La matematica di

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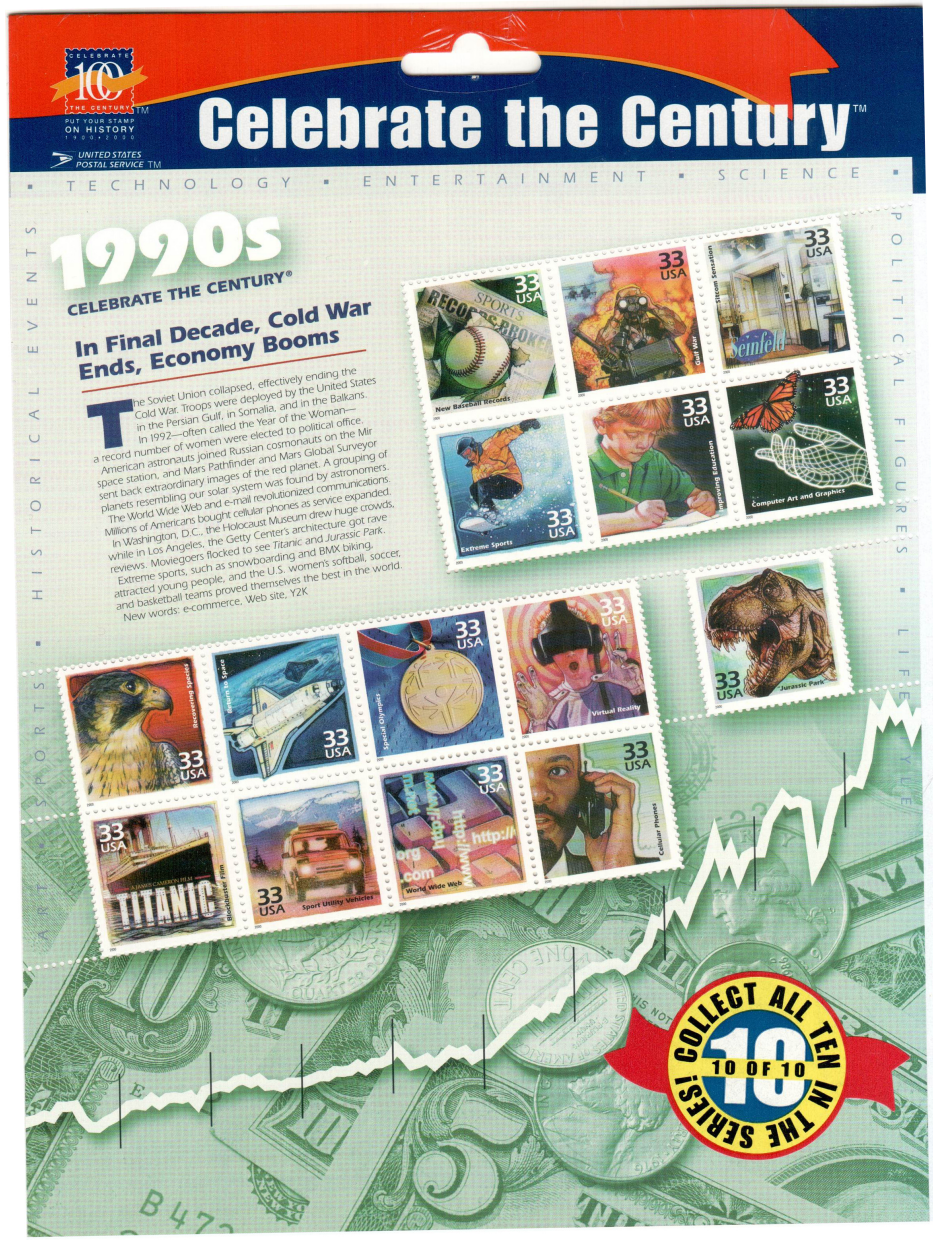
All'inizio degli anni '90....

"Il **World Wide Web** ravviva **Internet**, fino allora *solo-testo*, con immagini, suoni e filmati. Milioni di persone accedono a Internet per lavoro, studio e divertimento."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1990s
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In Final Decade, Cold War Ends, Economy Booms
The Soviet Union collapsed, effectively ending the Cold War. Troops were deployed by the United States in the Persian Gulf, in Somalia, and in the Balkans. In 1992—often called the Year of the Woman—a record number of women were elected to political office. American astronauts joined Russian cosmonauts on the Mir space station, and Mars Pathfinder and Mars Global Surveyor sent back extraordinary images of the red planet. A grouping of planets resembling our solar system was found by astronomers. The World Wide Web and e-mail revolutionized communications. Millions of Americans bought cellular phones as service expanded. In Washington, D.C., the Holocaust Museum drew huge crowds, while in Los Angeles, the Getty Center's architecture got rave reviews. Moviegoers flocked to see Titanic and Jurassic Park. Extreme sports, such as snowboarding and BMX biking, attracted young people, and the U.S. women's softball, soccer, and basketball teams proved themselves the best in the world. New words: e-commerce, Web site, Y2K
New Baseball Records
Gulf War
Seinfeld
Extreme Sports
Improving Education
Computer Art and Graphics
Reopening Space
Special Olympics
Virtual Reality
Jurassic Park
TITANIC
Blockbuster Film
Sport Utility Vehicles
World Wide Web
Cellular Phones
33 USA
COLLECT ALL TEN IN THE SERIES!
10 OF 10

Marzo-aprile 1994: il **World Wide Web Worm**

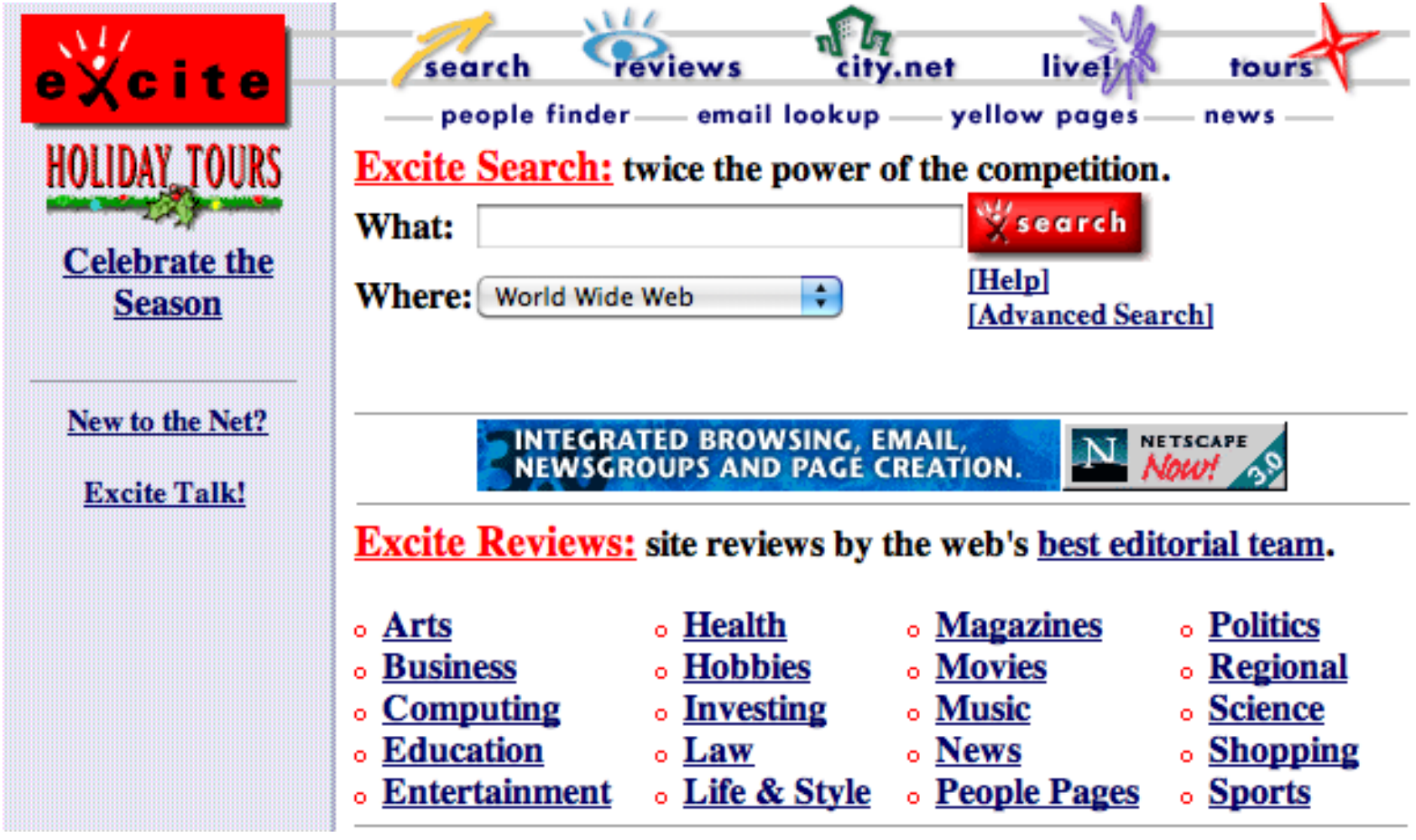
un indice di 150.000 pagine

circa 1500 ricerche al giorno

Novembre 1997: i migliori motori di ricerca

un indice fra 2.000.000 e 100.000.000 pagine

circa 20.000.000 ricerche al giorno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SEARCH
for more options use SuperSearch
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ONSALE Auctions
1-800-FLOWERS
GetSmart Mortgage Finder
Kodak
Music Boulevard
Microsoft® CarPoint™
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Oggi:

il World Wide Web contiene milioni di milioni di documenti.

Senza un buon criterio per ordinare i risultati di una ricerca

l'informazione cercata è sommersa

in un mare di risultati irrilevanti.

Il 27 settembre 1998, appare Google

Gli inventori: Larry Page & Sergey Brin

Presto Google diventa il motore di ricerca più usato.

Google presenta i risultati più rilevanti in testa.

Il segreto di Google ?

l'algoritmo PageRank,

che calcola il grado di rilevanza di ogni pagina del web.

Due miti da sfatare

Le pagine importanti sono le più visitate?
FALSO!

Sono quelle che hanno più link da altre pagine?
FALSO!

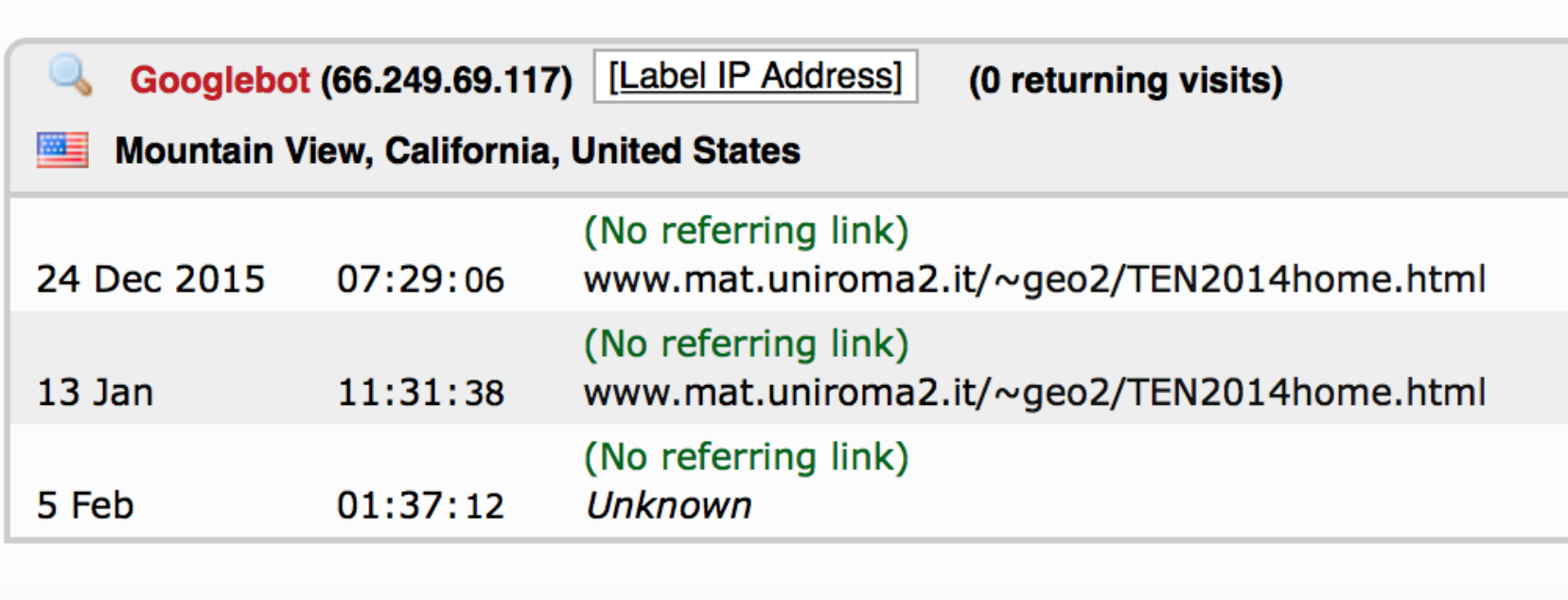
Da cosa dipende la rilevanza di una pagina?

Dipende dalla rilevanza

delle pagine che hanno un link verso di essa.

Google scansiona e indicizza le pagine del Web

Google conta i link in entrata e in uscita da ogni pagina



Googlebot (66.249.69.117) [Label IP Address] **(0 returning visits)**

Mountain View, California, United States

24 Dec 2015	07:29:06	(No referring link) www.mat.uniroma2.it/~geo2/TEN2014home.html
13 Jan	11:31:38	(No referring link) www.mat.uniroma2.it/~geo2/TEN2014home.html
5 Feb	01:37:12	(No referring link) *Unknown*

$P_1, \quad \ldots, P_m$ le pagine del web

L'algoritmo PageRank assegna ad ogni pagina un numero $x_i \geq 0$

$$x_i = \text{“grado di rilevanza” della pagina } P_i$$

$$x_1 + x_2 + \ldots + x_m = 1$$

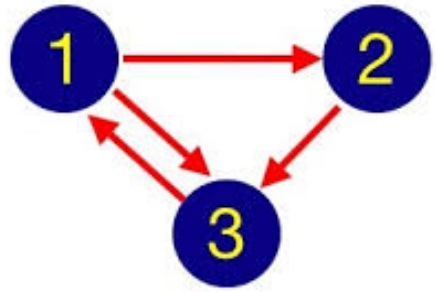
x_i grande $\quad \Leftrightarrow \quad$ pagina P_i importante.

La rilevanza di una pagina è data dalla *somma pesata delle rilevanze delle pagine che hanno un link verso di essa*.....
N.B.: i link a se stessi non contano...!!!

$$x_i = \sum_{j \neq i} \frac{1}{n_j} x_j,$$

dove j indica le pagine P_j che puntano verso P_i,

ed n_j è il numero di link da P_j verso altre pagine.



$$x_1 = 0 \cdot x_1 + 0 \cdot x_2 + 1 \cdot x_3$$
$$x_2 = \frac{1}{2} \cdot x_1 + 0 \cdot x_2 + 0 \cdot x_3$$
$$x_3 = \frac{1}{2} \cdot x_1 + 1 \cdot x_2 + 0 \cdot x_3$$

Soluzioni: $x_1 = x_3, \quad x_1 = \frac{1}{2}x_2$
Con la condizione $x_1 + x_2 + x_3 = 1$,
soluzione $x_1 = x_3 = 0.4, \quad x_2 = 0.2$.

I numeri

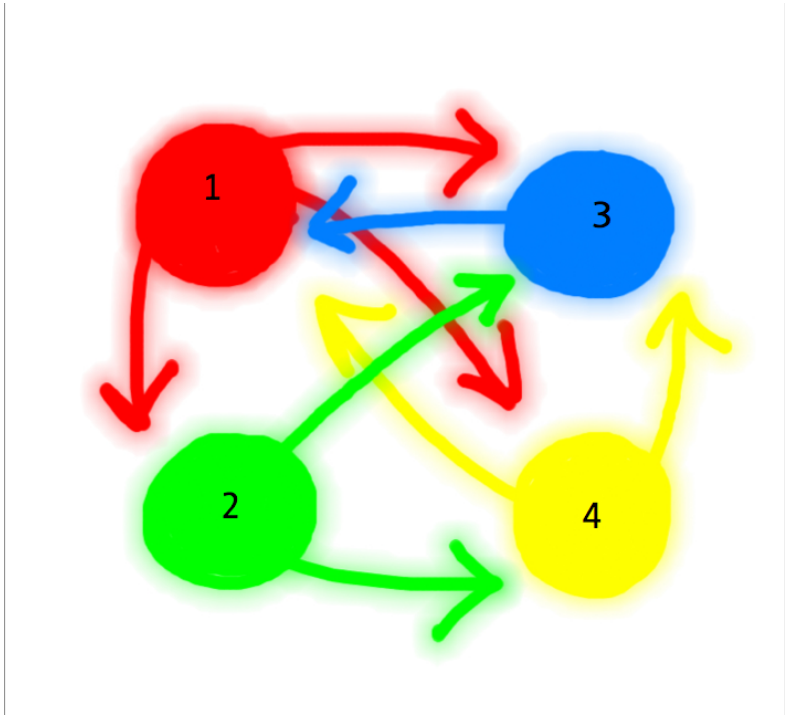
$$x_1, x_2, \ldots, x_m, \qquad x_1 + \ldots + x_m = 1$$

sono soluzioni di un sistema lineare:

$$\begin{aligned}
x_1 &= \quad \tfrac{1}{n_2}x_2 + \tfrac{1}{n_3}x_3 + \ldots \\
x_2 &= \tfrac{1}{n_1}x_1 + \quad + \tfrac{1}{n_3}x_3 + \ldots \\
\vdots &= \quad \vdots \quad \vdots \quad \vdots \quad \ldots \\
x_m &= \tfrac{1}{n_1}x_1 + \tfrac{1}{n_2}x_2 + \tfrac{1}{n_3}x_3 + \ldots
\end{aligned}$$

LA TEORIA: *per i sistemi di questo tipo*
$x_1, x_2, \ldots, x_m$ sono univocamente determinati.

Proviamo l'algoritmo PageRank su un altro mini-esempio:

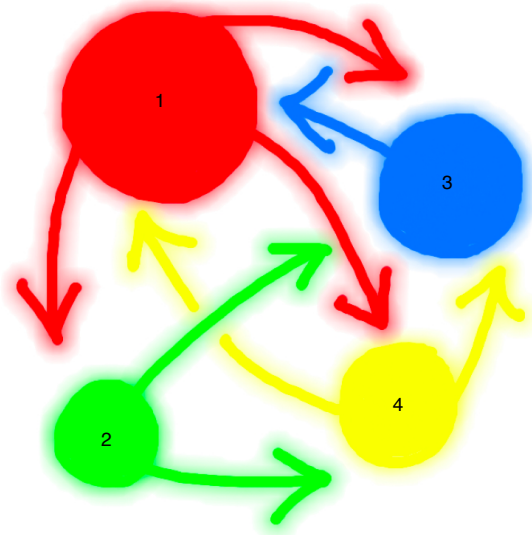


Ci sono 4 pagine: P_1, P_2, P_3, P_4

$n_1 = 3$, $n_2 = 2$, $n_3 = 1$, $n_4 = 2$.

$$
\begin{aligned}
x_1 &= & & & x_3 + \tfrac{1}{2}x_4 \\
x_2 &= \tfrac{1}{3}x_1 & & & \\
x_3 &= \tfrac{1}{3}x_1 + \tfrac{1}{2}x_2 + & & + \tfrac{1}{2}x_4 & \\
x_4 &= \tfrac{1}{3}x_1 + \tfrac{1}{2}x_2 & & &
\end{aligned}
$$

Risolvendo il sistema



$$x_1 = 12/31 \sim 0.387$$
$$x_2 = 4/31 \sim 0.129$$
$$x_3 = 9/31 \sim 0.290$$
$$x_4 = 6/31 \sim 0.194$$

Nel linguaggio dell'ALGEBRA LINEARE

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

è un AUTOVETTORE della matrice

$$\begin{pmatrix} 0 & 0 & 1 & 1/2 \\ 1/3 & 0 & 0 & 0 \\ 1/3 & 1/2 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{pmatrix}$$

THE $25,000,000,000* EIGENVECTOR
THE LINEAR ALGEBRA BEHIND GOOGLE

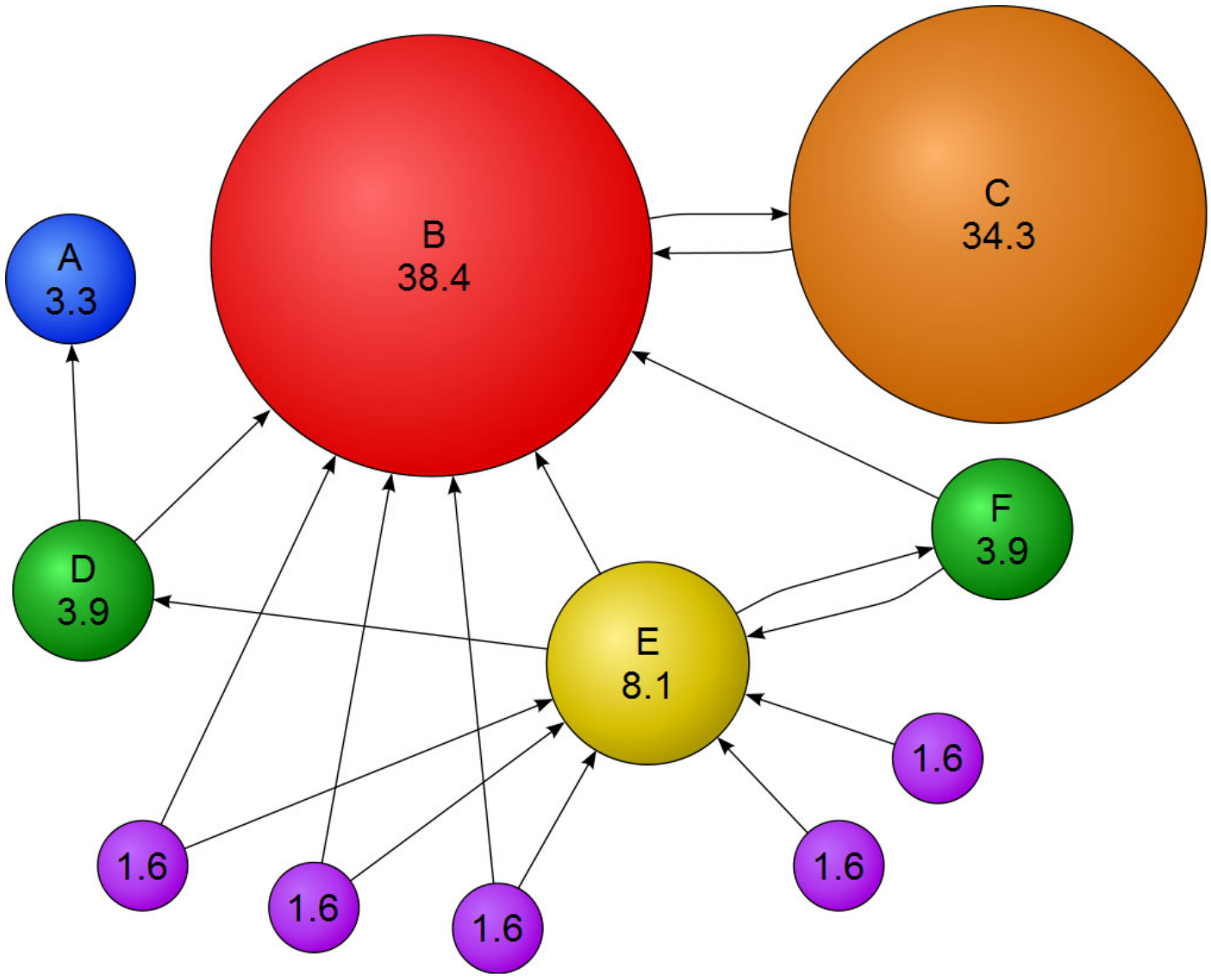
KURT BRYAN† AND TANYA LEISE‡

Abstract. Google's success derives in large part from its PageRank algorithm, which ranks the importance of webpages according to an eigenvector of a weighted link matrix. Analysis of the PageRank formula provides a wonderful applied topic for a linear algebra course. Instructors may assign this article as a project to more advanced students, or spend one or two lectures presenting the material with assigned homework from the exercises. This material also complements the discussion of Markov chains in matrix algebra. Maple and Mathematica files supporting this material can be found at www.rose-hulman.edu/∼bryan.

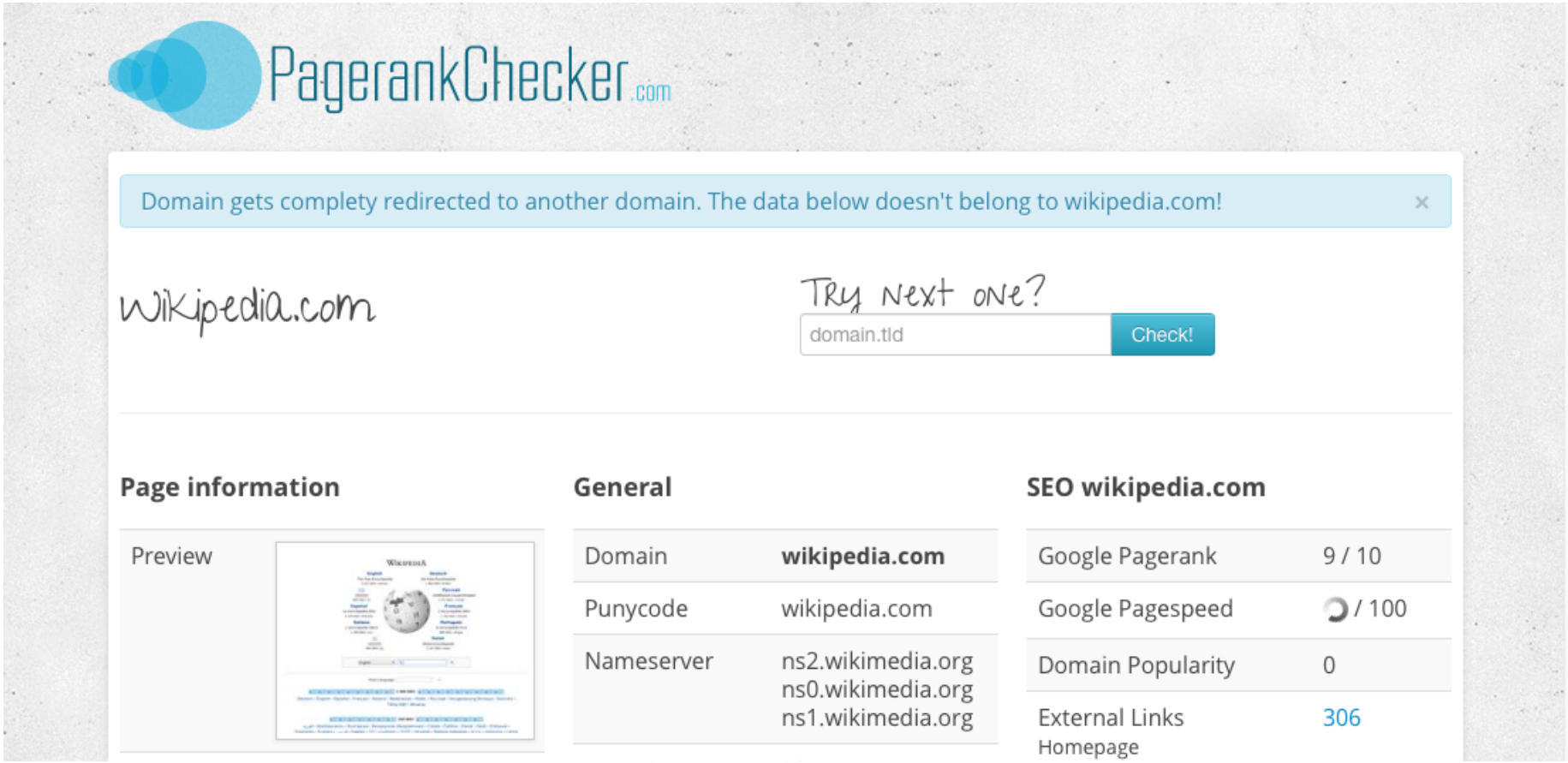
Key words. linear algebra, PageRank, eigenvector, stochastic matrix

AMS subject classifications. 15-01, 15A18, 15A51

Il secondo classificato ha solo un link in arrivo...ma molto importante...



Sul web si trovano diversi programmi che dicono di saper calcolare il PageRank di qualunque sito:



whitehouse.gov

Try next one?

domain.tld Check!

Page information

Preview	

General

Domain	**whitehouse.gov**
Punycode	whitehouse.gov
Nameserver	usw1.akam.net use6.akam.net ns1-176.akam.net usw5.akam.net

SEO whitehouse.gov

Google Pagerank	10 / 10
Google Pagespeed	/ 100
Domain Popularity	0
External Links Homepage	8

mat.uniroma2.it

Try next one?

domain.tld Check!

Page information

Preview	

General

Domain	**mat.uniroma2.it**
Punycode	mat.uniroma2.it
Nameserver	dns.uniroma2.it dns1.uniroma2.it
HTTP Status	200

SEO mat.uniroma2.it

Google Pagerank	5 / 10
Google Pagespeed	/ 100
Domain Popularity	0
External Links Homepage	0

27 settembre 2015: Google ha compiuto 17 anni.