

MULTIPARAMETER QUANTUM GENERAL LINEAR SUPERGROUP

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ABSTRACT. We introduce uniparametric and multiparametric quantisations of the general linear supergroup, in the form of “quantised function algebras”, both in a *formal* setting — yielding “quantum formal series Hopf superalgebras”, à la Drinfeld — and in a *polynomial* one — closer to Manin’s point of view.

In the uniparametric setting, we start from quantised universal enveloping superalgebras over $\mathfrak{gl}_n$ (endowed with a super-structure), as in [Ya1] and [Zha]: through a direct approach, we construct their linear dual, thus finding the quantum formal series Hopf superalgebras mentioned above, which are described in detail via an explicit presentation. Starting from the latter, then, we perform a deformation by a well-chosen 2-cocycle, thus getting a multiparametric quantisation, described again by an explicit presentation: this is, in turn, the dual to the multiparametric quantised universal enveloping algebra over $\mathfrak{gl}_n$ from [GGP].

We also provide some “polynomial versions” of these quantisations, both for the uniparametric and the multiparametric case. In particular, we compare the latter to Manin’s quantum function algebras from [Ma].

Finally, both for the uniparametric and the multiparametric setting, we provide suitable PBW-like theorems, in “formal” and in “polynomial” versions alike.

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