1. Calcolare i seguenti residui:

$$Res_f(2i),\quad f(z)=\frac{z^2}{(z-2)(z+3)},\qquad Res_f(i+1),\quad f(z)=\frac{e^z}{(z-i-1)^3}.$$

$$Res_f(\pi/2),\quad f(z)=\tan z,\qquad Res_f(0),\quad f(z)=\frac{e^z}{\sin z},\qquad Res_f(0),\quad f(z)=\frac{e^z}{1-e^z}$$

$$Res_f(0),\quad f(z)=\frac{e^z}{z^3},\qquad Res_f(0),\quad f(z)=\frac{e^{1/z}}{z^3}.$$

2. Sia $f(z)=\frac{z+1}{z^2(z+3)^2}$. Calcolare $Res_f(0)+Res_f(-3)$. Spiegare il risultato.

3. Calcolare

$$\int_\gamma \frac{e^z}{z^3}dz \qquad \gamma=\{e^{i\theta},\quad \theta\in[0,6\pi]\}\quad \text{oppure}\quad \gamma=\{e^{i\theta},\quad \theta\in[0,2\pi]\}.$$

Confrontare il risultato con la formula integrale di Cauchy.

4. Calcolare i seguenti integrali col metodo dei residui:

$$\int_\gamma \frac{z}{(z+1)(z+2i)}dz \qquad \gamma=\{5e^{i\theta},\quad \theta\in[0,2\pi]\},$$

$$\int_\gamma \frac{e^z}{\sin z}dz \qquad \gamma=\{e^{i\theta},\quad \theta\in[0,6\pi]\},$$

$$\int_\gamma \frac{e^z}{z(z+2)(z+1)}dz \qquad \gamma=\text{triangolo di vertici } -3, 1\pm i, \text{ percorso in senso orario.}$$

5. Calcolare i seguenti integrali reali col metodo dei residui:

$$\int_0^{2\pi}\frac{\sin 2x}{5+3\cos x}dx,\qquad \int_0^{+\infty}\frac{\log(1+x^2)}{1+x^2}dx,$$

$$\int_0^{+\infty}\frac{x^4}{1+x^{10}}dx,\qquad \int_0^{+\infty}\frac{1}{1+x^3}dx,$$

$$\int_0^{+\infty}\frac{x\sin x}{1+x^2}dx,\qquad \int_{-\infty}^{+\infty}\frac{\sin^2 x}{x^2}dx,\qquad \int_0^{+\infty}\frac{x^{1/4}}{1+x^3}dx,$$

6. Far vedere che

$$\sum_{n\geq 0}\frac{1}{n^2+a^2}=\frac{\pi}{2a}\coth(\pi a)+\frac{1}{2a^2},\quad a>0.$$

7. Sommare le serie

$$\sum_{n\geq 0}\frac{1}{n^4+1},\qquad \sum_{n\in\mathbf{Z}}\frac{1}{n^3+3},\qquad \sum_{n\geq 0}\frac{n^2+1}{n^4+4}.$$