1. Compute the Hermite constant in dimension 1 and 2.

2. Let L be an integral lattice in $\mathbf{R}^n$. For a sublattice $\Lambda \subset L$, define

$$\Lambda^\perp = \{x \in L : \langle x, y \rangle = 0 \text{ for all } y \in \Lambda\}.$$

   (a) Let $\mathbf{v} \in L$ be a vector of length 1. Put $\Lambda = \mathbf{Z}\mathbf{v}$. Show that $L = \Lambda \oplus \Lambda^\perp$.
   (b) Let $\Lambda \subset L$ be a unimodular sublattice. Show that $L = \Lambda \oplus \Lambda^\perp$.

3. Let L be a unimodular integral lattice of dimension $n \leq 4$.
   (a) Prove that the ball of radius $\sqrt{2}$ centered in the origin of $\mathbf{R}^n$ contains a non-zero vector $\mathbf{v} \in L$.
   (b) Prove that L contains a vector length 1.
   (c) Prove that L is isomorphic to $\mathbf{Z}^n$.

4. Let L be an integral lattice generated by vectors of length 1. Prove it is isomorphic to $\mathbf{Z}^n$.

5. Let $\ell \subset \mathbf{C}$ be a line with slope between -1 and $+1$. Show $\int_\ell e^{-\pi z^2} dz = 1$.

6. For $m, n \geq 1$ put $\sigma_m(n) = \sum_{d|n} d^m$. For even $k \geq 2$ let E_k denote the Eisenstein series of weight k.
   (a) Show that $E_4 = 1 + 240 \sum_{n \geq 1} \sigma_3(n) q^n$ and $E_8 = 1 + 480 \sum_{n \geq 1} \sigma_7(n) q^n$.
   (b) Show that $E_4^2 = E_8$.
   (c) Show that $\sigma_7(n) - \sigma_3(n) = 120 \sum_{0<m<n} \sigma_3(m) \sigma_3(n-m)$ for all $n \geq 1$.