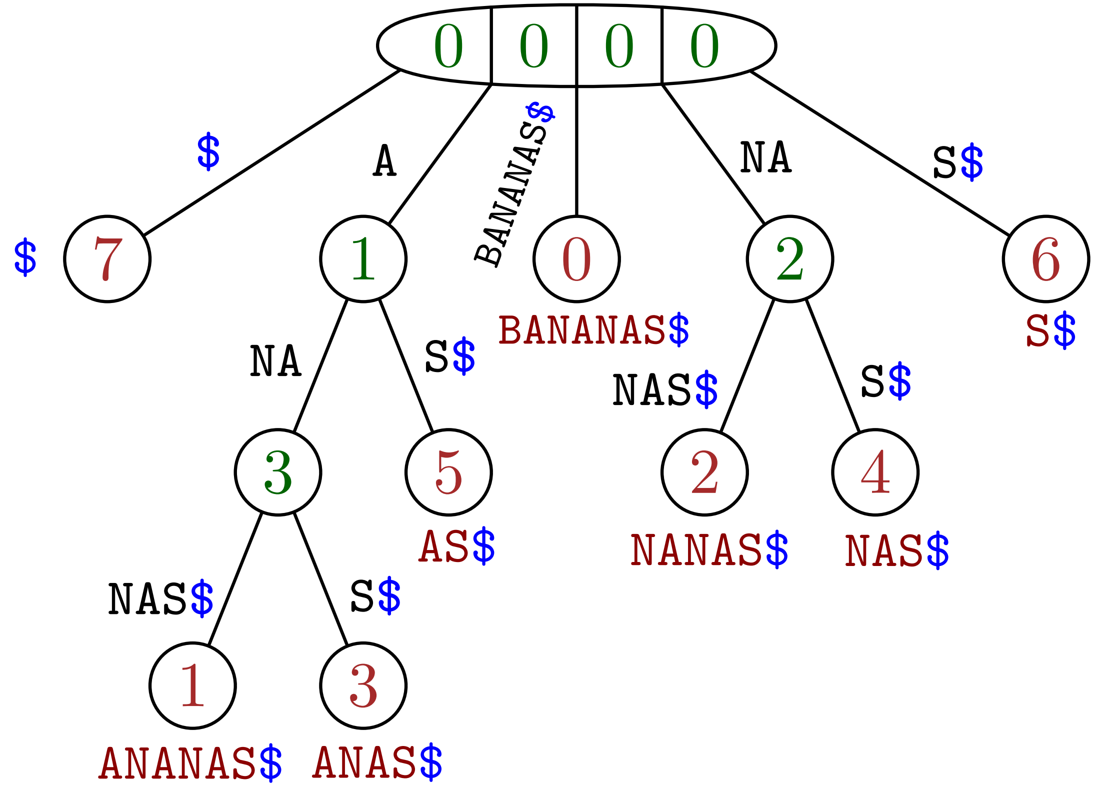

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Suffix + LCP Arrays can be built in $O(|T|)$ time

[J. Kärkkäinen, P. Sanders, ICALP'03]

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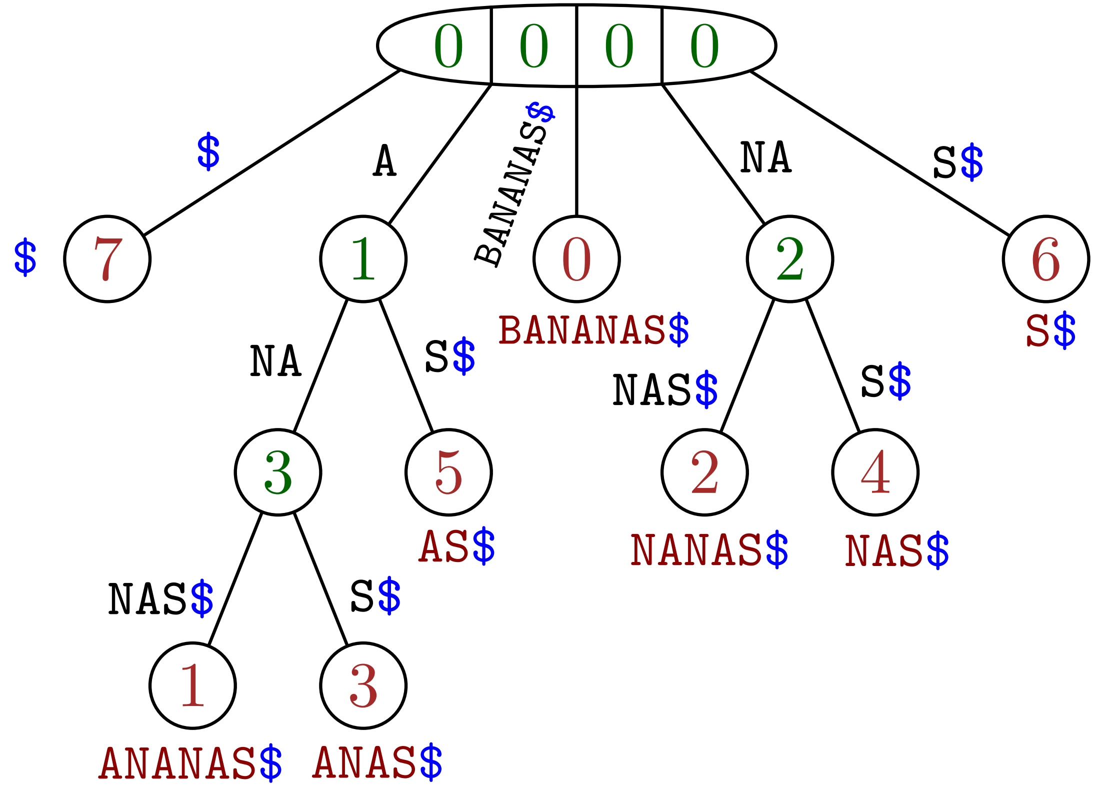

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$\Downarrow$

Suffix trees can be built in $O(|T|)$ time!